Magnetism and Electromagnetic Induction

CHAPTER 4

4 Electromagnetic Induction

INTRODUCTION

From previous chapters, we know that current produces magnetic field. Is reverse possible i.e., can magnetic field produce electric current? The answer is "yes". It is found that currents were induced in closed coils when subjected to changing magnetic fields.

The phenomenon in which electric current is generated by changing magnetic fields is known as electromagnetic induction. The current so produced is known as **induced current**.

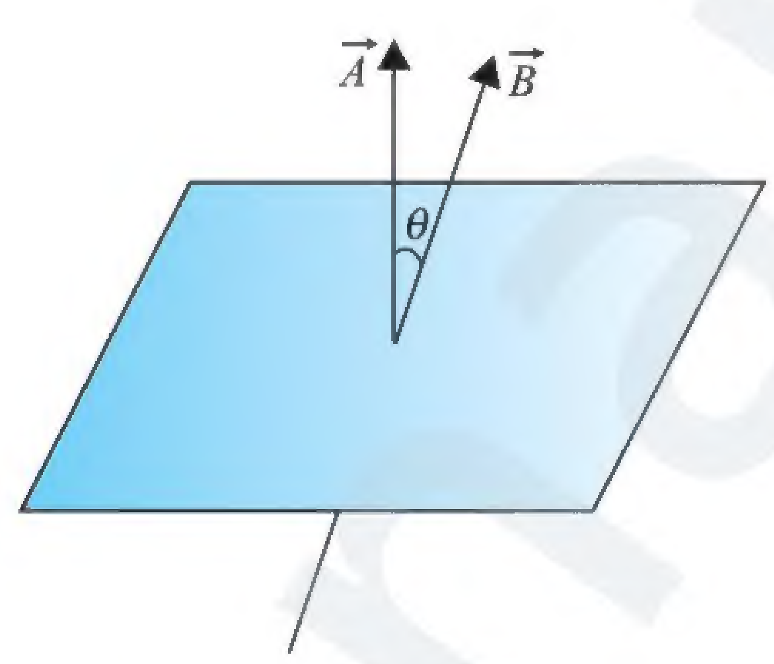
If current is produced in the circuit, this must be due to some emf produced in the circuit. This emf produced as a result of change in magnetic field is known as **induced emf**.

The phenomenon of electromagnetic induction (EMI) is the basis of the working of power generators, dynamos, transformers, etc.

MAGNETIC FLUX

Magnetic flux through any surface held in a magnetic field is measured as the total number of magnetic field lines crossing the surface. It is a scalar quantity.

Consider a plane area A placed in a magnetic field B as shown in figure. Area vector $\vec{A}$ makes an angle θ with the direction of magnetic field.



Now the magnetic flux passing through the area is defined as

$$\phi = \vec{B}\cdot\vec{A} \qquad \text{[for uniform } \vec{B}\text{]}$$

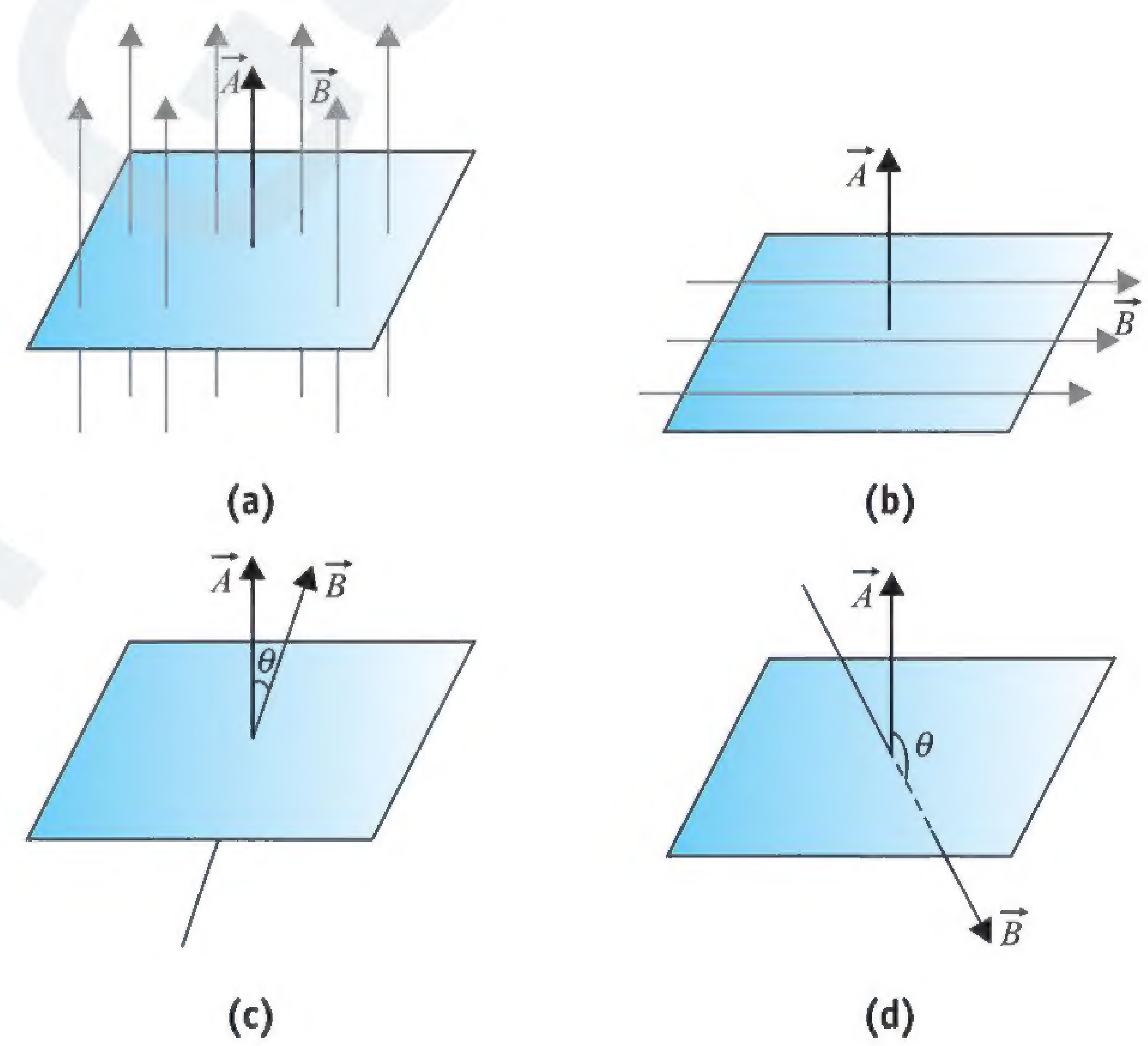
or $\quad \phi = BA\cos\theta = (B\cos\theta)A = B_{\perp}A$

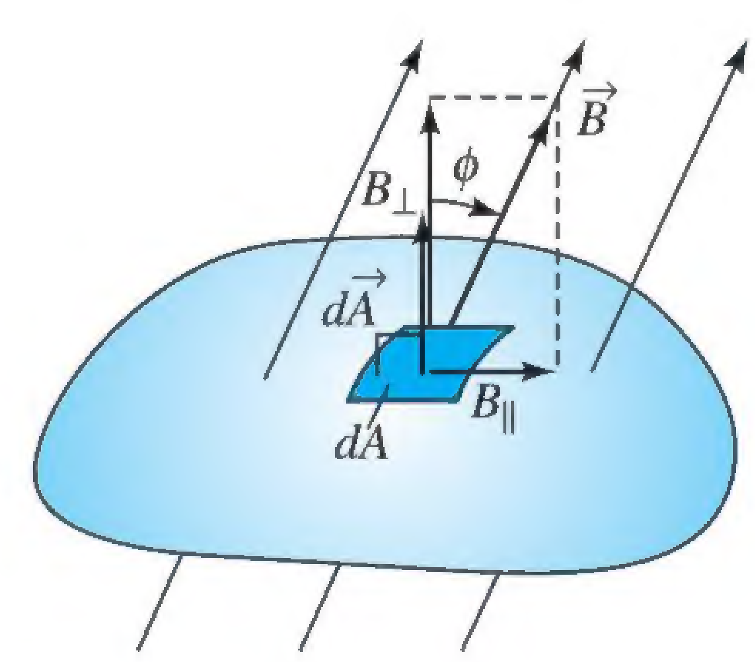
where $B_{\perp} = B\cos\theta$ is the component of the magnetic field B perpendicular to the face of the area. Direction of area vector is normal to the face of the area.

Special cases:

1. if $\theta = 0°$, then $\phi = BA\cos 0° \Rightarrow \phi = BA$ (maximum flux)
2. if $\theta = 90°$, then $\phi = BA\cos 90° \Rightarrow \phi = 0$
3. if $\theta < 90°$, then $\cos\theta > 0 \Rightarrow \phi > 0$ (positive flux)
4. if $\theta > 90°$, then $\cos\theta < 0 \Rightarrow \phi < 0$ (negative flux)

If the surface is not plane, we can divide any surface into elements of area dA (as shown in figure). For each element we determine $B_{\perp}$, the component of normal to the surface at the position of that element, as shown. From figure, $B_{\perp} = B\cos\phi$, where ϕ is the angle between the direction of B and a line perpendicular to the surface. In general, this component varies from point to point on the surface.





We define the magnetic flux $d\Phi_B$ through the area element dA as

$$d\phi = B_{\perp}\, dA$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\phi = \int B_{\perp} dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A}$$

(magnetic flux through a surface)

The unit of magnetic flux is tesla-meter^2 which is called weber (Wb) in honour of Wilhelm Weber. 1 Wb = 1 T m^2. Clearly, B can be measured in Wb m^{-2}. 1 Wb m^{-2} = 1 T. Sometimes it is referred to as flux density.

Flux Linkage

Flux linkage is the linking of the magnetic field with the conductors of a coil when the magnetic field passes through the loops of the coil, expressed as a value.The flux linkage of a coil is simply an alternative term for total flux. If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$ If the coil has N turns, the total flux linkage $\phi = NBA\cos\theta$

Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2] \quad \because B = \frac{F}{IL\sin\theta}$$

$$[\because F = BIL\sin\theta]$$

$$\therefore \quad [\phi] = \left[\frac{MLT^{-2}}{AL}\right][L^2] = [ML^2T^{-2}A^{-1}]$$

SI unit of magnetic flux:

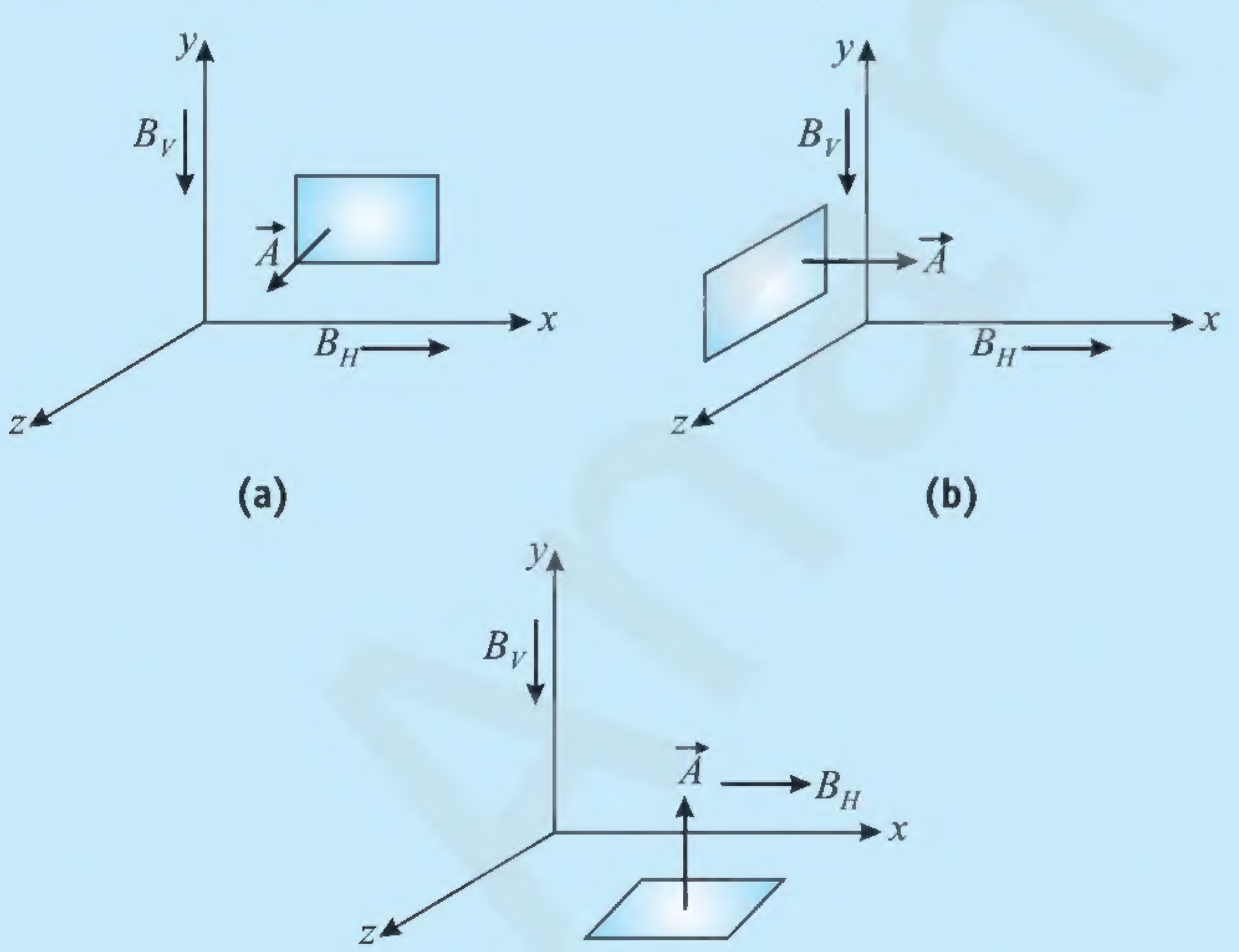
$\because$ $[ML^2T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{weber}(\text{Wb})$$

or $\quad \text{T-m}^2 (\text{as tasla} = \text{Wb/m}^2)\left[\text{ampere} = \frac{\text{coulomb}}{\text{second}}\right]$

Illustration 4.1

At a given place, horizontal and vertical components of earth's magnetic field B_H and B_V are along positive x and negative y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area A, if the area A is in **(a)** x-y plane **(b)** y-z plane and **(c)** z-x plane?



(a) (b) (c)

Sol. The magnetic field in given place, $\vec{B} = B_H(\hat{i}) - B_V(\hat{j})$ which is constant.

The flux associated with a closed surface in a constant magnetic field

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

(a) In case (a), the area is in x-y plane $\vec{A} = A\hat{k}$,

$$\phi_{xy} = (B_H\hat{i} - B_V\hat{j}).(A\hat{k}) = 0$$

(b) In case (b), the area is in y-z plane $\vec{A} = A\hat{i}$,

$$\phi_{yz} = (B_H\hat{i} - B_V\hat{j}).(A\hat{i}) = B_H A$$

(c) In case (b), the area is in z-x plane $\vec{A} = A\hat{j}$,

$$\phi_{zx} = (B_H\hat{i} - B_V\hat{j}).(A\hat{j}) = -B_V A$$

Illustration 4.2

A long solenoid with radius 2.0 cm carries a current of 5.0 A. The solenoid is 1.0 m long and is composed of 1000 turns of wire. Assuming ideal solenoid model, calculate the flux linked with a circular surface if

(a) it has a radius of 1 cm and is perpendicular to the axis of the solenoid:
(i) inside, **(ii)** outside.

(b) it has a radius of 3 cm and is perpendicular to the axis of the solenoid with its center lying on the axis of the solenoid.

(c) it has a radius greater than 2 cm and axis of the solenoid subtends an angle of 60° with the normal to the area (the center of the circular surface being on the axis of the solenoid).

(d) the plane of the circular area is parallel to the axis of the solenoid.

Sol. For an ideal solenoid magnetic field outside the solenoid is zero and inside the solenoid is constant.

Here $B_{\text{out}} = 0$ and $B_{\text{in}} = \mu_0 ni$,

n = number of turns per unit length $= \dfrac{N}{l} = \dfrac{1000}{1.0} = 1000$ turn/m

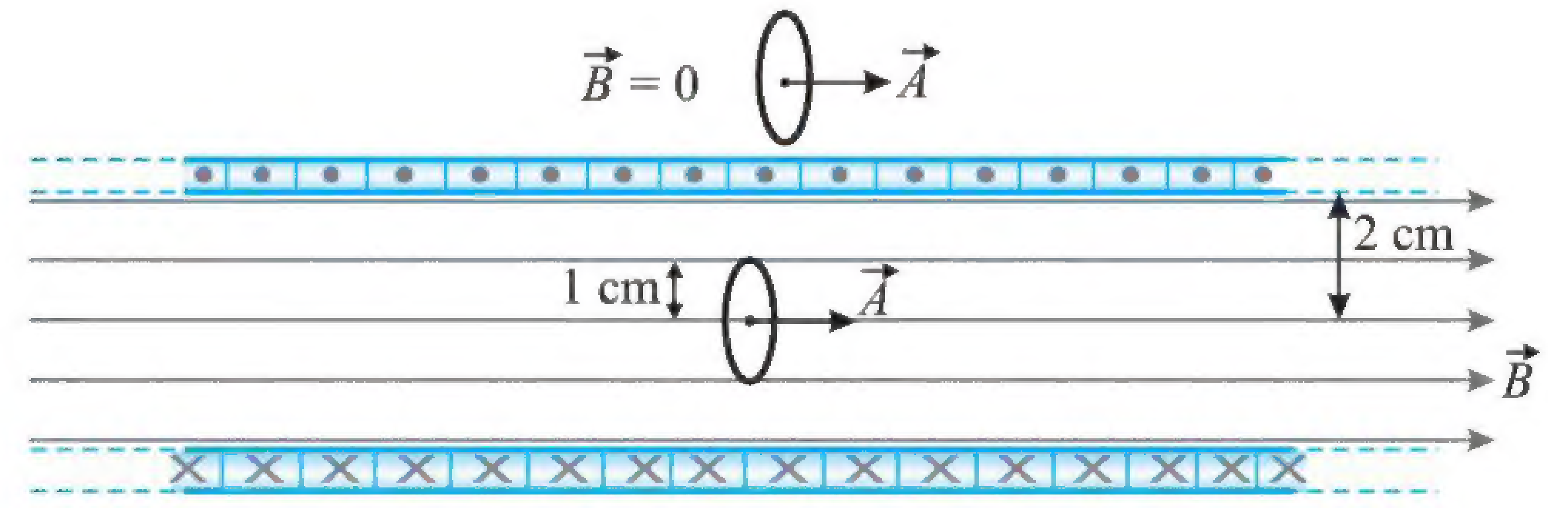
Hence $B_{\text{in}} = \mu_0 ni = (4\pi \times 10^{-7}) \times 1000 \times 5 = 2\pi \times 10^{-3}$ T

The magnetic flux in case of constant field is given by

$$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta \quad \ldots\text{(i)}$$

θ is the angle between $\vec{B}$ and $\vec{A}$.

(a)



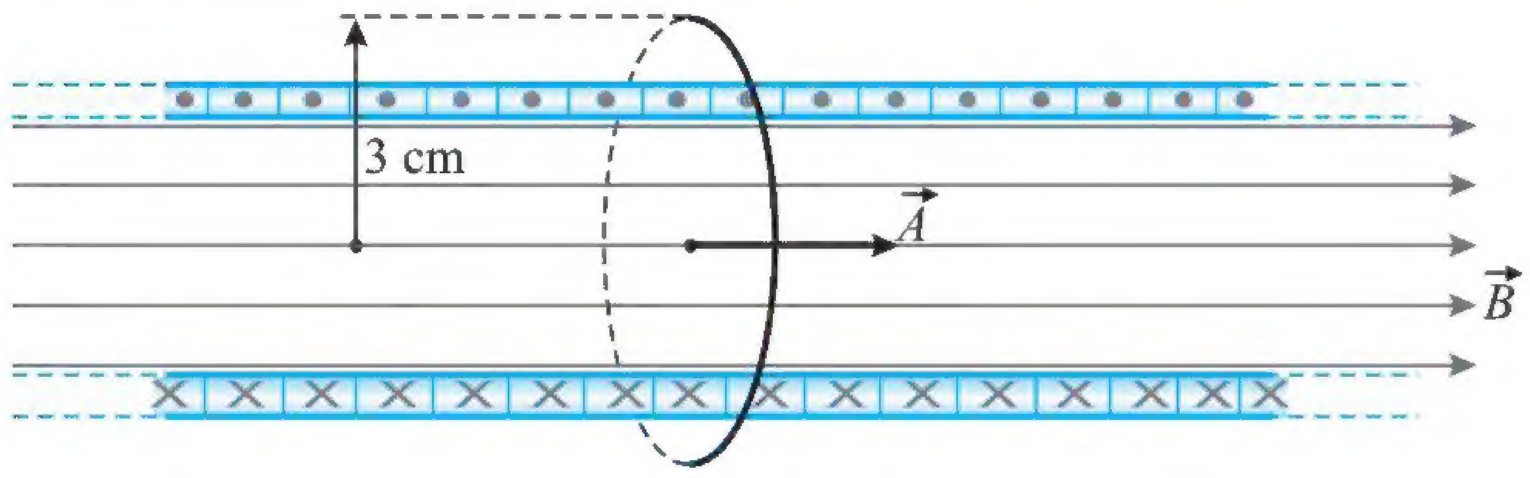
(i) In this case angle between $\vec{A}$ and $\vec{B}$ is zero.

$$\phi = BA\cos\theta = (2\pi \times 10^{-3}) \times \pi \times \left(\frac{1}{100}\right)^2 \times \cos 0°$$

$$= 2\pi^2 \times 10^{-7} \text{ Wb}$$

(ii) The magnetic field outside the solenoid is zero, hence the flux linked with a circular ring should be zero.

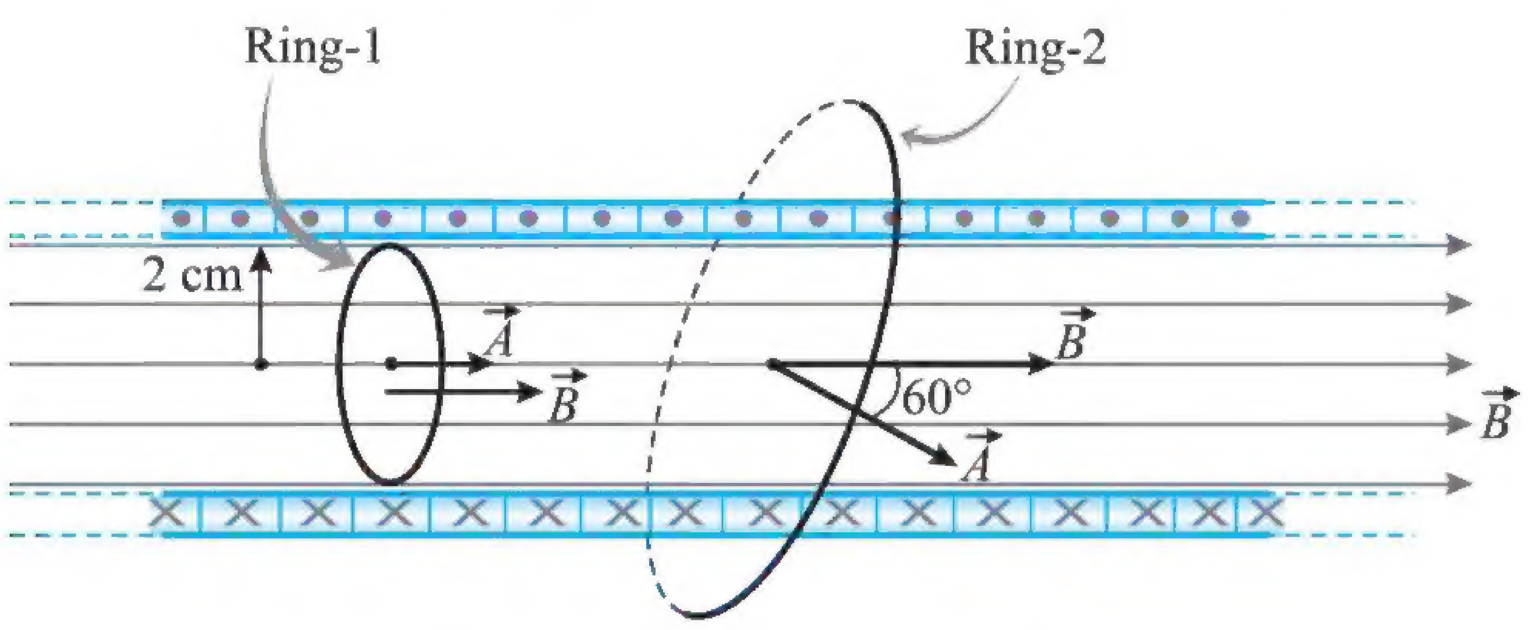
(b) No flux should be linked with the area outside the solenoid. The flux associated should be with the area common to solenoid and ring.



Hence $\phi = BA\cos\theta = (2\pi \times 10^{-3}) \times \pi \times \left(\frac{2}{100}\right)^2 \times \cos 0°$

$\phi = 8\pi^2 \times 10^{-7}$ Wb

(c) In this case the angle between $\vec{B}$ and $\vec{A}$ is 60°.



The flux is proportional to number of magnetic field lines. The number of magnetic field lines crossing in ring-1 and ring-2 are same. Hence the flux passing through ring-2 is equal to the flux associated with ring-1.

Hence $\phi = 2\pi \times 10^{-3} \times \pi \times \left(\frac{2}{100}\right)^2 \times \cos 0° = 8\pi^2 \times 10^{-7}$ Wb

(d) In this case the angle between $\vec{A}$ and $\vec{B}$ is 90°. Hence the flux associated with the ring should be zero.

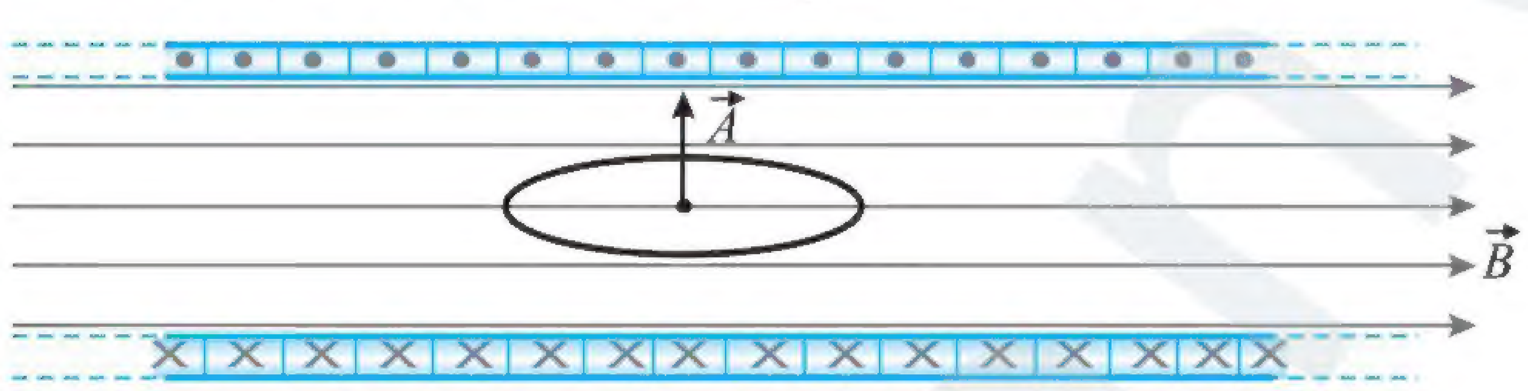
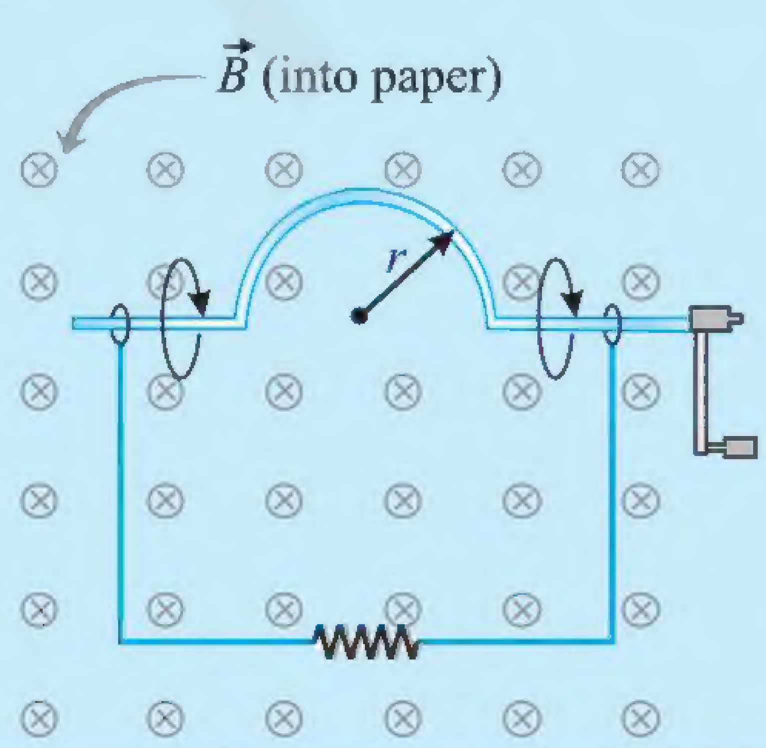


Illustration 4.3

Aclosed loop of wire having shape as shown in the figure, the top part of the wire is bent into a semicircle of radius r. The normal to the plane of the loop is parallel to a constant magnetic field ($\theta = 0°$) of magnitude B. What is the change in the magnetic flux ($\Delta\Phi$) that passes through the loop, when starting with the position shown in the drawing, the semicircle is rotated through half a revolution?



Sol. As the handle turns, the angle between the normal to the loop and the direction of the magnetic field changes. At any given moment, the flux Φ that passes through the loop is given by $\Phi = BA\cos\theta$, where B is the magnitude of the magnetic field and A is the area of the loop.

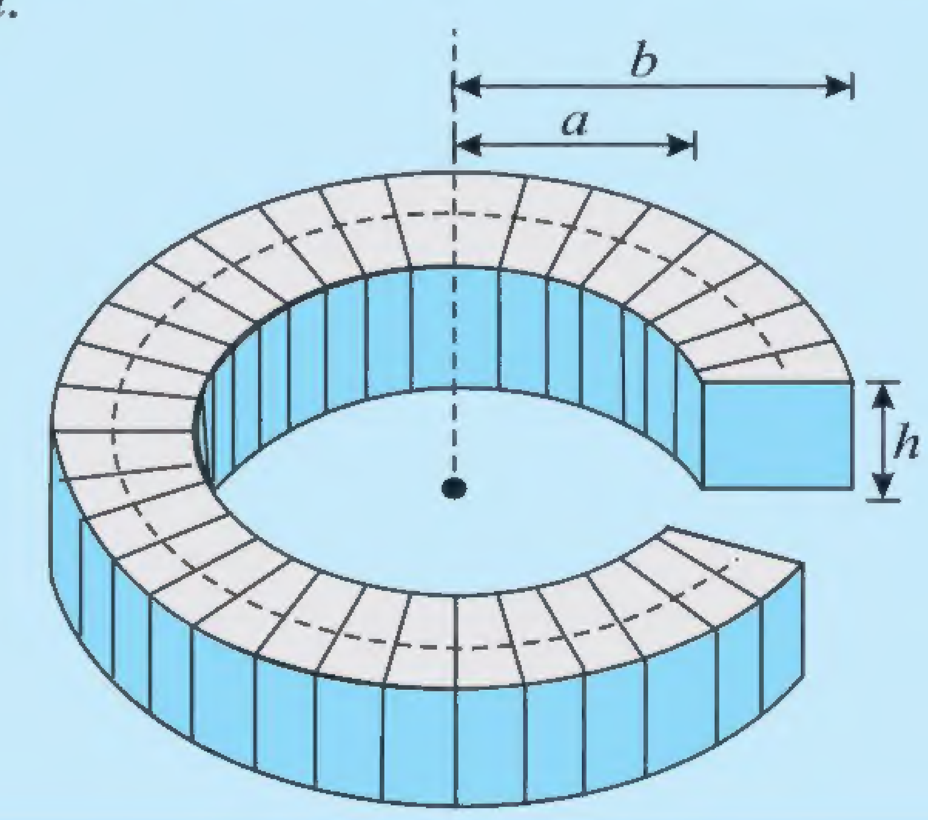
Here we can consider of the loop as being divided into a rectangular portion and a semicircular portion. Initially let A_0 be the area of the loop which is equal to the rectangular area A_{rec} plus the area $A_{semi} = \frac{1}{2}\pi r^2$ of the semicircle, where r is the radius of the semicircle: $A_0 = A_{rec} + \frac{1}{2}\pi r^2$. After half a revolution, the semicircle is once again within the plane of the loop, but now as a reduction of the area of the rectangular portion. Therefore, the final area A of the loop is equal to the area of the rectangular portion minus the area of the semicircle: $A = A_{rec} - \frac{1}{2}\pi r^2$. The angle between the normal to the loop and the direction of the magnetic field both directed into the page i.e., angle between both the vectors is zero degree ($\theta = 0°$).

$$\Delta\Phi = \Phi - \Phi_0 = BA\cos\theta - BA_0\cos\theta = B\cos\theta(A - A_0)$$

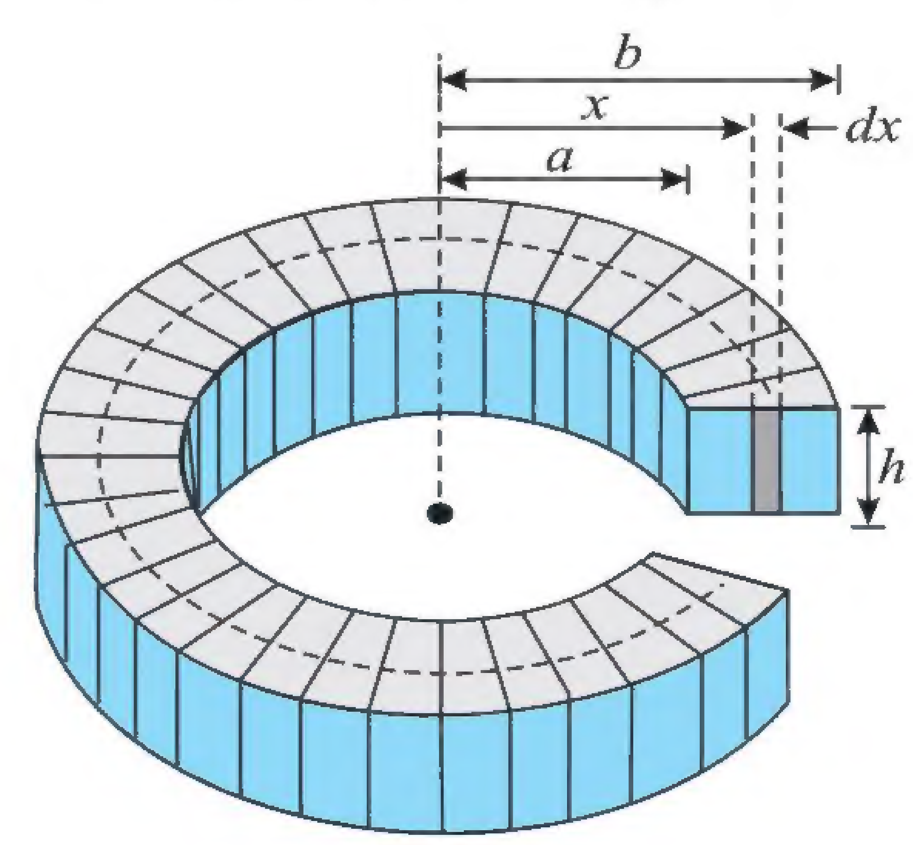
$$= B\cos 0^0\left[\left(A_{rec} - \frac{1}{2}\pi r^2\right) - \left(A_{rec} + \frac{1}{2}\pi r^2\right)\right] = -\pi r^2 B$$

Illustration 4.4

Figure shows a toroidal solenoid whose cross-section is rectangular in shape. Find the magnetic flux through this cross-section if the current through the toroidal winding is I, total number of turns in winding is N, the inside and outside radii of the toroid are a and b respectively and the height of toroid is equal to h.



Sol. To find the magnetic flux through the cross section shown in figure, we consider an elemental strip of width dx at a distance x from the axis of toroid as shown in figure.



The magnetic field at a distance x from the axis of the toroid is given as

$$B=\mu_0\left(\frac{N}{2\pi x}\right)I=\frac{\mu_0 NI}{2\pi x}$$

Magnetic flux through an elemental strip of radial thickness dx and height h as shown in figure is given as

$$d\phi=\vec{B}\cdot\vec{dA}=BdA$$

$$\Rightarrow \qquad d\phi=\frac{\mu_0 NI}{2\pi x}hdx$$

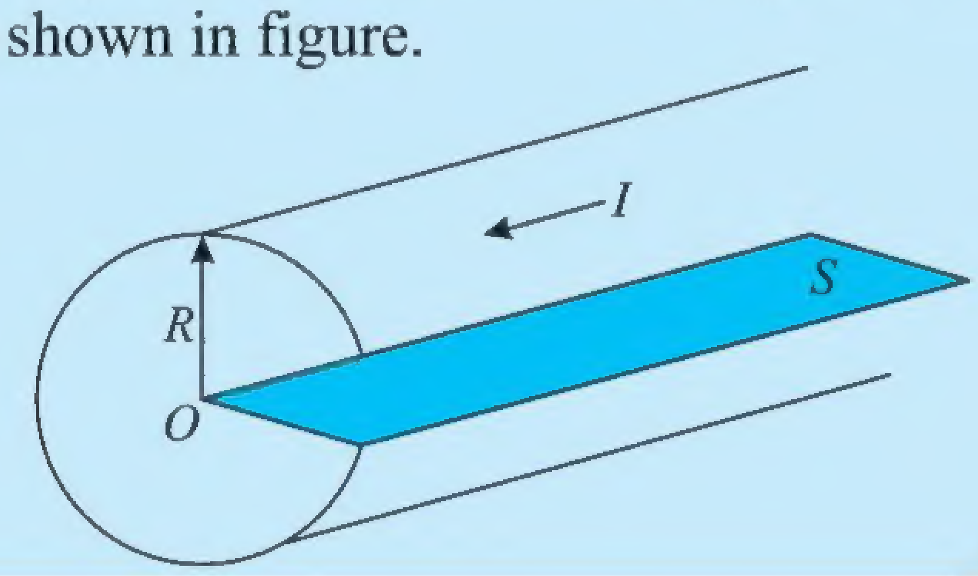
Total magnetic flux through the cross section of the toroid is given by integrating the above elemental flux within limits from a to b which is given as

$$\phi=\int_a^b\frac{\mu_0 NIh}{2\pi x}\,dx=\frac{\mu_0 NIh}{2\pi x}\int_a^b\frac{1}{x}\,dx=\frac{\mu_0 NIh}{2\pi x}[\ln x]_a^b$$

$$\phi=\frac{\mu_0 NIh}{2\pi x}[\ln b-\ln a]=\frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

Illustration 4.5

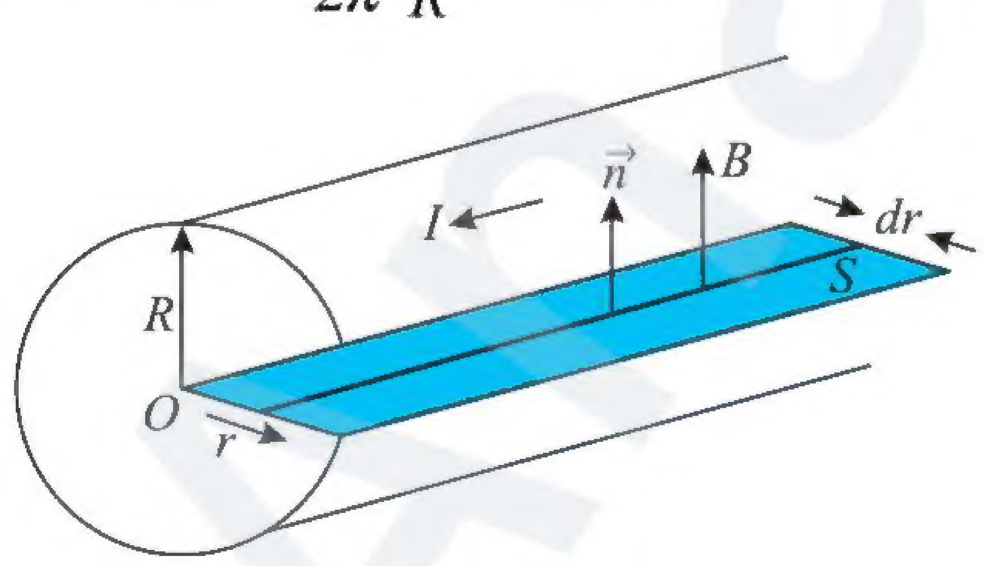
A long copper wire carries a current of I ampere. Calculate the magnetic flux per meter of the wire for a plane surface S inside the wire as shown in figure.



Sol. Consider an element of width dr at a distance r from the axis of the wire. The field due to the current I in the wire at the position of element will be $B=\dfrac{\mu_0 Ir}{2\pi R^2}$.

And as its direction is perpendicular to the plane surface, flux linked with the element is given by

$$d\phi=B\,ds\cos\theta=\frac{\mu_0}{2\pi}\frac{I}{R^2}r\;(1\,dr)\cos\theta$$



So, the flux linked with the plane surface is given by

$$\phi=\frac{\mu_0}{2\pi}\frac{I}{R^2}\int_0^R r\,dr=\frac{\mu_0 I}{4\pi}$$

Work Done in Changing Orientation of a Current Carrying Coil in Magnetic Field

When a current carrying coil is displaced in a uniform magnetic field in such a way that its orientation changes with angle between magnetic induction and magnetic moment of coil from θ_1 to θ_2 then the potential energy of the coil in magnetic field in initial and final state are given as

To change the orientation of coil externally we need to apply a torque against the magnetic torque of equal magnitude thus work done in changing the orientation in above situation can also be given as

$$W=\int dW=\int\tau d\theta\Rightarrow W=\int_{\theta_1}^{\theta_2}MB\sin\theta.d\theta$$

$$\Rightarrow \qquad W=MB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W=MB(\cos\theta_1-\cos\theta_2) \qquad \text{...(i)}$$

Above expression of work can be rewritten as

$$W=IAB(\cos\theta_1-\cos\theta_2)=I(BA\cos\theta_1-BA\cos\theta_2)$$

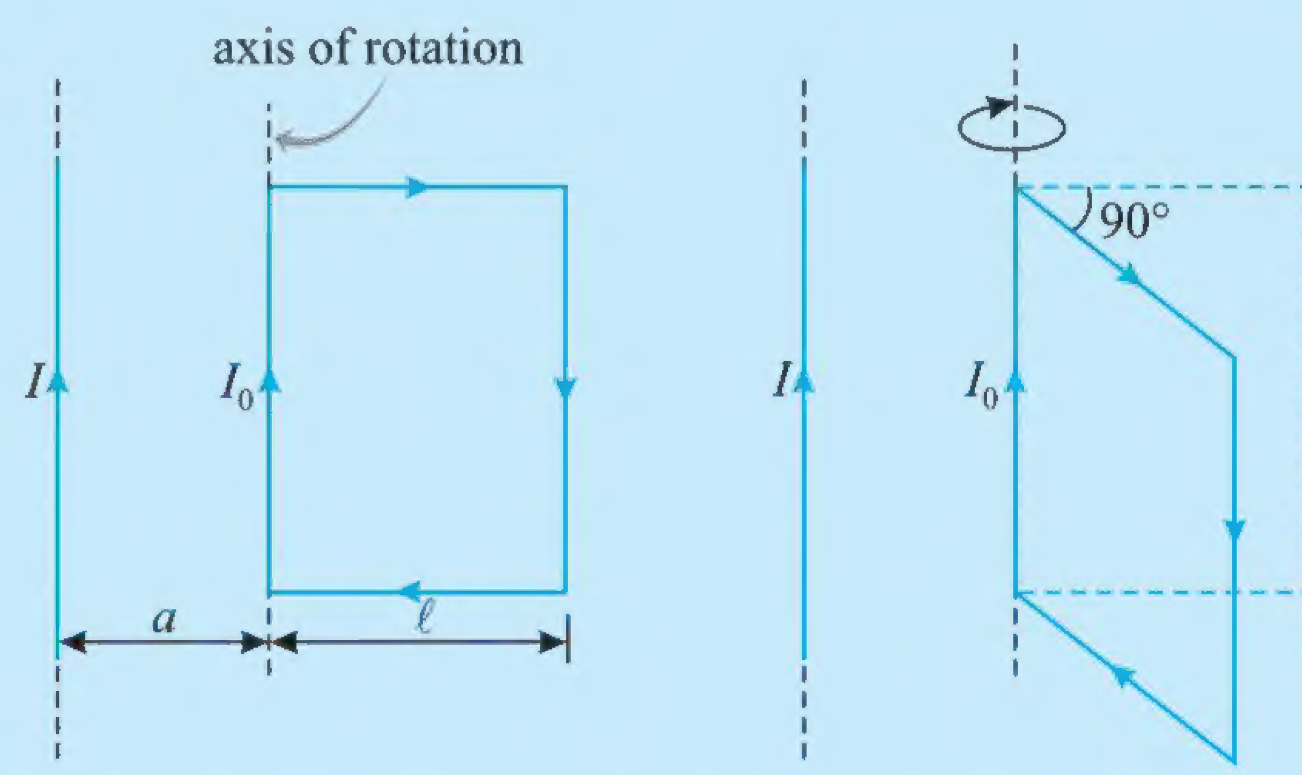
$$\Rightarrow \qquad W=I(\phi_{\text{initial}}-\phi_{\text{final}})$$

$$\Rightarrow \qquad W=-I(\phi_{\text{final}}-\phi_{\text{initial}})$$

$$\Rightarrow \qquad W=-I\Delta\phi \qquad \text{...(ii)}$$

Illustration 4.6

A square frame carrying current I_0 is located in the same plane as a long straight wire carrying currents I [Fig. (a)]. The frame side has a length l. The axis of the frame is parallel to the wire as shown. Find



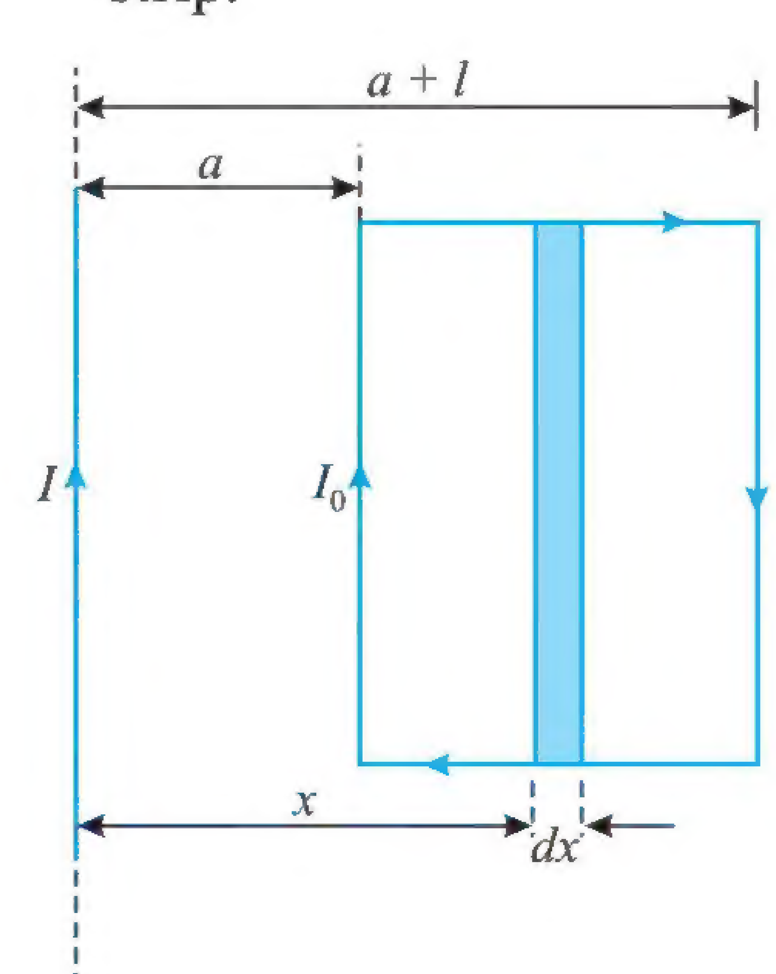
(a) (b)

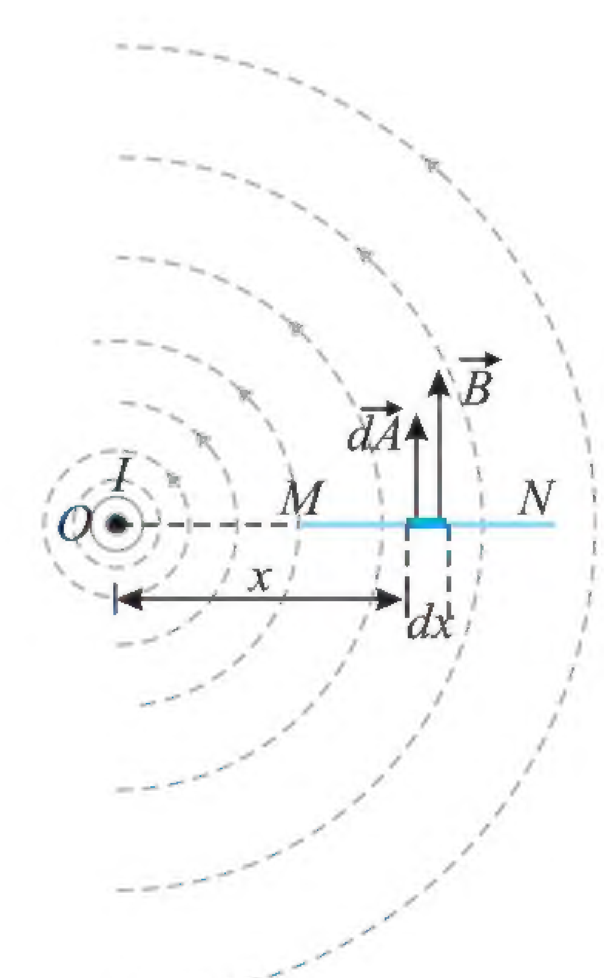
(a) the magnetic flux passing through the frame.

(b) the mechanical work to be performed to the frame through 90° about its axis [Fig. (b)].

Sol.

(a) Let us consider an elemental strip of thickness dx, at a distance x from the wire as shown. As this strip is parallel to the wire the magnitude field should be constant on this strip.





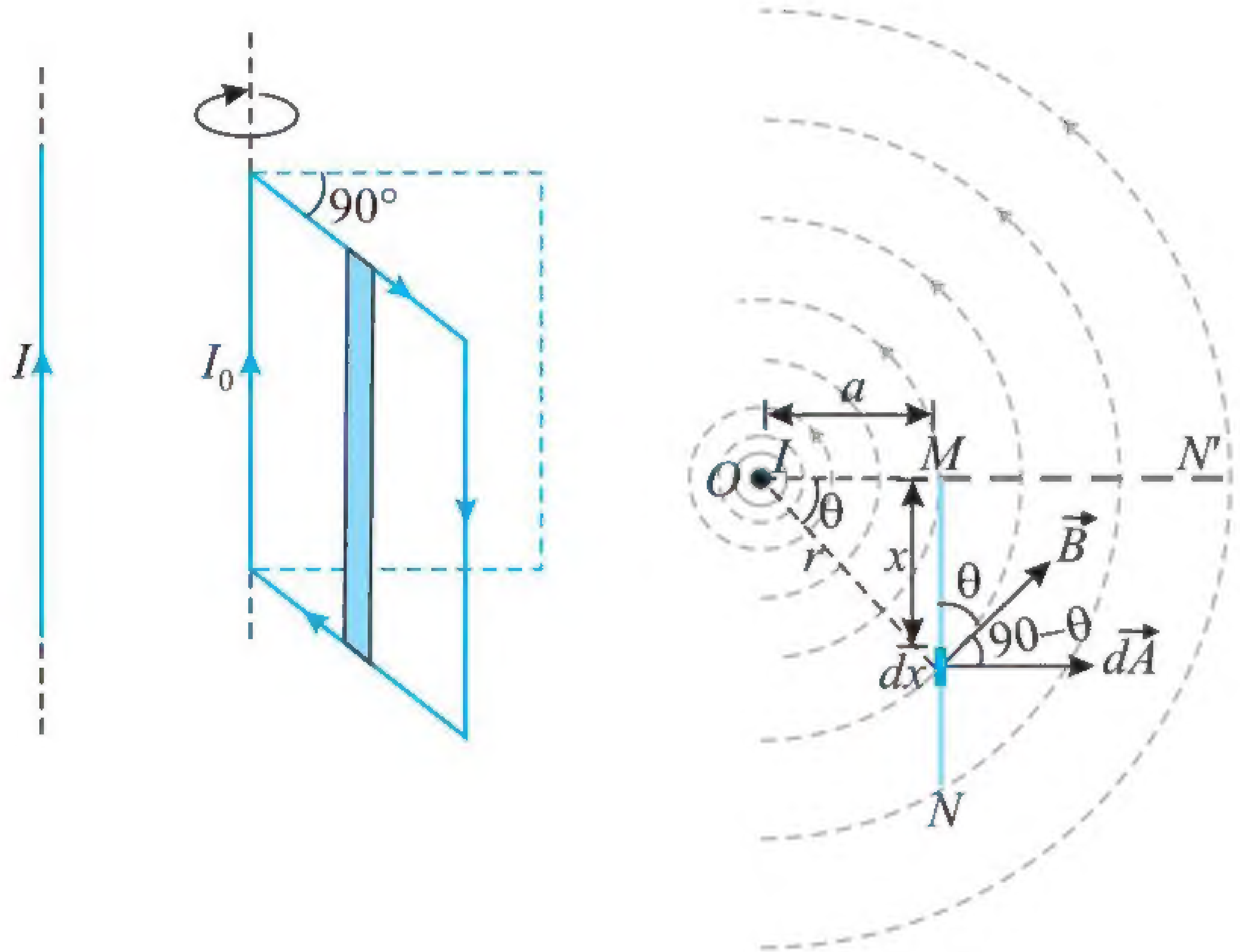
The magnetic flux through the elemental strip

$$d\phi_1 = \vec{B}\cdot\vec{dA} = B\cdot dA\cos 0° \Rightarrow d\phi_1 = \frac{\mu_0 I}{2\pi x}(l\,dx)$$

Total magnetic flux through the coil

$$\phi_1 = \int d\phi = \frac{\mu_0 Il}{2\pi}\int_a^{a+l}\frac{dx}{x} = \frac{\mu_0 Il}{2\pi}\ln\frac{(a+l)}{a} \qquad \text{...(i)}$$

(b) Now the coil is rotated by 90°.



In this situation the magnetic field vector will not be normal to the plane of the coil at every point. Let us again consider an elemental strip of the coil as shown in figure. In this situation the flux passing through the strip is

$$d\phi_2 = \vec{B}\cdot\vec{dA} = \left(\frac{\mu_0 I}{2\pi r}\right)(l\,dx)\cdot\cos(90° - \theta)$$

$$\Rightarrow \quad d\phi_2 = \frac{\mu_0 Il}{2\pi\sqrt{a^2+x^2}}\,\frac{x}{\sqrt{a^2+x^2}}\cdot dx$$

For calculating total flux through the coil we need to integrate above equation

$$\phi_2 = \int d\phi_2 = \frac{\mu_0 Il}{2\pi}\int_a^l \frac{x\,dx}{(a^2+x^2)}$$

Let $a^2 + x^2 = t \Rightarrow 2x\,dx = dt$

where $x = 0, t = a^2$ and when $x = l, t = a^2 + l^2$

Changing the limits in above equation we get

$$\phi_2 = \frac{\mu_0 Il}{4\pi}\int_{a^2}^{a^2+l^2}\frac{dt}{t} = \frac{\mu_0 Il}{4\pi}[\ln t]_{a^2}^{a^2+l^2}$$

$$\Rightarrow \quad \phi_2 = \frac{\mu_0 Il}{4\pi}\ln\frac{(a^2+l^2)}{a^2} \qquad \text{...(ii)}$$

The work done in rotation of the coil is given as

$$W = -I_0\Delta\phi = -I_0(\phi_2 - \phi_1) \qquad \text{...(iii)}$$

Substituting the values of ϕ_1 and ϕ_2 from (i) and (ii), and substituting in (iii) we get

$$W = -I_0\left[\frac{\mu_0 Il}{4\pi}\ln\frac{(a^2+l^2)}{a^2} - \frac{\mu_0 Il}{2\pi}\ln\frac{(a+l)}{a}\right]$$

$$= \frac{\mu_0 II_0 l}{4\pi}\left[\ln\frac{(a^2+l^2)}{a^2} - 2\ln\frac{(a+l)}{a}\right]$$

$$\Rightarrow \quad W = \frac{\mu_0 II_0 l}{4\pi}\left[\ln\frac{(a^2+l^2)}{(a+l)^2}\right]$$

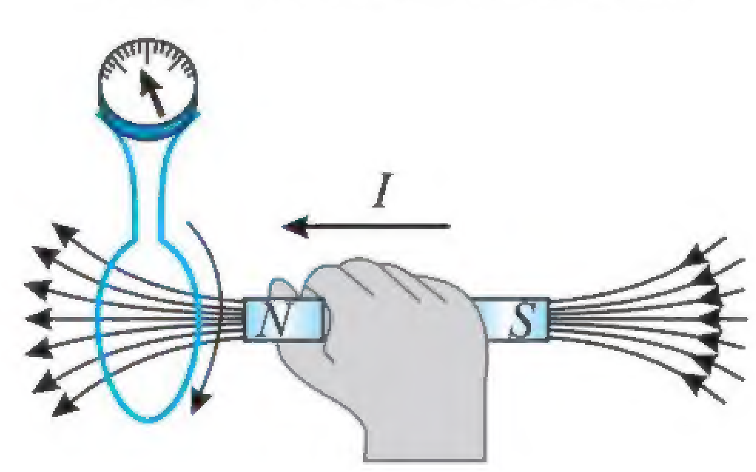
FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

British scientist Michael Faraday based on various experiment demonstrated that a changing magnetic field could generate a potential difference in a conductor, strong enough to produce an electric current. This discovery is of basic importance to all the electrical and magnetic devices we use every day, from computers to cell phones, from television to credit cards, from the tiniest batteries to the largest electrical power grid.

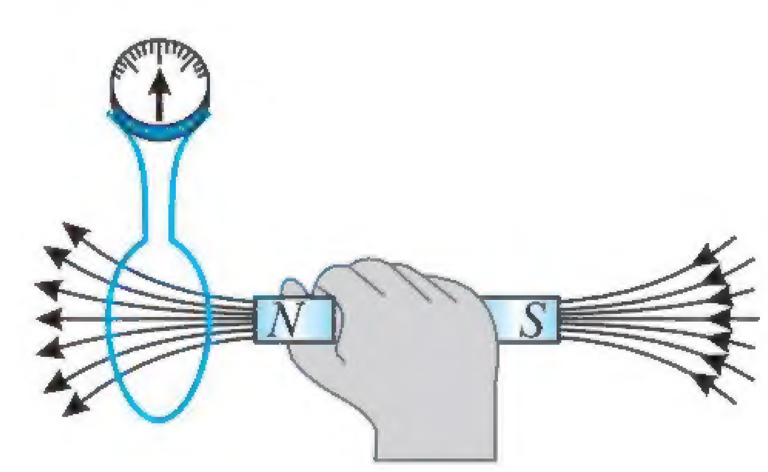
Let us examine two simple experiments to prepare for our discussion of Faraday's law of induction.

First experiment: Figure shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuitas shown in Fig. (a). The current disappears when the magnet stops as shown in Fig. (b). If we then move the magnet away, a current again suddenly appears, but now in the opposite direction as shown in Fig. (c). If we experimented for a while, we would discover the following:

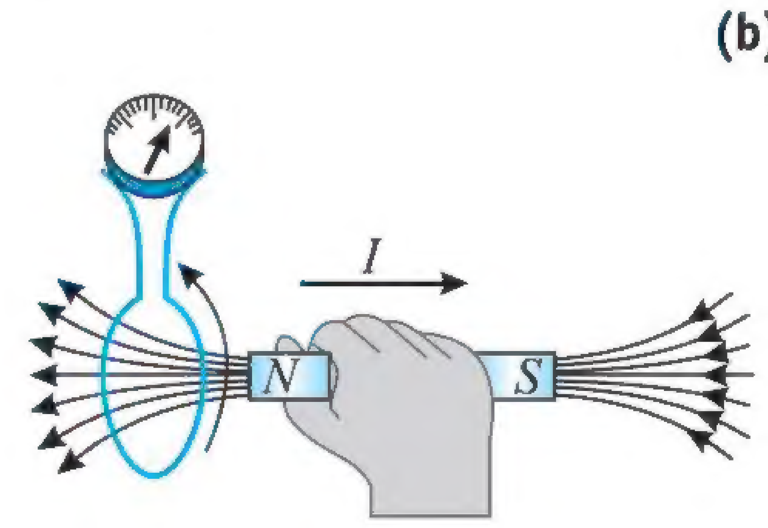
1. A current appears in the loop only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion of the magnet produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counter clockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.



(a)



(b)

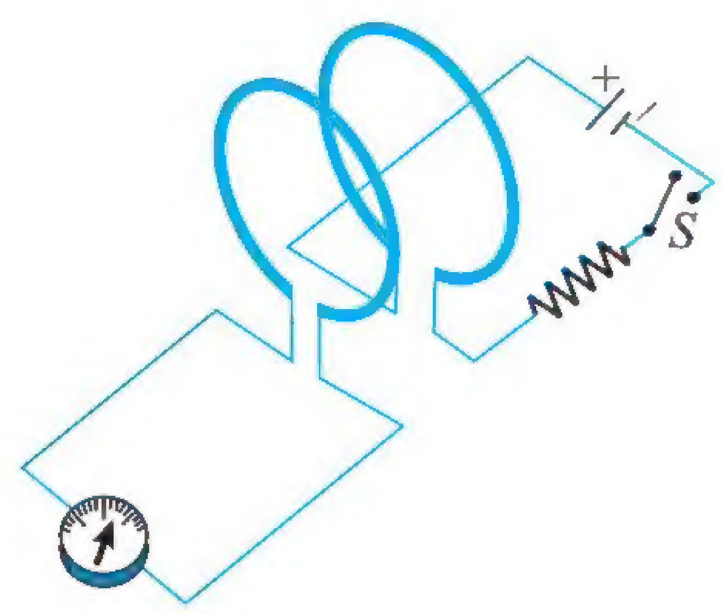


(c)

The current appeared in the loop is called an **induced current**; the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf**; and the process of producing the current and emf is called **induction**.

Second Experiment: In this experiment two conducting loops are placed close to each other but not touching as shown in figure. If we close switch *S*, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—

an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction. We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).



In both the experiments discussed above have one thing in common: in each case, an emf is induced in a loop when the magnetic flux through the loop changes with time. In general, this emf is directly proportional to the time rate of change of the magnetic flux through the loop. This statement can be written mathematically as **Faraday's law of induction:**

$$\varepsilon = -\frac{d\Phi_B}{dt} \qquad \text{...(i)}$$

If a coil consists of N loops with the same area and Φ_B is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series, so their emfs add; therefore, the total induced emf in the coil is given by

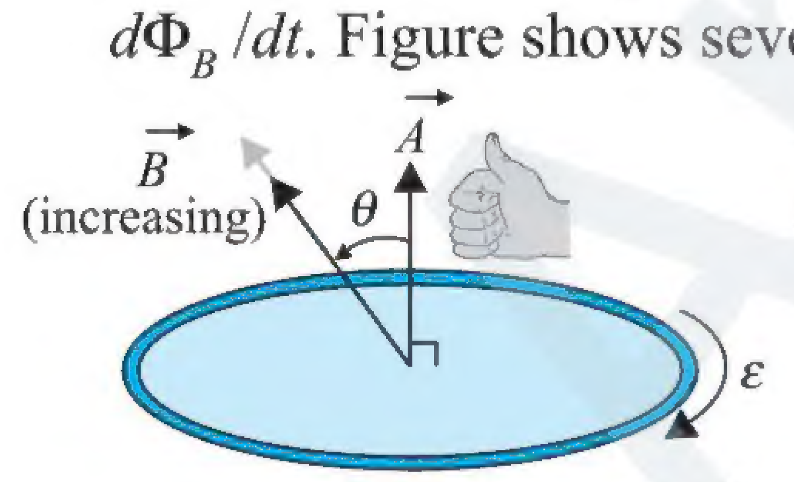
$$\varepsilon = -N\frac{d\Phi_B}{dt} \qquad \text{...(ii)}$$

As such, Faraday's law in itself is complete to tell the magnitude and polarity of induced emf. But Lenz's rule is commonly used to determine the polarity of induced emf or direction of induced current.

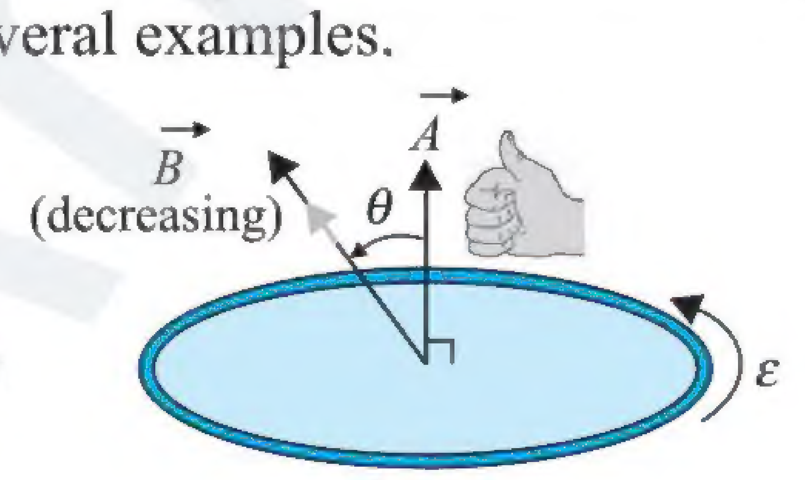
Direction of Induced EMF

We can find the direction of an induced emf or current by using $\varepsilon = -d\Phi_B/dt$ together with some simple sign rules. Here is the procedure:

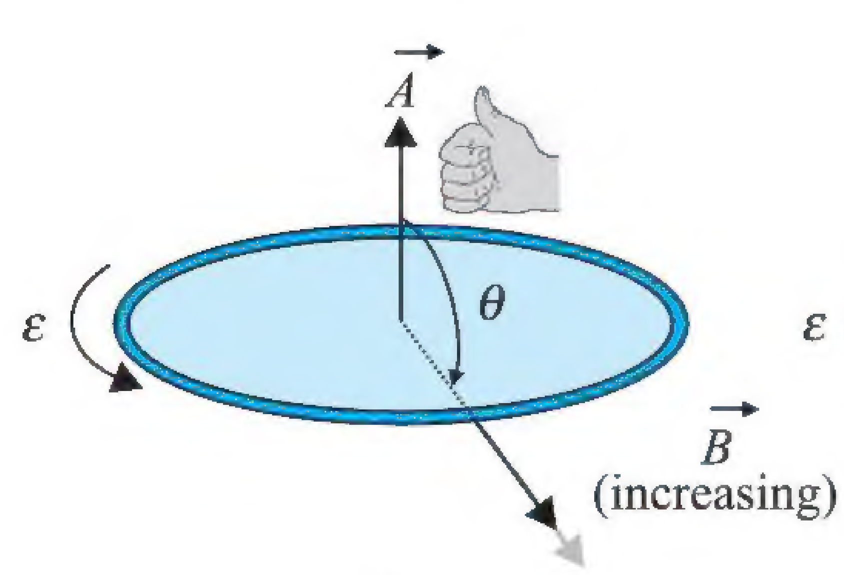
1. Define a positive direction for the area vector $\vec{A}$.
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$. Figure shows several examples.



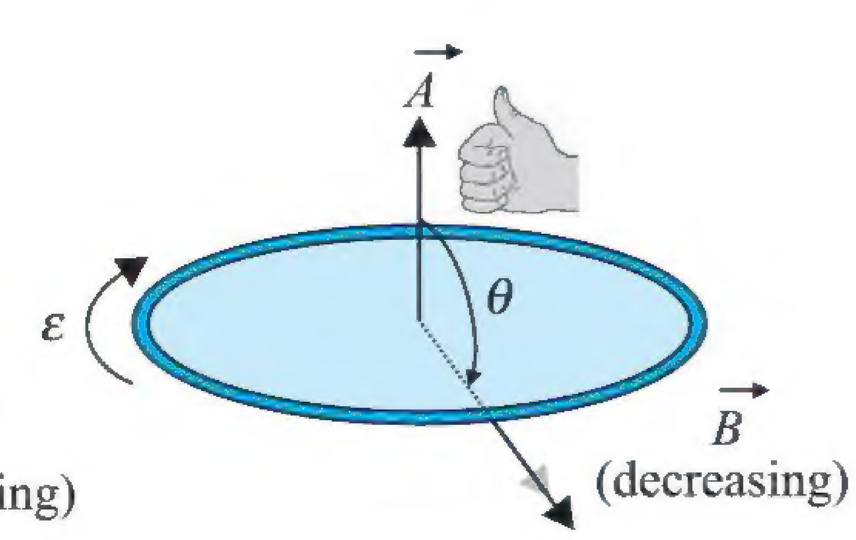
(a)



(b)



(c)



(d)

For Fig. (a):

1. $\vec{A}$ is upward so anticlockwise direction is positive.
2. $\phi = BA\cos\theta$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\theta) \Rightarrow \varepsilon = -A\cos\theta\frac{dB}{dt} \qquad \text{...(i)}$$

Here θ is acute, so cos θ is positive. B is increasing, so dB/dt is positive. Then we find that ε is negative. So ε is clockwise as shown.

For Fig. (b):

Here B is decreasing, so dB/dt is negative. Then from Eq. (i), ε is positive. So ε is anticlockwise as shown.

For Fig. (c):

Here θ is obtuse, so cos θ is negative. B is increasing, so dB/dt is positive. Then from Eq. (i), ε is positive. So ε is anticlockwise as shown.

For Fig. (d):

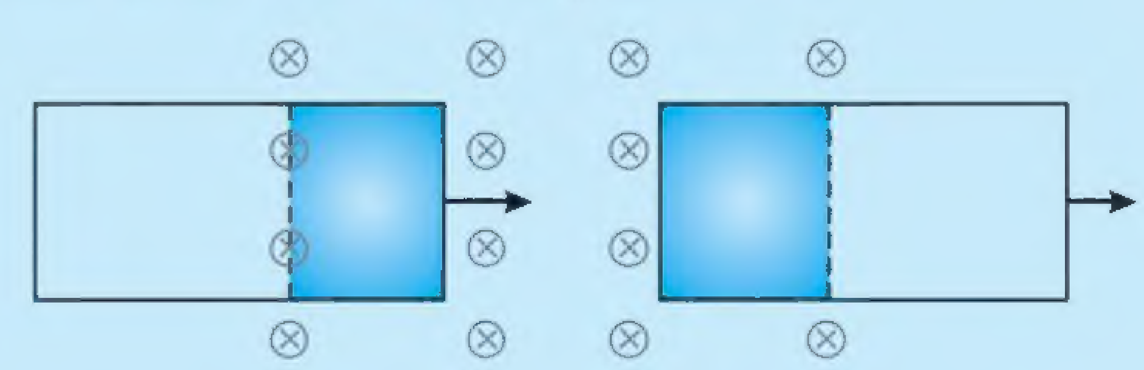
Here θ is obtuse, so cos θ is negative, B is decreasing so dB/dt is negative. Then from Eq. (i), ε is negative. So ε is clockwise as shown.

Important Points:

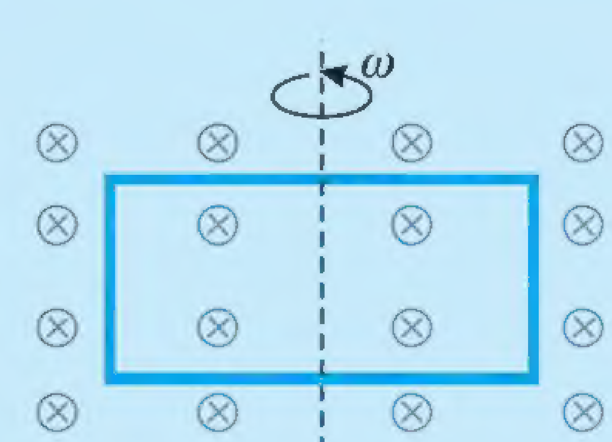
The induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by $\phi = BA\cos\theta$.

Thus, flux can be changed in several ways:

- The magnitude of $\vec{B}$ can change with time. In the problems if magnetic field is given as a function of time, it implies that the magnetic field is changing. Thus, $B = B(t)$.
- The area enclosed by the loop can change with time.
- This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by the loop (hatched area) can be changed.



- The angle θ between $\vec{B}$ and the normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.

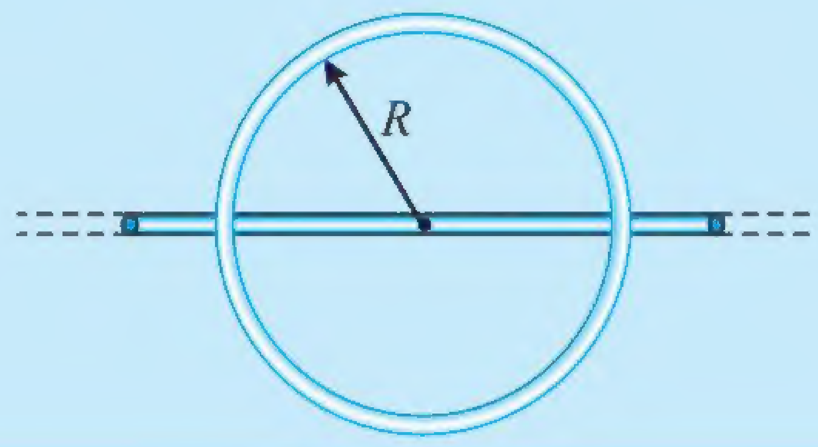


- Any combination of the above can occur.

Illustration 4.7

In figure shown below, a wire forms a closed circular loop, of radius R = 1.0 m and resistance 5.0 Ω. The circle is centered on a long straight wire; at time t = 0, the current in the long

straight wire is 10.0 A rightward. Thereafter, the current changes according to i = 10.0 A + (0.5 A/s^2)t^2. (The straight wire is insulated; so there is no electrical contact between it and the wire of the loop.) What is the magnitude of the current induced in the loop at times $t > 0$?



Sol. The magnetic field due to long straight current wire is out of the page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

ILLUSTRATION 4.8

A coil is placed in a constant and uniform magnetic field of induction $10^{-3}\,\text{Wb/m}^2$ acting normal to the plane of the coil. If the radius of a coil decreases steadily at the rate of 10^{-2} m/s, what will be the radius of the coil when the induced e.m.f. in the 1 μV.

Sol. According to Faraday's law of induction, the magnitude of induced emf in the coil,

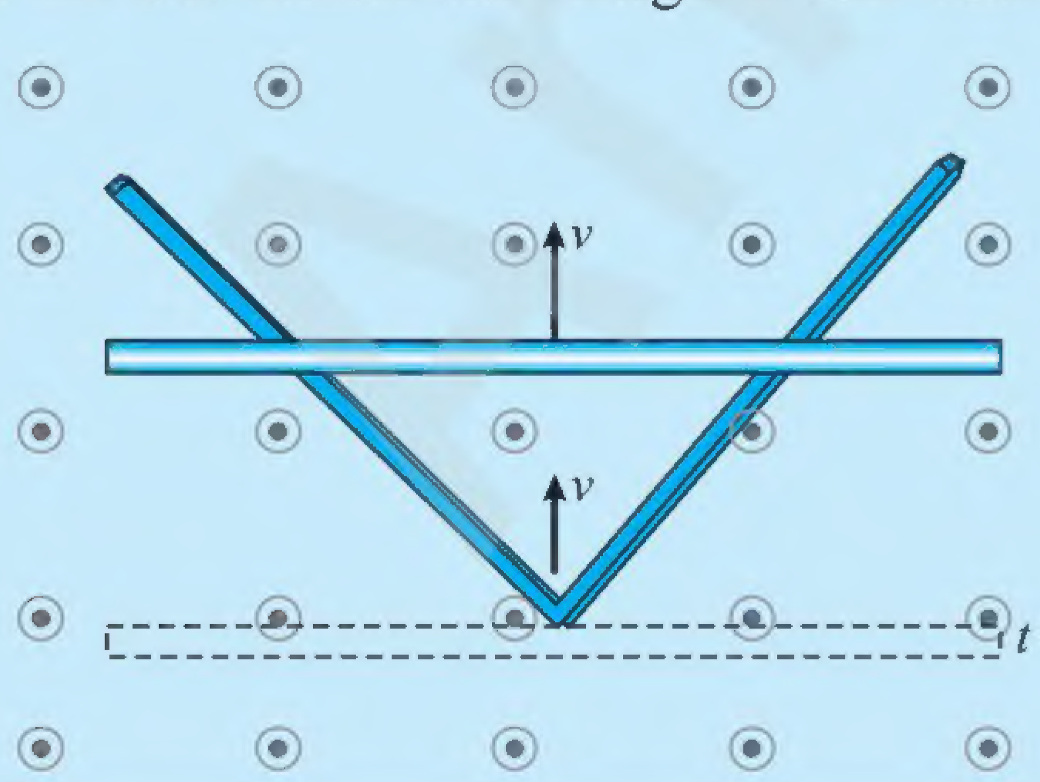
$$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r\frac{dr}{dt}$$

Hence the radius of coil,

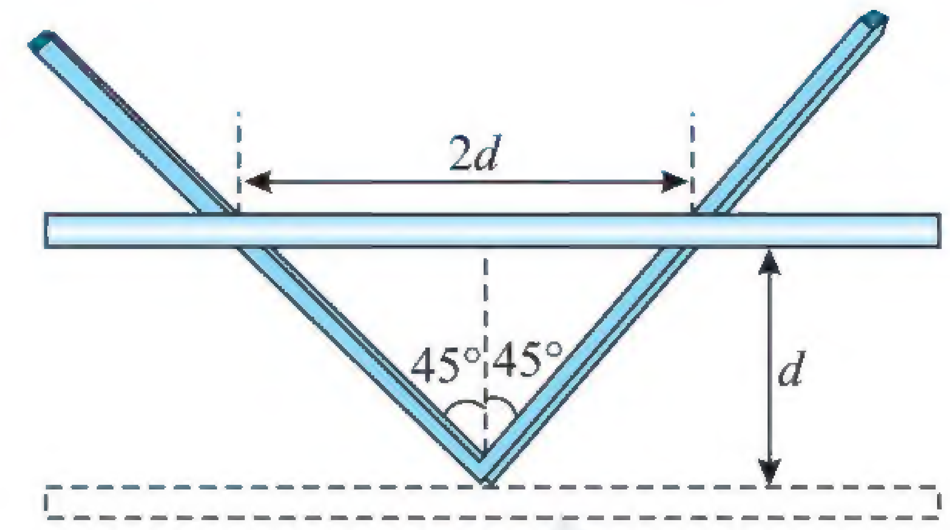
$$r = \frac{e}{2B\pi\left(\dfrac{dr}{dt}\right)} = \frac{1\times10^{-6}}{2\times10^{-3}\times\pi\times10^{-2}} = \frac{1}{20\pi}\text{ m} = \frac{5}{\pi}\text{ cm}$$

ILLUSTRATION 4.9

A structure formed from two straight conducting rods forming a right angle, is placed in a uniform magneticfield with B = 0.40 T is directed out of the page. A straight conducting bar in contact with the structure, starts at the vertex at time $t = 0$ and moves with a constant velocity of 5.0 m/s along them. Calculate the emf around the triangle at that time t = 4.0 s.



Sol. The perpendicular length of the triangular area enclosed by the structure and bar is the same as the distance travelled by it in time t: $d = vt$, where v = 5.0 m/s.



Here the "base" of that triangle (the distance between the intersection points of the bar with the structure) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2.$$

Since the field is a uniform B = 0.350 T, then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.40)(5.0)^2t^2 = 10.0t^2.$$

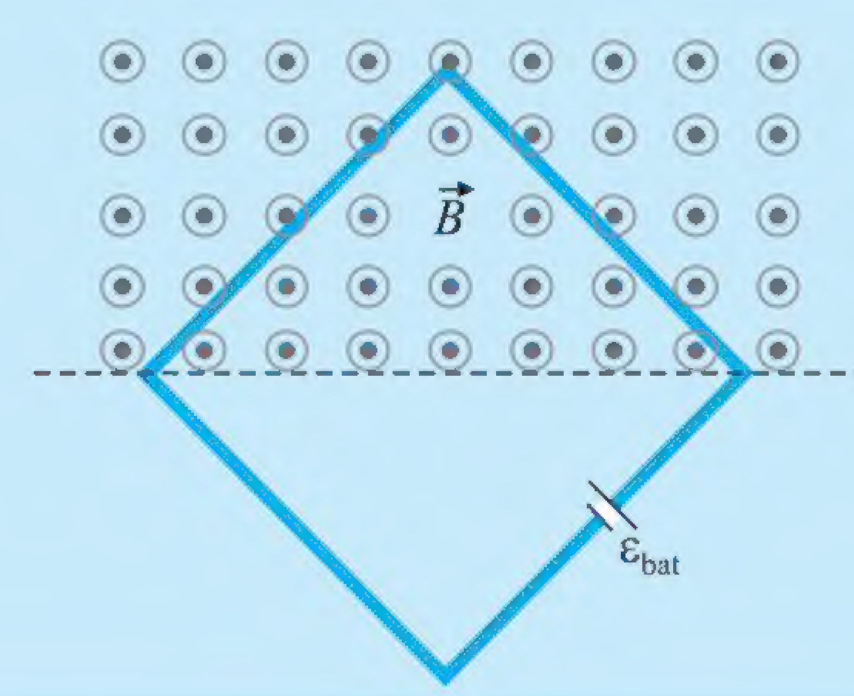
The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 10.0\frac{dt^2}{dt} = 20.0t$$

At t = 4.0 s, this yields ε = 80.0 V.

ILLUSTRATION 4.10

A square wire loop with side L = 1.0 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in figure. The resistance of the loop is 35 Ω and the loop contains an ideal battery with emf ε = 6.0 V. If the magnitude of the field varies with time according to $B = 5.0 - 2.0t$, with B in teslas and t in seconds, what are **(a)** the net emf in the circuit and **(b)** the direction of the (net) current around the loop?



Sol.

(a) Let magnetic field at any time be B. Let upward direction of area vector as positive, the magnetic flux associated with the circuit is

$$\Phi_B = \frac{BL^2}{2} \text{ and the induced emf is}$$

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

As, $B = 5.0 - 2.0t$, hence, $\dfrac{dB}{dt} = -2.0T/s$

It means the induced emf $\varepsilon_i = -\dfrac{(1.0)^2}{2}(-2.0) = 1.0$ V.

As ε_i is positive, so the induced emf is counter clockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon_{\text{total}} = \varepsilon + \varepsilon_i = 6.0 \text{ V} + 1.0 \text{ V} = 7.0 \text{ V}$$

(b) The current is in the sense of the total emf (counter clockwise). The magnitude of the current in the loop

$$i = \frac{\varepsilon_{\text{total}}}{R} = \frac{7.0}{35.0} = 0.2 \text{ A}$$

Induced Charge

When the magnetic field passing through a loop is changed, an induced emf, and hence an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

Current starts flowing in the circuit, i.e., flow of charge takes place. Charge flown in the circuit in time dt will be given by

$$dq = i\,dt = \frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt, we can write

$$\Delta q = i\Delta t = \frac{1}{R}(-\Delta\phi_B)$$

From these equations we can see that e and i are inversely proportional to Δt while Δq is independent of Δt. It depends on the magnitude of change in flux, not the time taken by it.

Important Points:

- Induced charge in any coil (or circuit) does not depends on time in which change in flux occurs i.e., it is independent from rate of change of flux or relative speed of coil–magnet system.
- Induced charge depends on change in flux through the coil and nature of the coil (or circuit) i.e. resistance.

Illustration 4.11

A closed coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 30 ohms. If the normal of the coil is inclined at 60° to the direction of a magnetic field of intensity 1.5 Wb/m^2, the coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

Sol. The total flux linked with the coil having turns N and area A is

$$\phi_{\text{initial}} = N(\vec{B}.\vec{A}) = NBA \cos\theta = NBA \cos 60° = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_{\text{final}} = 0$

The change in flux is $|\Delta\phi| = \dfrac{NBA}{2}$

Hence the charge flow through the circuit is

$$q = \frac{|\Delta\phi|}{R} = \frac{NBA}{2R} = \frac{20\times1.5\times10^{-4}}{2\times30} = 5.0\times10^{-5} \text{ C}$$

Lenz's Law

Direction of the Induced Current in a Circuit

Lenz's law states that "when the magnetic flux through a loop changes, a current is induced in the loop such that the magnetic field due to the induced current opposes the change in the magnetic flux through the loop".

The above rule can be systematically applied as follows to determine the direction of the induced currents.

- Identify the loop in which the induced current is to be determined.
- Determine the direction of the magnetic field in this loop (i.e., in or out of the loop).
- The direction of flux is the same as the direction of the magnetic field. Determine if the flux through the loop is increasing or decreasing (because of change in area or change in B).

Choose the appropriate current in the loop that will oppose the change in flux.

- If the flux is into the paper and increasing then the flux due to the induced current should be out of the paper.
- If the flux is into the paper and decreasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and increasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and decreasing, the flux due to the induced current should be out of the paper.

The above description is the physical interpretation of Lenz's law. We can determine the direction of the induced current mathematically by simply applying Lenz's law, $\xi_{\text{ind}} = -\dfrac{d\Phi_B}{dt}$ with the appropriate conventions.

The right hand sign convention used is as follows.

- For counterclockwise current, emf is positive.
- For clockwise current, emf is negative.
- Magnetic flux out of the paper is positive.
- Magnetic flux into the plane of the paper is negative.
- The rate of change of an increasing positive flux is positive.
- The rate of change of a decreasing positive flux is negative.
- The rate of change of an increasing negative flux is negative.
- The rate of change of a decreasing negative flux is positive.

Illustration 4.12

A circular loop of wire is placed on flat horizontal plane. An external magnetic field is directed perpendicular to the plane of the loop. The magnitude of the applied magnetic field is increasing with time. Because of this increasing magnetic field, an induced current is flowing clockwise in the loop, as viewed from above. What is the direction of the external magnetic field?

Sol. As external magnetic field is perpendicular to the plane of the horizontal loop, so it must point either upward or downward. The external magnetic field B is increasing in magnitude, it means the magnetic flux through the loop also increases with time. In order to oppose the increase in magnetic flux, the induced magnetic field B_{ind} must be directed opposite to the external

magnetic field B. The drawing shows the loop as viewed from above, with an induced current I_{ind} flowing clockwise. According to Right-Hand Rule, this induced current creates an induced magnetic field B_{ind} that is directed into the page at the center of the loop (and all other points of the loop's interior). Therefore, the external magnetic field B must be directed out of the page. Because we are viewing the loop from above, "out of the page" corresponds to upward toward the viewer.

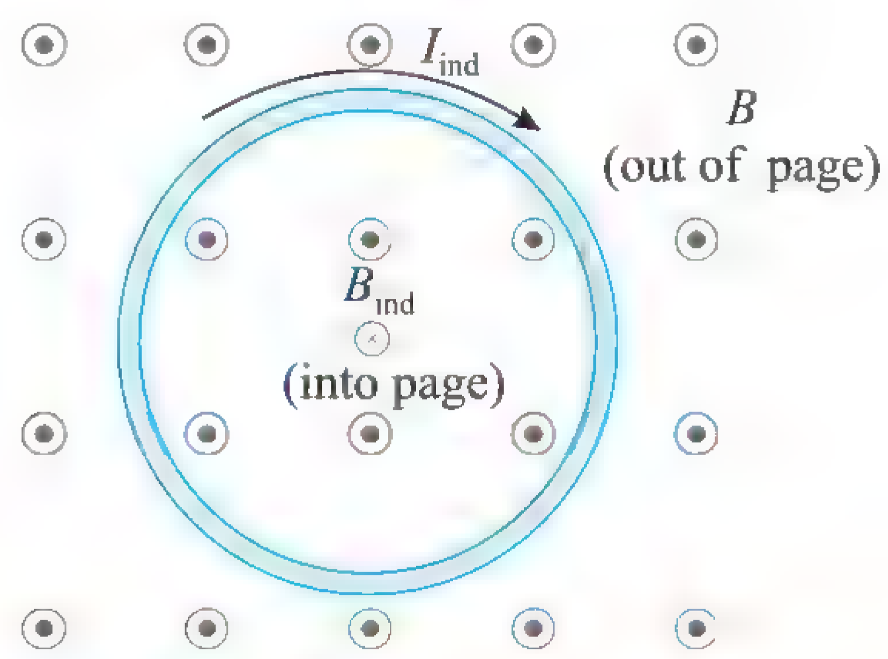
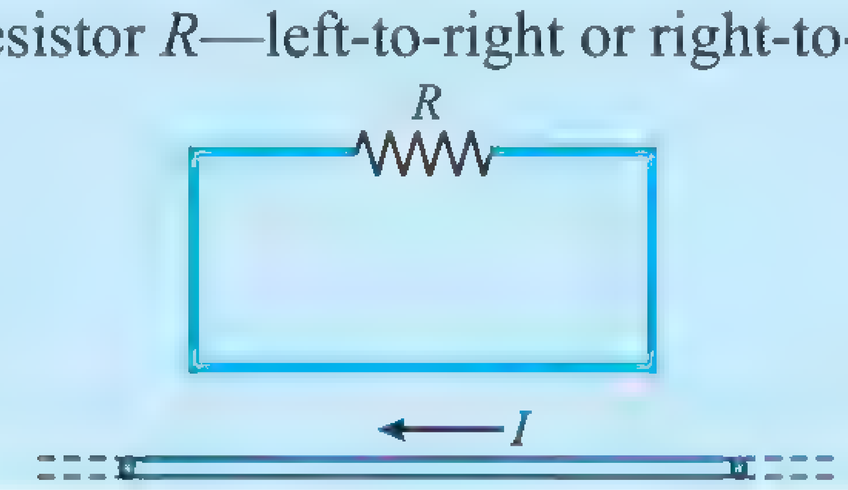


ILLUSTRATION 4.13

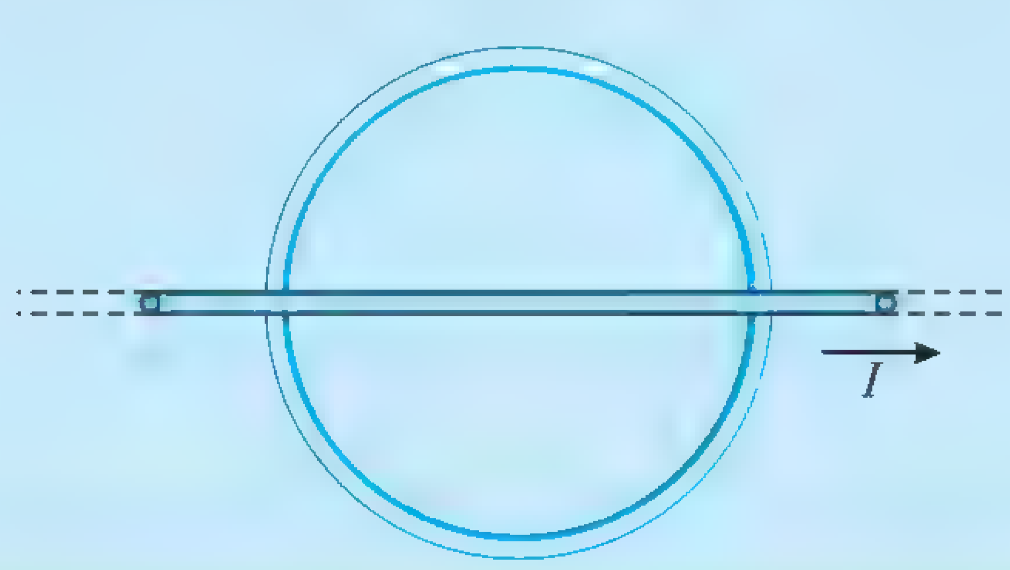
A rectangular loop that contains a resistor R is placed near a long straight wire carrying a current I. The wire and the loop are placed in same plane as shown in figure. If the current I is decreasing in time, what is the direction of the induced current through the resistor R—left-to-right or right-to-left?



Sol. At the location of the loop, the magnetic field produced by the current I is directed into the page .This can be verified by using right hand rule. The current is decreasing, so the magnetic field is decreasing. Therefore, the magnetic flux that penetrates the loop is decreasing. According to Lenz's law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes this flux change. The induced magnetic field will oppose this decrease in flux by pointing into the page, in the same direction as the field produced by I. According to RHR-2, the induced current must flow clockwise around the loop in order to produce such an induced field. The current then flows from left-to-right through the resistor.

ILLUSTRATION 4.14

An insulated circular loop of wire rests upon a long, straight wire such that the center of the loop lies on the wire. Both the wire and loop are in same horizontal plane as shown in figure. The current I in the straight wire is decreasing. In what direction is the induced current, if any, in the loop?



Sol. The current I in the straight wire produces a circular pattern of magnetic field lines around the wire. The magnetic field at any point is tangent to one of these circular field lines. Thus, the field points perpendicular to the plane of the table. Furthermore, according to Right-Hand Rule, the field is directed up out of the table surface in region 1 above the wire and is directed down into the table surface in region 2 below the wire.

As the current I decreases, the magnitude of the field that it produces also decreases. Since the fields in these two regions always have opposite directions and equal magnitudes at any given radial distance from the straight wire, the flux through the regions add up to give zero for any value of the current. With the flux remaining constant as time passes, Faraday's law indicates that there is no induced emf in the coil. Since there is no induced emf in the coil, there is no induced current.

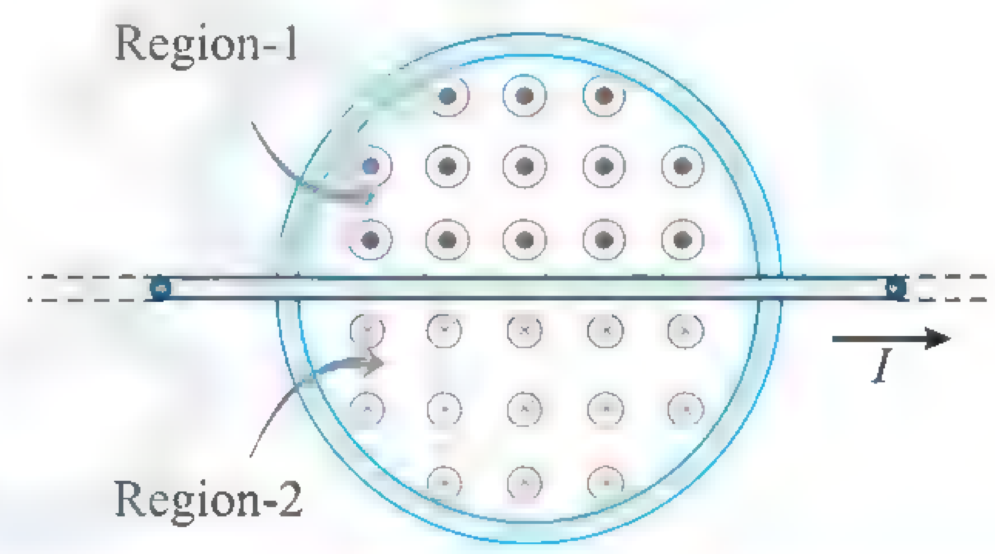
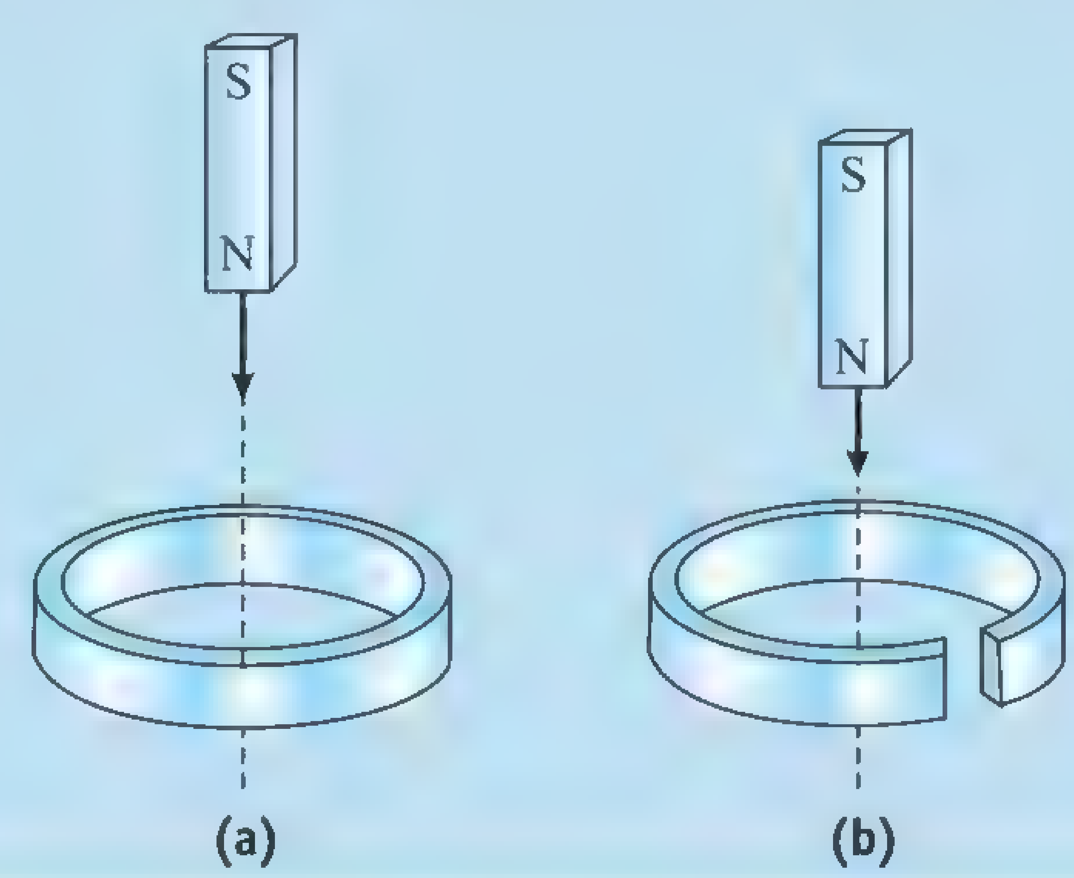


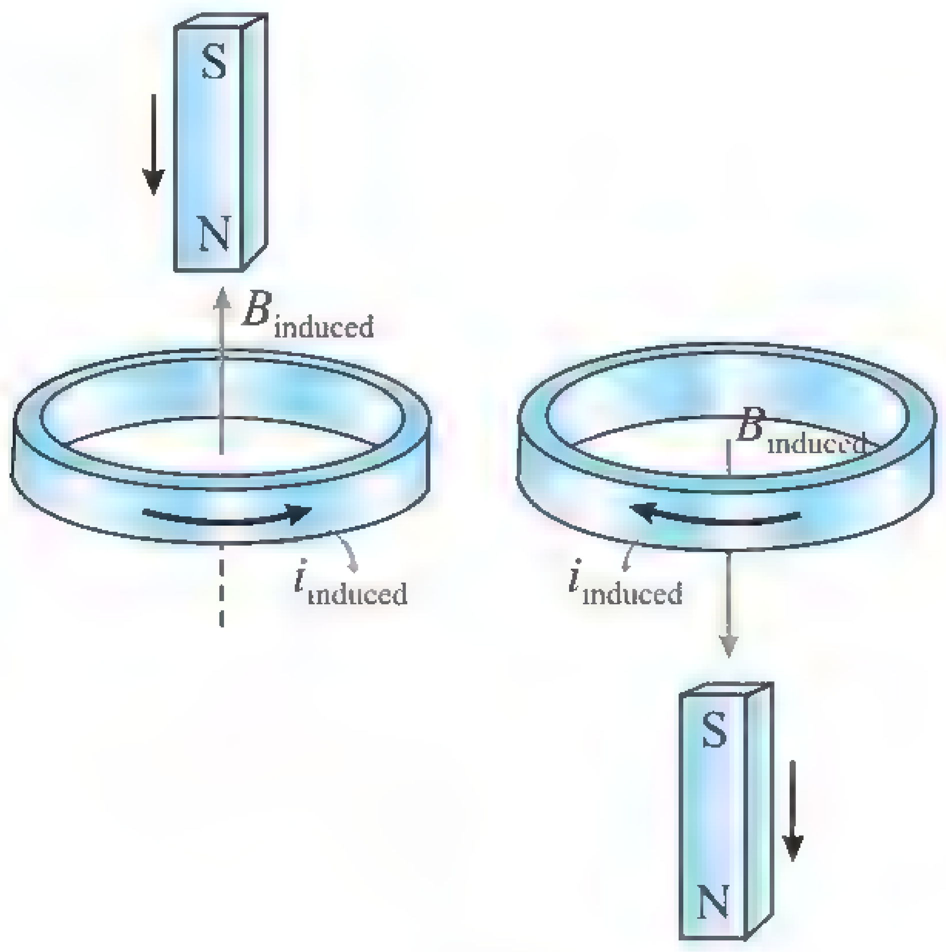
ILLUSTRATION 4.15

A bar magnet is released from rest and it starts falling through a metal ring as shown in the figure. Consider two cases, in case (I) the ring is solid all the way around, but in case (II) it has been cut through. Discuss the motion of the magnet in both the cases when it is above the ring and below the ring as well.



Sol. In case (I), when the magnet is above the ring its magnetic field points down through the ring and is increasing in strength as the magnet falls. According to Lenz's law, an induced magnetic field appears that attempts to reduce the increasing field. Therefore, the induced field must point up. Using Right-Hand Rule, we can see that the induced current in the ring is as shown in the drawing. Because of the induced current, the ring looks like a magnet with its north pole at the top. The north pole of the loop repels the falling magnet and retards its motion. When the magnet is below the ring, its magnetic field still points down through the ring but is decreasing as the magnet falls. According to Lenz's law, the induced magnetic field attempts to bolster the decreasing field and, therefore, must point down. The induced current in the ring is as shown in the drawing, and the ring then looks like a

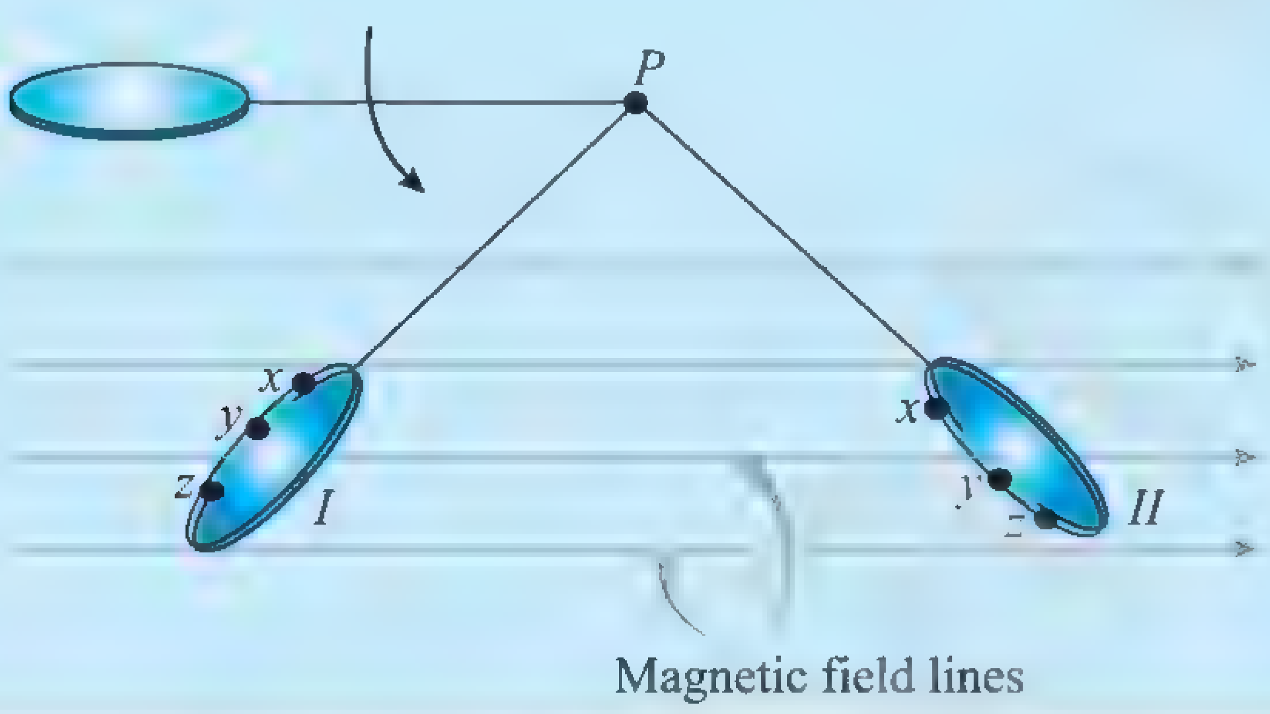
magnet with its north pole at the bottom, attracting the south pole of the falling magnet and retarding its motion.



In case (II), the motion of the magnet is unaffected, since no induced current can flow in the cut ring. No induced current means that no induced magnetic field can be produced to repel or attract the falling magnet.

ILLUSTRATION 4.16

A wire loop is suspended from a string that is attached to point P in the drawing. When released, the loop swings downward, from left to right, through a uniform magnetic field, with the plane of the loop remaining perpendicular to the plane of the paper at all times. Determine the direction of the current induced in the loop as it swings past the locations labeled I and II. Specify the direction of the current in terms of the points x, y, and z on the loop (e.g., $x \to y \to z$ or $z \to y \to x$). The points x, y, and z lie behind the plane of the paper.



Sol. **Location I:** As the loop swings downward, the normal to the loop makes a smaller angle with the applied field. Hence, the flux through the loop is increasing. The induced magnetic field must point generally to the left to counteract this increase. The induced current flows $\boxed{x \to y \to z}$

Location II: The angle between the normal to the loop and the applied field is now increasing, so the flux through the loop is decreasing. The induced field must now be generally to the right, and the current flows $\boxed{z \to y \to x}$

ILLUSTRATION 4.17

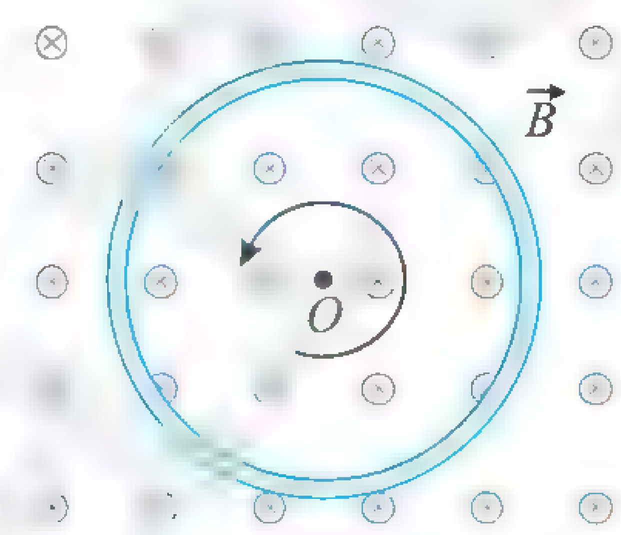
A circular loop of radius a having n turns is kept in a horizontal plane. A uniform magnetic field B exists in a vertical direction as shown in figure. Find the emf induced in the loop if the loop is rotated with a uniform angular velocity ω about

(a) an axis passing through the center and perpendicular to the plane of the loop.

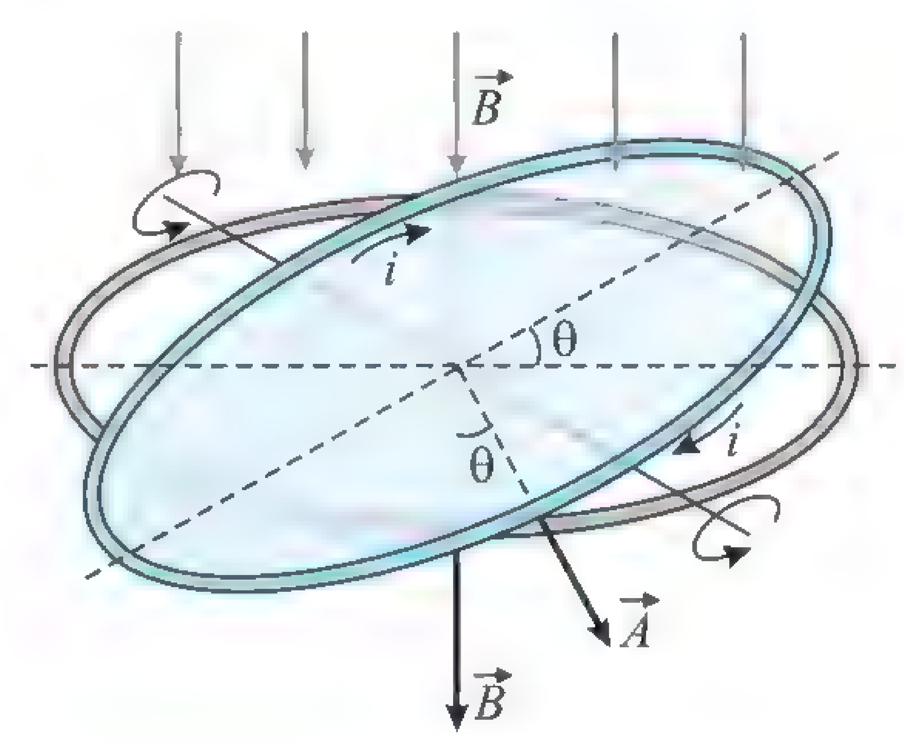
(b) the diameter.

Sol.

(a) The emf induces when there is change of flux. As in this case (figure), there is no change of flux, hence no emf will be induced in the coil.



(b) If the loop is rotated about the diameter there will be change of flux with time. In this case, emf, will be induced in the coil. The area of the loop is $A = \pi a^2$. If the normal of the loop makes an angle $\theta = 0$ with the magnetic field at $t = 0$, this angle will become $\theta = \omega t$ at time t. The flux of the magnetic field at this time is



$$\phi = nB\pi a^2 \cos\theta = nB\pi a^2 \cos\omega t$$

The induced emf is $\varepsilon = -\dfrac{d\phi}{dt} = \pi n a^2 B\omega \sin\omega t$.

ε is coming out to be positive, so the direction of induced emf will be as shown in figure. Because here this sense is positive if we look at the direction of area vector.

ILLUSTRATION 4.18

Figure (a) shows two circular rings of radii a and b ($a > b$) joined together with wires of negligible resistance. Figure (b) shows the pattern obtained by folding the small loop in the plane of the large loop.

(a) **(b)** **(c)**

The pattern shown in Fig. (c) is obtained by twisting the small loop of Fig. (a) through 180°.

All the three arrangements are placed in a uniform time varying magnetic field $dB/dt = k$, perpendicular to the plane of the loops. If the resistance per unit length of the wire is λ, then determine the induced current in each case.

Sol.

(a) $\phi_B = \pi(a^2 + b^2)B$

$$|\varepsilon| = \frac{d\phi_B}{dt} = \pi(a^2 + b^2) \times \frac{dB}{dt} = \pi(a^2 + b^2)k$$

$$\text{Induced current, } I = \frac{|\varepsilon|}{R} = \frac{\pi(a^2 + b^2)k}{\lambda[2\pi(a+b)]} = \frac{k(a^2 + b^2)}{2\lambda(a+b)}$$

(b) $\phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$

$$|\varepsilon| = \frac{d\phi_B}{dt} = \frac{\pi dB}{dt} \times (a^2 - b^2) = \pi k(a^2 - b^2)$$

$$\text{Induced current, } I = \frac{|\varepsilon|}{R} = \frac{\pi k(a^2 - b^2)}{2\pi\lambda(a+b)} = \frac{k(a-b)}{2\lambda}$$

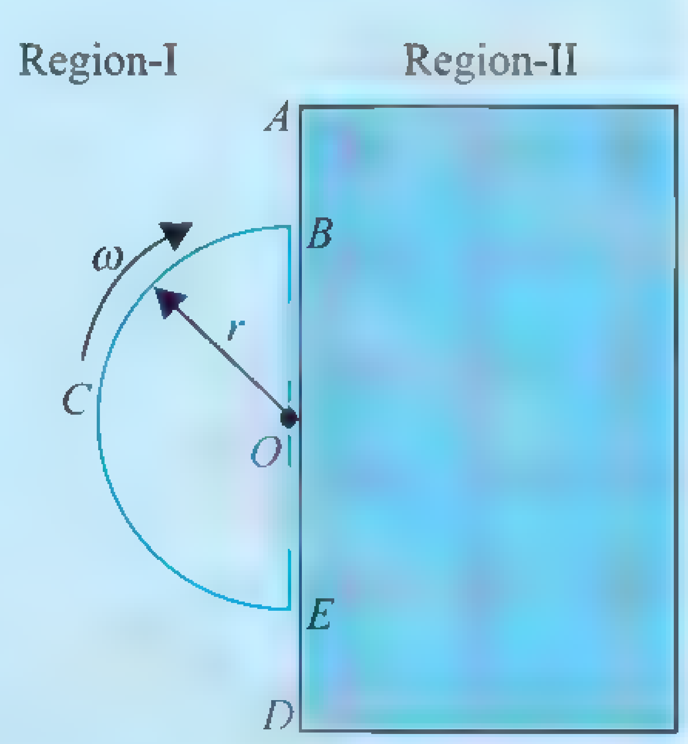
(c) Here induced emf in both loops will oppose each other, hence effective flux is given by

$$\Phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$$

$$\Rightarrow \quad |\varepsilon| = \frac{d\Phi_B}{dt} = \pi k(a^2 - b^2)$$

$$\text{Induced current, } I = \frac{|\varepsilon|}{R} = \frac{k(a-b)}{2\lambda}$$

ILLUSTRATION 4.19



Space is divided by the line AD into two regions. Region I is field free and region II has a uniform magnetic field B directed into the plane of the paper. BCE is a semicircular conducting loop of radius r with center at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R.

(a) Obtain an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into region II.

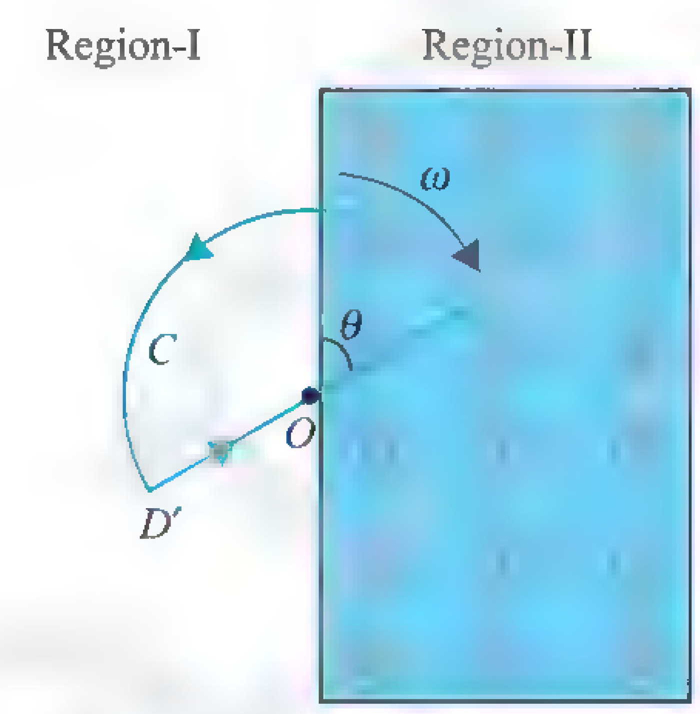
(c) Plot a graph between the induced emf and the time of rotation for the two periods of rotation.

Sol.

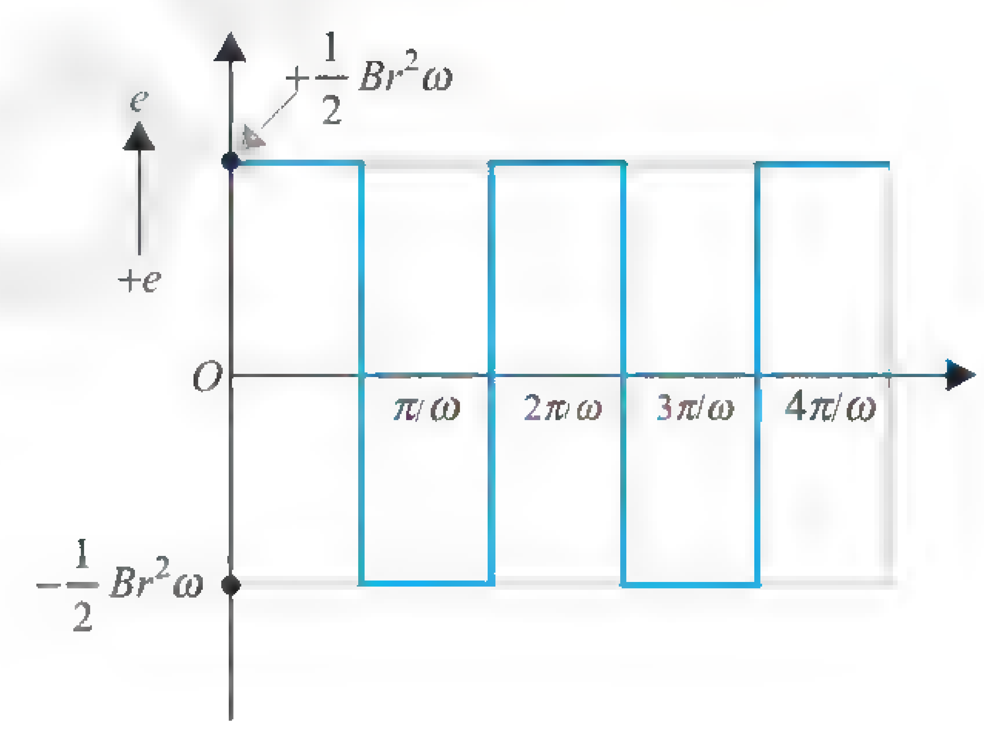
(a) When the loop is rotated about an axis passing through center O and perpendicular to the plane of the paper, the angle between magnetic field vector $\vec{B}$ and area $\vec{A}$ is always 0°. When the loop is in region I, the magnetic flux linked with loop = BA cos 0 = 0 the (since B = 0 in region I).

When the loop enters the magnetic field in region II, the magnetic flux linked with it is given by $\phi = BA$ where $A = \frac{1}{2}r^2\theta$. Therefore, emf induced

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = \frac{-Br^2}{2}\frac{d\theta}{dt} = -\frac{Br^2}{2}\omega$$



(a)



(b)

As resistance of the loop is R, the current induced is given by

$$i = \frac{e}{R} = \frac{1}{2}\frac{Br^2\omega}{R}$$

This is the required expression for current induced in the loop.

(b) According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. So, when entering into region II the field produced by the current induced must be upward. For this, the current in the loop must be anticlockwise as shown in Fig. (a).

(c) When the loop enters the magnetic field, the magnetic flux linked with it increases and the emf $e = \frac{1}{2}Br^2\omega$ is induced in one direction. When the loop comes out of the field, the flux decreases and emf is induced in opposite sense. The graph for representing the emf induced versus time for two periods ($T = 2\pi/\omega$) is shown in Fig. (b). Here we have taken anticlockwise direction as positive.

ILLUSTRATION 4.20

A current $i = 2.5(1+2t)\times10^3$ A increases at a steady rate in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and its center is placed at a distance of 1m from the wire. The resistance of the loop is $5\pi\times10^{-4}\ \Omega$. Find the magnitude and the direction of the induced current in the loop.

Sol. The field due to the wire at the center of loop is

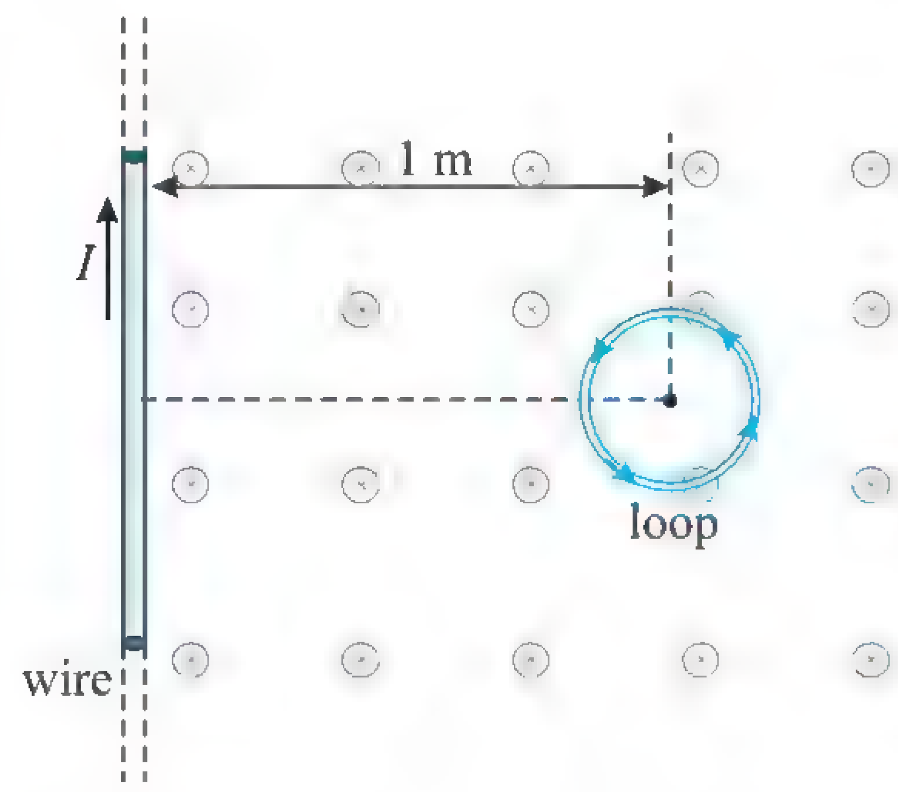
$$B = \frac{\mu_0 I}{2\pi d} = \frac{4\pi \times 10^{-7} \times I}{2\pi \times 1} = 2I \times 10^{-7}\ \text{W/m}^2$$

So the flux linked with the loop wire

$$\phi = BA = B \times \pi r^2 = 2I \times 10^{-7} \times \pi \times (10^{-3})^2 = 2\pi I \times 10^{-13}\ \text{W}$$

The magnitude of emf induced in the loop due to change of current

$$|e| = \frac{d\phi}{dt} = 2\pi \times 10^{-13} \frac{dI}{dt}\ \text{V} \qquad \text{...(i)}$$



$$\because\ I = 2.5(1+2t) \times 10^3 \quad \therefore\ \frac{dI}{dt} = 5.0 \times 10^3\ \text{A/s} \qquad \text{...(ii)}$$

From (i) and (ii), we get $e = 2\pi \times 10^{-13} \times 5.0 \times 10^3 = 10\pi \times 10^{-10}$ V

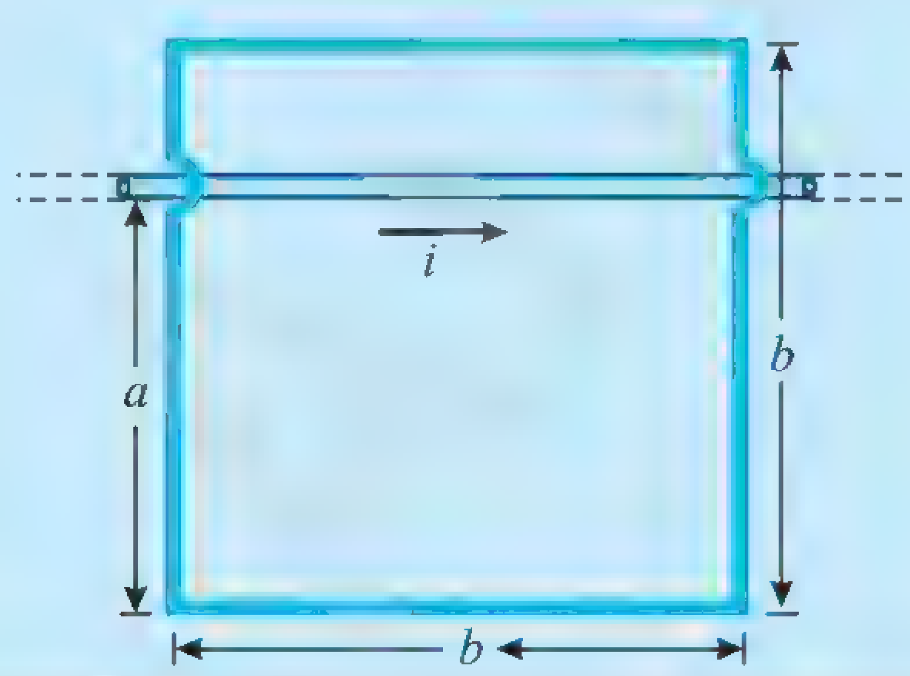
And the induced current in the loop

$$i = \frac{e}{R} = \frac{10\pi \times 10^{-10}}{5\pi \times 10^{-4}} = 2.0 \times 10^{-6}\ \text{A}$$

Due to increase in current in the wire the flux linked with the loop will increase, so in accordance with Lenz's law the direction of current induced in the loop will be inverse of that in wire, i.e., anticlockwise.

ILLUSTRATION 4.21

A wire loop with a = 10.0 cm and b = 15.0 cm and a long straight wire is arranged as shown in figure. The current in the wire is changing according with a relation $i = 5.0t^2 - 10.0t$, where i is in amperes and t is in seconds. **(a)** Find the emf in the square loop at t = 4.0 s. **(b)** What is the direction of the induced current in the loop?



Sol.

(a) First, we observe that a large portion of the figure contributes flux that "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b - a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b - a$ away from it) has exactly the equal but opposite flux that cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B\,dA = \int_{b-a}^{a} \left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right).$$

According to Faraday's law, the e.m.f induced

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right)\right]$$

$$= -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a}\right)\frac{di}{dt}$$

$$= -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a}\right)\frac{d}{dt}(5t^2 - 10t)$$

$$= \frac{-\mu_0 b(10t - 10)}{2\pi} \ln\left(\frac{a}{b-a}\right).$$

With a = 0.10 m and b = 0.15 m, then at t = 4.0 s, the magnitude of the emf induced in the rectangular loop is

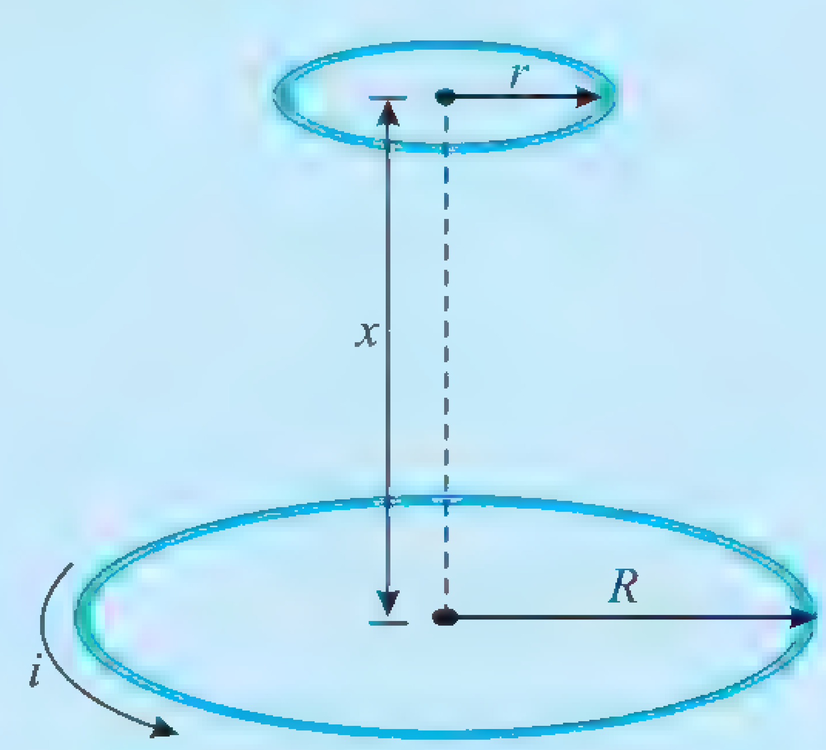
$$|\varepsilon| = \frac{(4\pi \times 10^{-7})(0.15)(10(4) - 10)}{2\pi} \ln\left(\frac{0.10}{0.15 - 0.10}\right)$$

$$= 4.16 \times 10^{-7}\ \text{V}.$$

(b) We note that $di/dt > 0$ at $t = 2.0$ s. From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

ILLUSTRATION 4.22

Two circular loops are placed with common vertical axis one above other as shown in figure. The smaller loop (radius r) is above the larger loop (radius R) by a distance $x >> R$. Consequently, the magnetic field due to the counterclockwise current i in the larger loop is nearly uniform throughout the smaller loop. Suppose that x is increasing at the constant rate $dx/dt = v$. Find the induced emf and the direction of the induced current in the smaller loop,



Sol. Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis.

The magnetic field at the centre of smaller loop, $\vec{B} = \frac{\mu_0 i R^2}{2x^3}\hat{i}$,

where the $+x$ direction is upward. The area of the smaller loop is $A = \pi r^2$.

The magnetic flux through the smaller loop is,

$$\Phi_B = BA = \frac{\pi \mu_0 i r^2 R^2}{2x^3}.$$

The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi \mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right).$$

$$= -\left(\frac{\pi \mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi \mu_0 i r^2 R^2 v}{2x^4}.$$

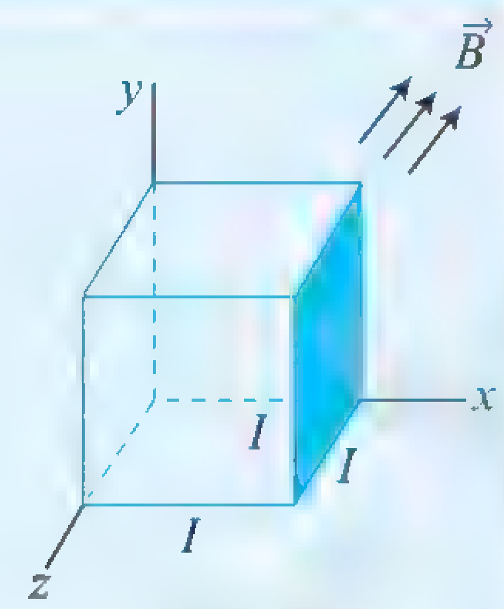
As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

Concept Application Exercise 4.1

1. A cube of edge length $l = 2.50$ cm is positioned as shown in figure. A uniform magnetic field given by

$\vec{B} = (5.00\hat{i} + 4.00\hat{j} + 3.00\hat{k})$ T

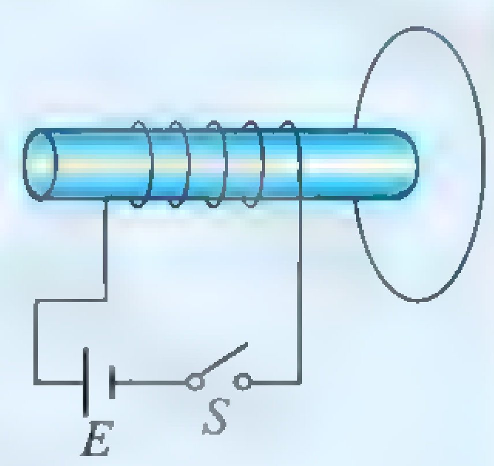
exists throughout the region.



(a) Calculate the flux through the shaded face.

(b) What is the total flux through the six faces?

2. A conducting ring is placed near a solenoid as shown in figure. Find the direction of the induced current in the ring.

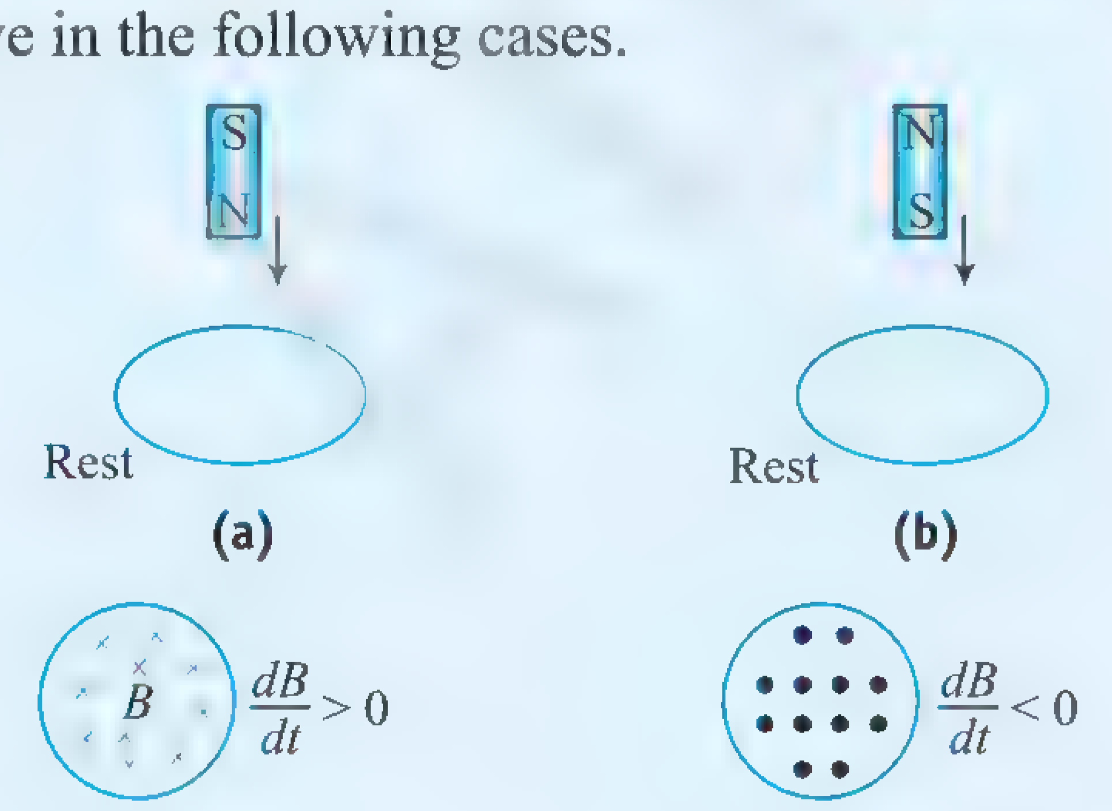


(a) At the instant the switch in the circuit containing the solenoid is closed.

(b) After the switch has been closed for a long time.

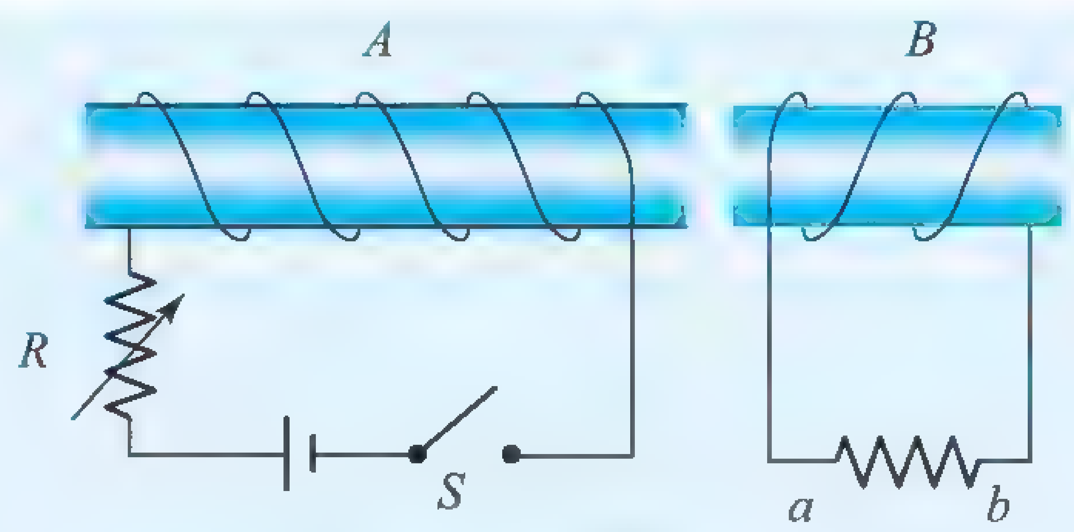
(c) At the instant the switch is opened.

3. Identify the direction of induced current as seen from the above in the following cases.



(a) **(b)** **(c)** **(d)**

4. Using Lenz's law, determine the direction of the current in resistor ab in figure when

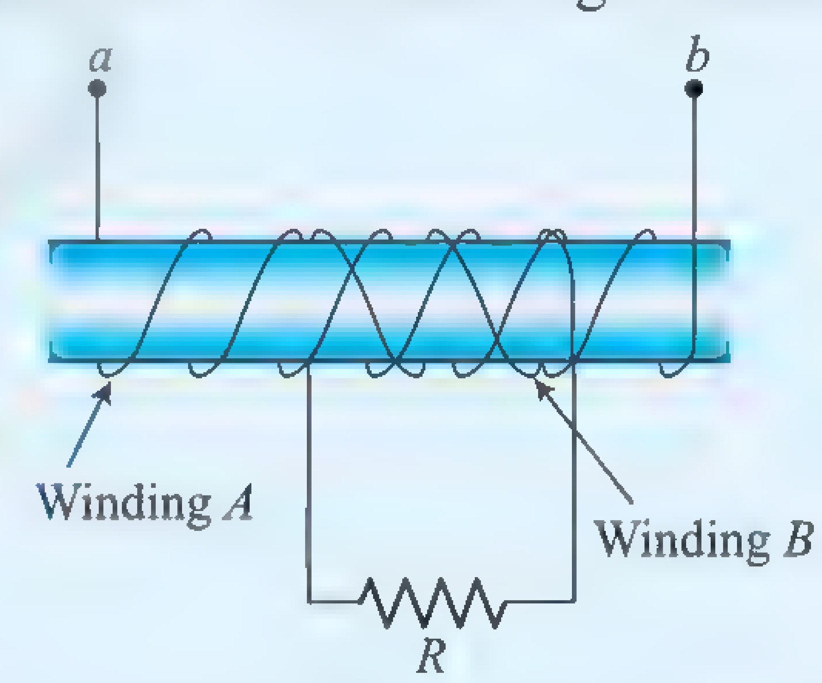


(a) switch S is opened after having been closed for several minutes.

(b) coil B is brought closer to coil A with the switch closed.

(c) the resistance of R is decreased while the switch remains closed.

5. A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions as shown in figure. Terminals a and b of winding A may be connected to a battery through a reversing switch. State whether the induced current in the resistor R is from left to right or from right to left in the following circumstances.

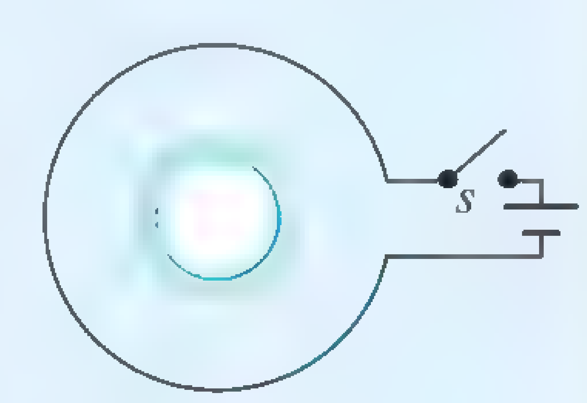


(a) The current in winding A is from a to b and is increasing.

(b) The current in winding A is from b to a and is decreasing.

(c) The current in winding A is from b to a and is increasing.

6. A small, circular ring is inside a larger loop that is connected to a battery and a switch as shown in figure. Use Lenz's law to find the direction of the current induced in the small ring

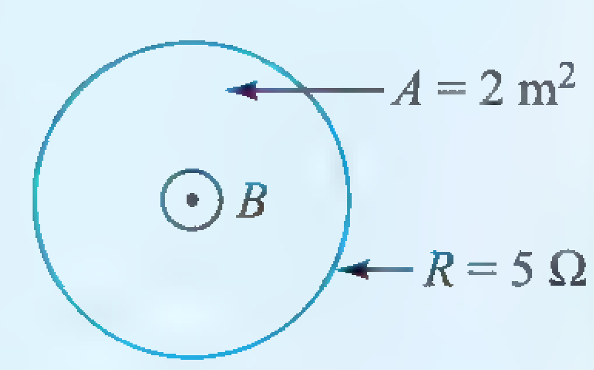


(a) just after switch S is closed;

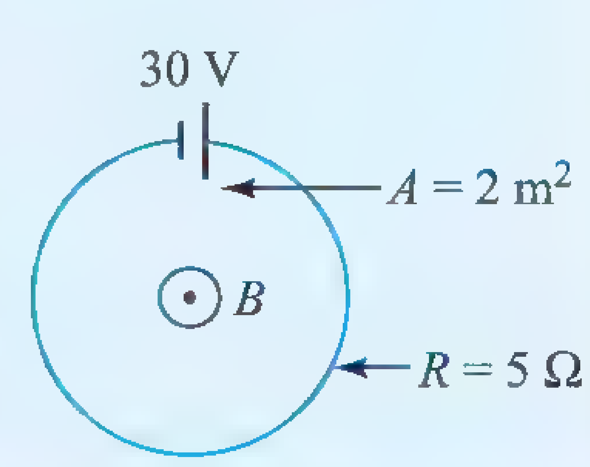
(b) after S has been closed for a long time;

(c) just after S has been reopened after being closed for a long time.

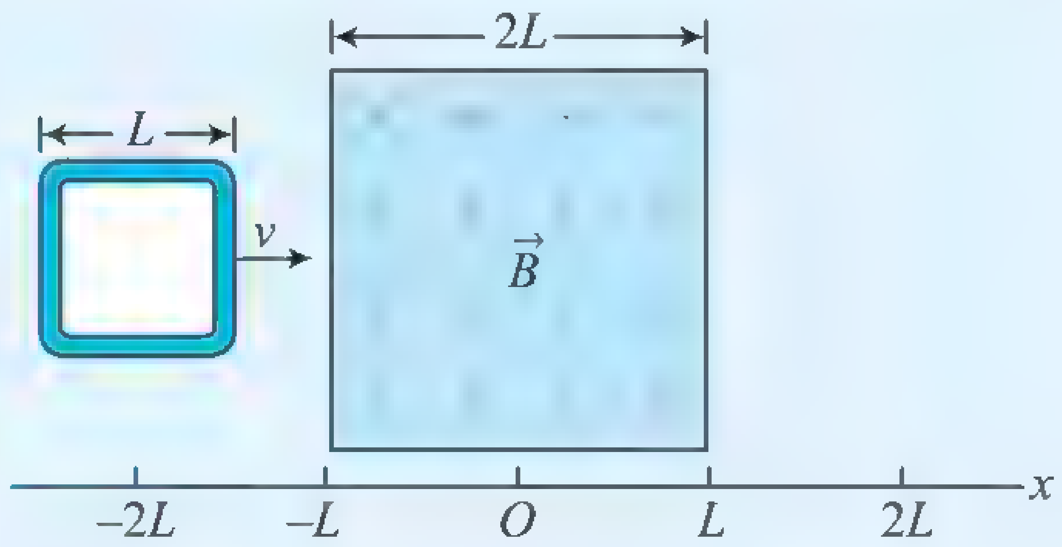
7. Figure shows a coil placed in a decreasing magnetic field applied perpendicular to the plane of the coil. The magnetic field is decreasing at a rate of 10 T s^{-1}. Find out current in magnitude and direction of current.



8. Figure shows a coil placed in a magnetic field decreasing at a rate of 10 T s^{-1}. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.



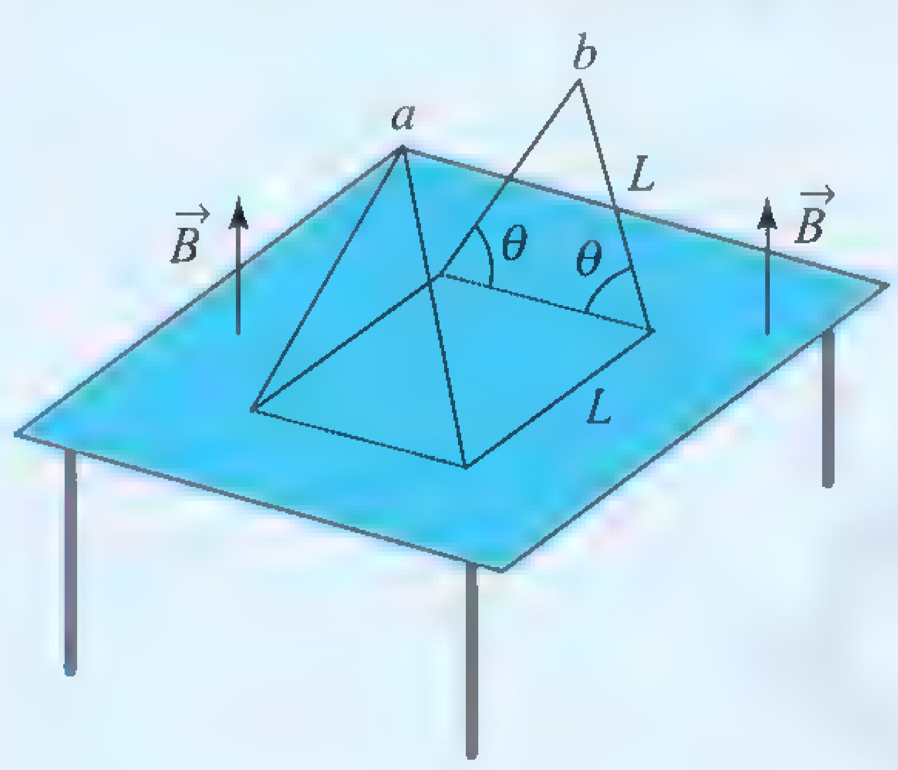
9. A square loop of wire with resistance R is moved at a constant speed v across a uniform magnetic field confined to a square region whose sides are twice the length of those of the square loop (as shown in figure).



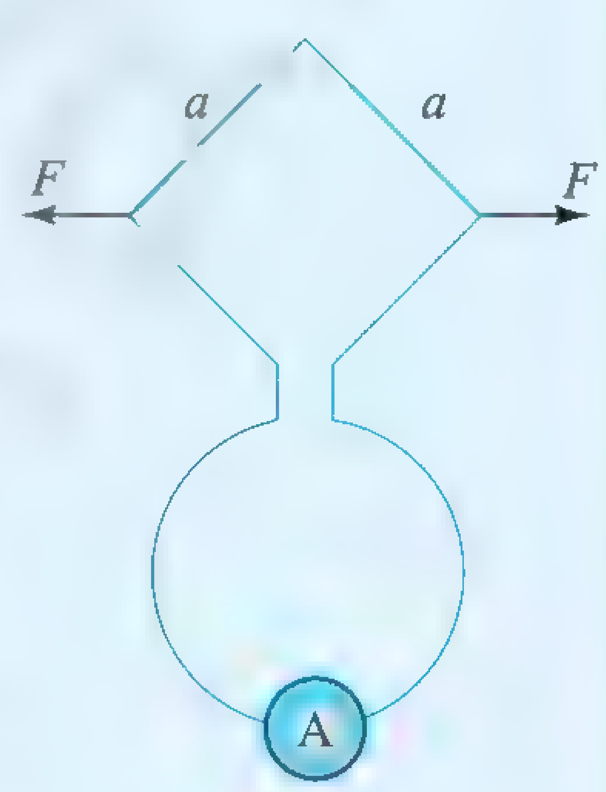
(a) Graph the external force F needed to move the loop at a constant speed as a function of the coordinate x from $x = -2L$ to $x = +2L$. (The coordinate x is measured from the center of the magnetic field region to the center of the loop. It is negative when the center of the loop is to the left of the center of the magnetic field region. Take positive force to be to the right.)

(b) Graph the induced current in the loop as a function of x. Take counterclockwise currents to be positive.

10. The wire shown in figure is bent in the shape of a tent, with $\theta = 60.0°$ and $L = 1.50$ m, and is placed in a uniform magnetic field of magnitude 0.300 T perpendicular to the tabletop. The wire is rigid but hinged at points a and b. If the tent is flattened out on the table in 0.100 s, what is the average induced emf in the wire during this time?



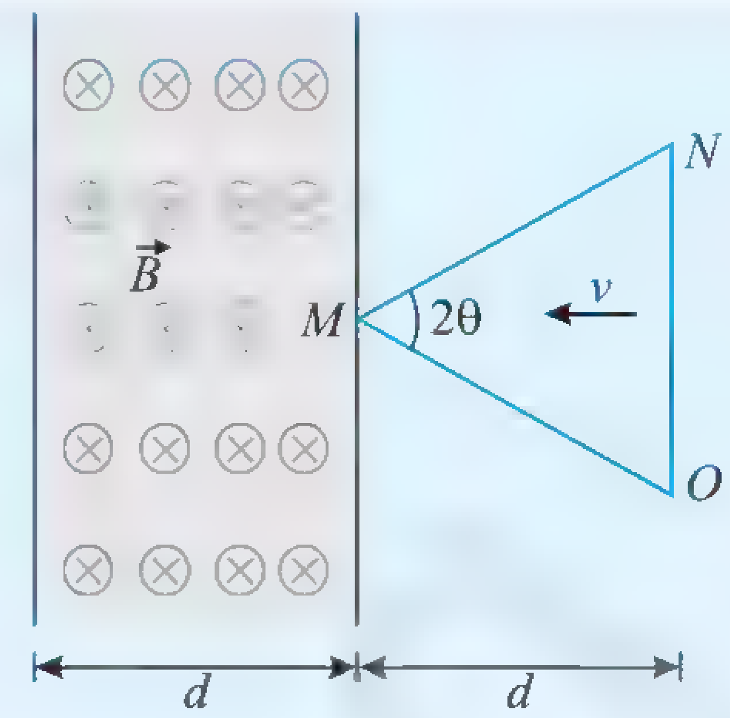
11. The plane of a square loop of wire with edge length $a = 0.200$ m is perpendicular to the Earth's magnetic field at a point where $B = 15.0\ \mu$T, as shown in figure. The total resistance of the loop and the wires connecting it to a sensitive ammeter is 0.50 Ω. If the loop is suddenly collapsed by horizontal forces as shown, what is the total charge passing through the ammeter?



12. A triangular conducting loop having resistance R is placed outside an uniform inward magnetic field B. At $t = 0$,the loop starts moving into the magnetic field with a uniform speed v. Find the

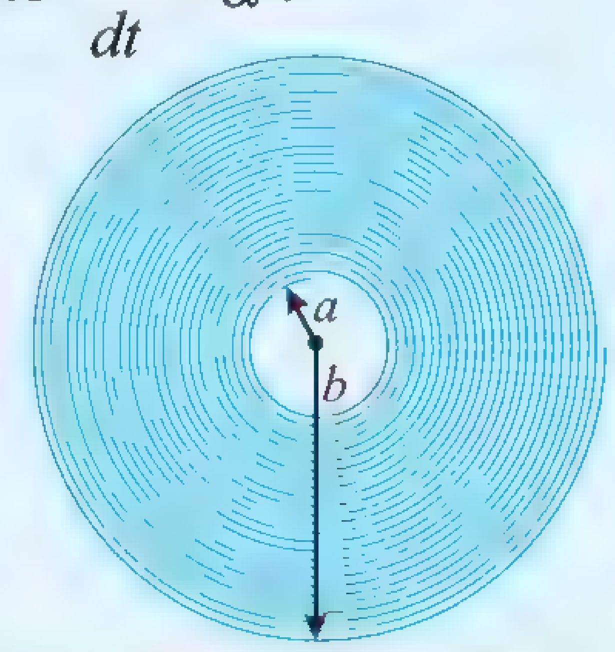
(a) magnitude of induced emf ε at any time t.

(b) the magnitude and direction of induced current at any time t.



13. A circular stretchable conducting ring is kept in a uniform magnetic field (B) that is perpendicular to the plane of the ring. The ring is pulled out uniformly from all sides so as to increase its radius at a constant rate $\frac{dr}{dt} = n$ while maintaining its circular shape. Calculate the rate of work done by the external agent against the magnetic force when the radius of the ring is r_0. Resistance of the ring remains constant at R.

14. A flat spiral coil has a large number of turns N. The turns are wound tightly and the inner and outer radii of the coil are a and b respectively. A uniform external magnetic field (B) is applied perpendicular to the plane of the coil. Find the emf induced in the coil when the field is made to change at a rate $\frac{dB}{dt} = \alpha$.



Answers

1. (a) 3.125×10^{-3} Wb (b) 0
2. (a) anticlockwise (b) no change (c) clockwise
3. (a) anticlockwise (b) clockwise (c) anticlockwise (d) anticlockwise
4. (a) to the right (b) to the left (c) to the left
5. (a) right to left (b) right to left (c) left to right
6. (a) clockwise (b) no change (c) counter-clockwise
7. 4A, anticlockwise **8.** 2A, clockwise **10.** 5.84 V
11. 1.2 μC **12.** (a) $2(Bv^2 \tan\theta)t$

(b) $\frac{2Bv^2 \tan\theta}{R}t$ (counter clockwise sense)

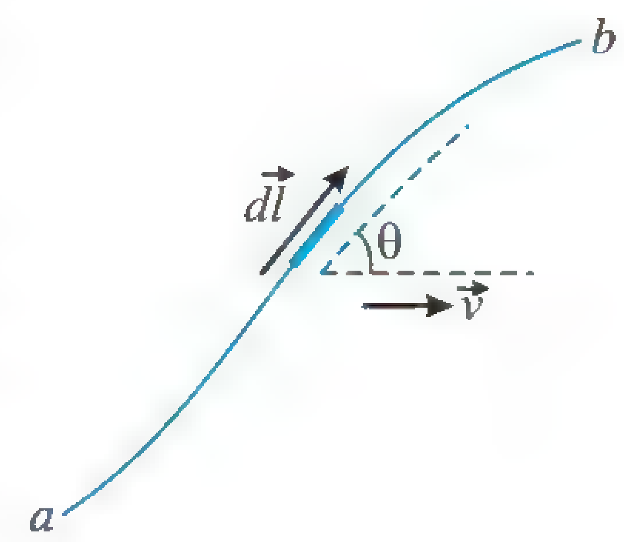
13. $\frac{4\pi^2 B^2 r^2 n^2}{R}$ **14.** $\frac{\pi N\alpha}{3}(a^2 + b^2 + ab)$

MOTIONAL ELECTROMOTIVE FORCE

When a conductor moves in a magnetic field, then charges inside conductor experience magnetic force given by $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{v}$ is the velocity of conductor (here we assume that

velocity of charges inside conductor is same as the velocity of conductor and random motion of charges inside the conductor is neglected). Due to this, force charges start moving in a particular direction in the conductor or we say that an emf is induced in the conductor. This emf is known as motional emf.

Consider a conductor of arbitrary shape as shown in figure moving in some magnetic field $\vec{B}$.



The different parts of the conductor may have different velocity. Consider an element $\vec{dl}$ having velocity $\vec{v}$.

Force due to magnetic field on a charge in conductor.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Work done by this force on charge q in passing through $\vec{dl}$

$$dW = \vec{F} \cdot \vec{dl} = q(\vec{v} \times \vec{B}) \cdot \vec{dl}$$

So emf induced within $\vec{dl}$: $\quad d\varepsilon = \dfrac{dW}{q} = (\vec{v} \times \vec{B}) \cdot \vec{dl}$

Integrate this to find net emf: $\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot \vec{dl}$

This emf is directed from a to b.

Other Approach

Due to the magnetic force, charges start moving in the conductor. An electric field is set up in the conductor. In equilibrium force of electric field balances the force of magnetic field. Let electric electric field induced be $\vec{E}$, then

$$q\vec{v} \times \vec{B} + q\vec{E} = 0 \quad \Rightarrow \quad \vec{E} = -\vec{v} \times \vec{B}$$

Now potential difference developed across the ends of the conductor: $V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dl} = \int_a^b (\vec{v} \times \vec{B}) \cdot \vec{dl}$

If circuit is incomplete, then this terminal potential difference will be equal to emf induced. Hence emf induced:

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

Note: We can also write $\varepsilon = \int_a^b \vec{B} \cdot (\vec{dl} \times \vec{v})$

As $\vec{dl} \times \vec{v}$ is the area swept per unit time by length $\vec{dl}$ and hence $\vec{B} \cdot (\vec{dl} \times \vec{v})$ is the flux of induction through the area. Therefore, the motional emf is equal to the flux of induction cut by the conductor per unit time.

We can write the expression for induced emf in various forms:

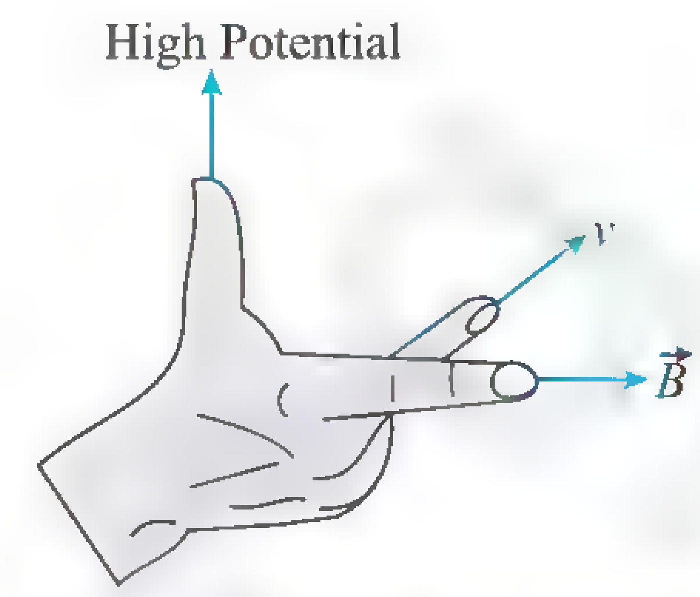
$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot \vec{dl} = \int_a^b (\vec{B} \times \vec{dl}) \cdot \vec{v} = \int_a^b (\vec{dl} \times \vec{v}) \cdot \vec{B}$$

if any two out of $\vec{v}$, $\vec{B}$ and $\vec{dl}$ become parallel or antiparallel, ε will become zero.

Finding Direction of Induced EMF

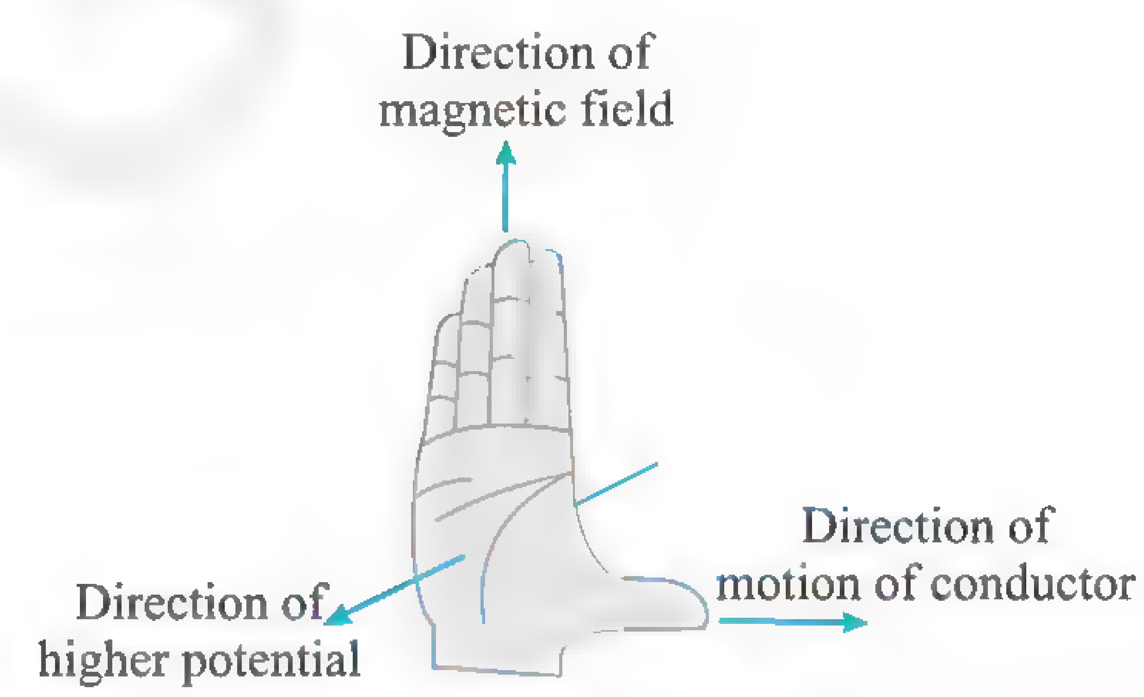
We can find the direction of induced EMF by Flemings Right Hand Rule as given below.

"If we stretch the index finger, middle finger and thumb of right hand in mutually perpendicular directions as shown in figure with index finger pointing toward the direction of magnetic induction and thumb along the velocity of conductor then middle finger will point toward the high potential end of the conductor."



The direction of induced EMF can also be find out by Right Hand Palm Rule.

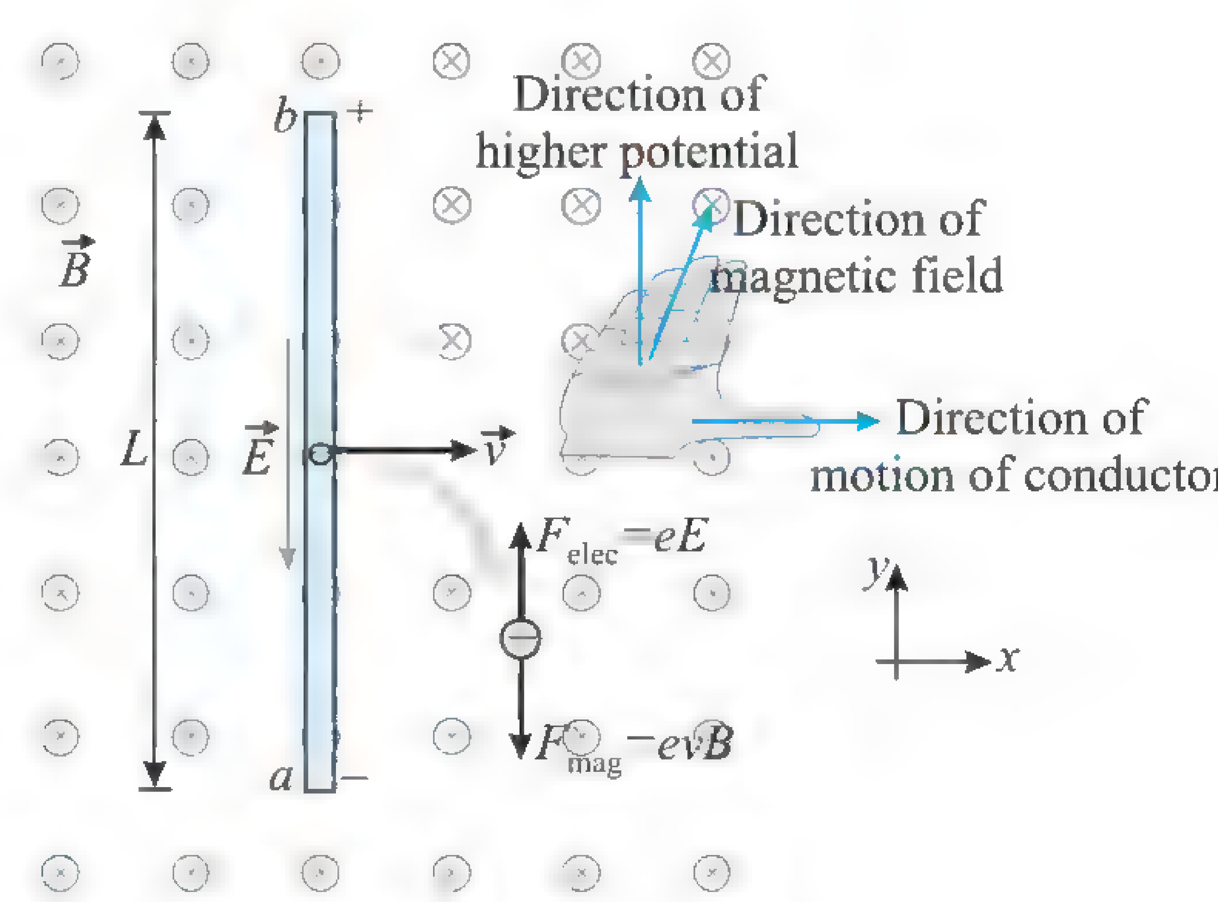
"If we stretch the fingers of right hand towards magnetic field and thumb along the velocity of conductor then the high potential end of the conductor is directed perpendicular to the right-hand palm."



Special Cases

A Straight Conductor Moving in a Magnetic Field

Consider a straight conductor of length L is moving in a magnetic field B with velocity v. Here all three L, B, and v are mutually perpendicular.



We have $\vec{v} = v\hat{i}$, $\vec{B} = -B\hat{k}$, $\vec{dl} = dl\,\hat{j}$

Induced emf in the rod is given by

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot \vec{dl} = \int_a^b [v\hat{i} \times (-B\hat{k})] \cdot dl\,\hat{j}$$

$$= \int_a^b (Bv\hat{j})] \cdot dl\,\hat{j} = Bv\int_a^b dl = BVL \quad \Rightarrow \quad \varepsilon = BVL$$

Here end b will be positive w.r.t. end a.

Other Approach

Consider an electron inside the conductor. Magnetic force $F_b = evB$ on the electron will be toward a. So electrons will start accumulating near end a and end b will become positive w.r.t. end a. This will create an electric field E from b to a. Thus E will apply electric force $F_e = eE$ on electrons opposite to F_b.

In equilibrium: $F_b = F_e$

$$\Rightarrow \quad evB = eE \quad \Rightarrow \quad E = vB$$

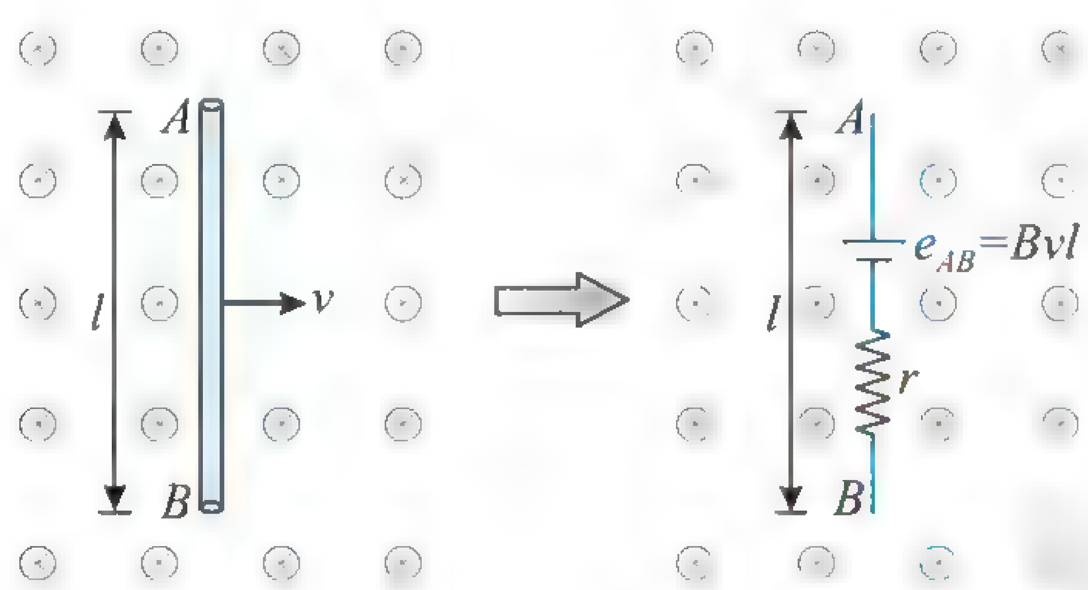
Potential difference between b and a: $V_b - V_a = EL = vBL$

If ends a and b are not connected in external circuit, then this terminal potential diffrence will be equal to emf induced. Hence

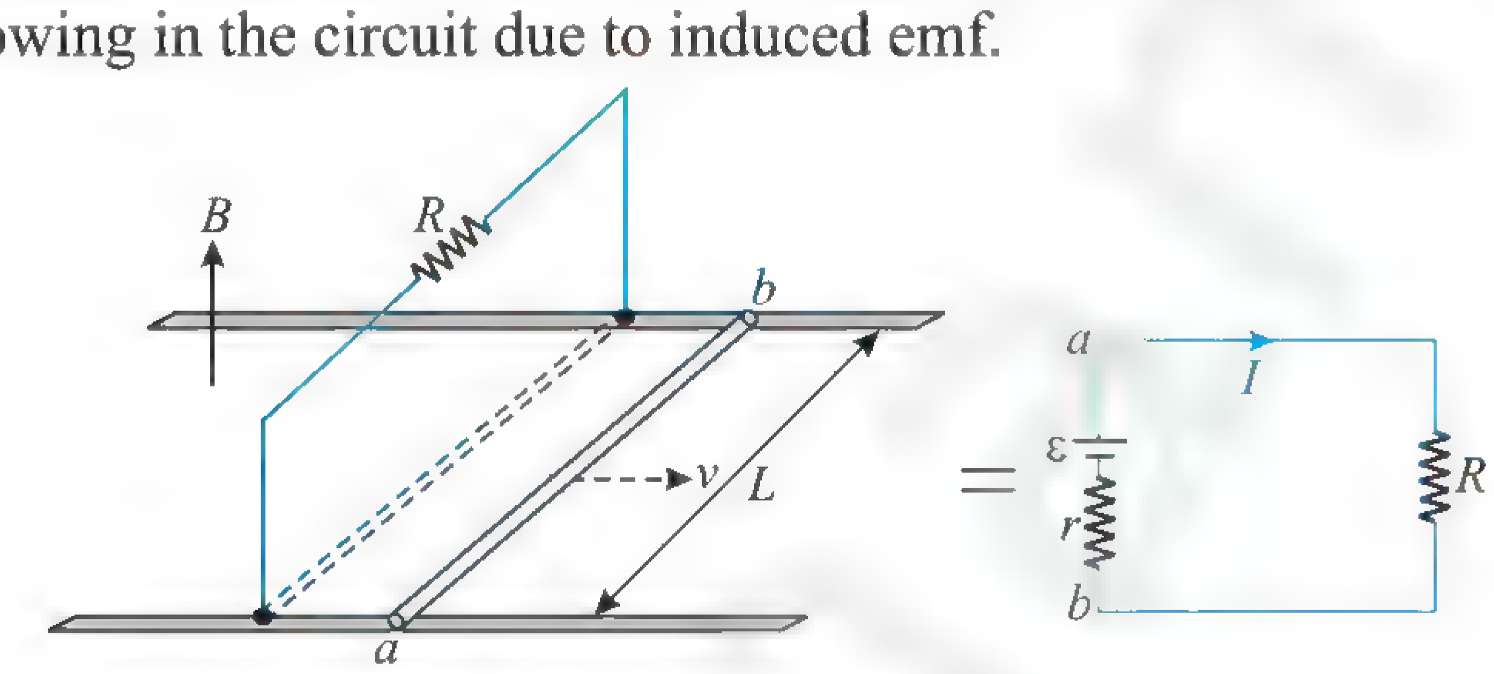
$$\varepsilon = BvL$$

Motional EMF as an Equivalent Battery

When a conductor moves in magnetic field, the EMF may develop across its ends. The conductor can be considered like an equivalent battery with internal resistance equal to the resistance of the conductor which can supply current. In the figure a conducting rod of length l, resistance r moving at a velocity v in a uniform magnetic field B .In this case its length (l), velocity (v) and magnetic field (B) are perpendicular to each other and it can be replaced by an equivalent battery of EMF Bvl as shown in figure.



Now what will happen if ends a and b are connected externally through resistance R as shown in Fig. (a). A current will start flowing in the circuit due to induced emf.

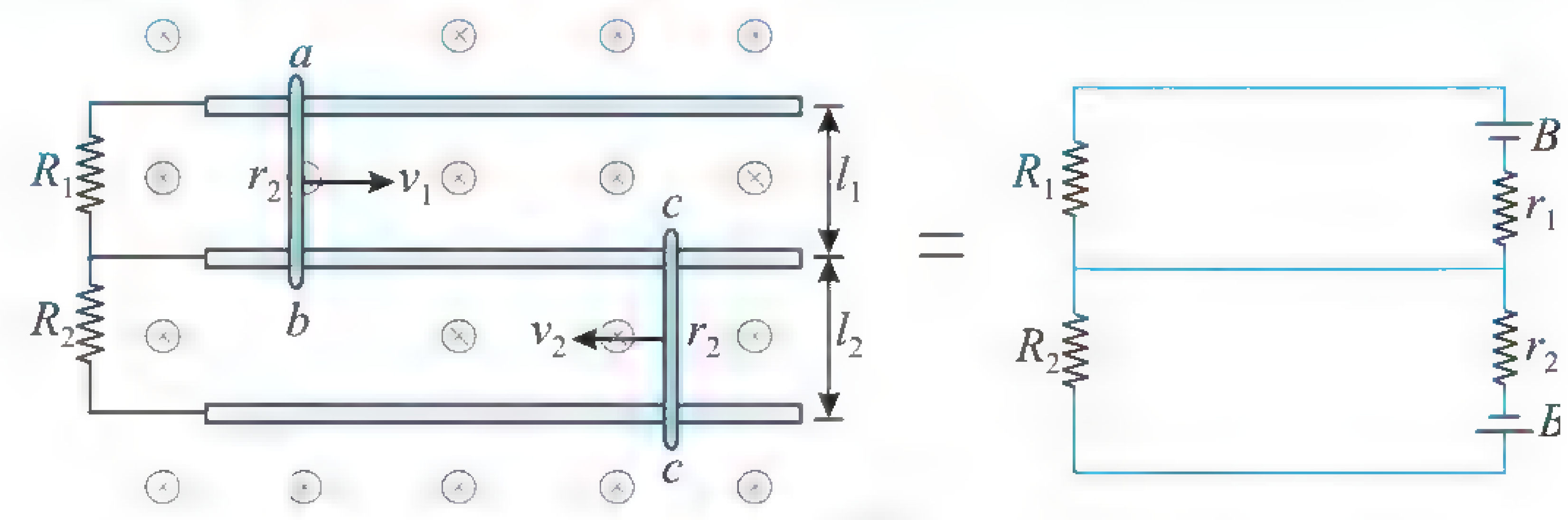


The equivalent circuit is shown in right Fig. (b). The rod will act as a battery of emf $\varepsilon = BvL$ and r is the resistance of the rod which will act as internal resistance of the battery. Current in the circuit is given by: $I = \dfrac{E}{R+r}$

pd between ends a and b:

$$V_b - V_a = \varepsilon - Ir = \varepsilon - \frac{\varepsilon r}{R+r} = \frac{\varepsilon R}{R+r}$$

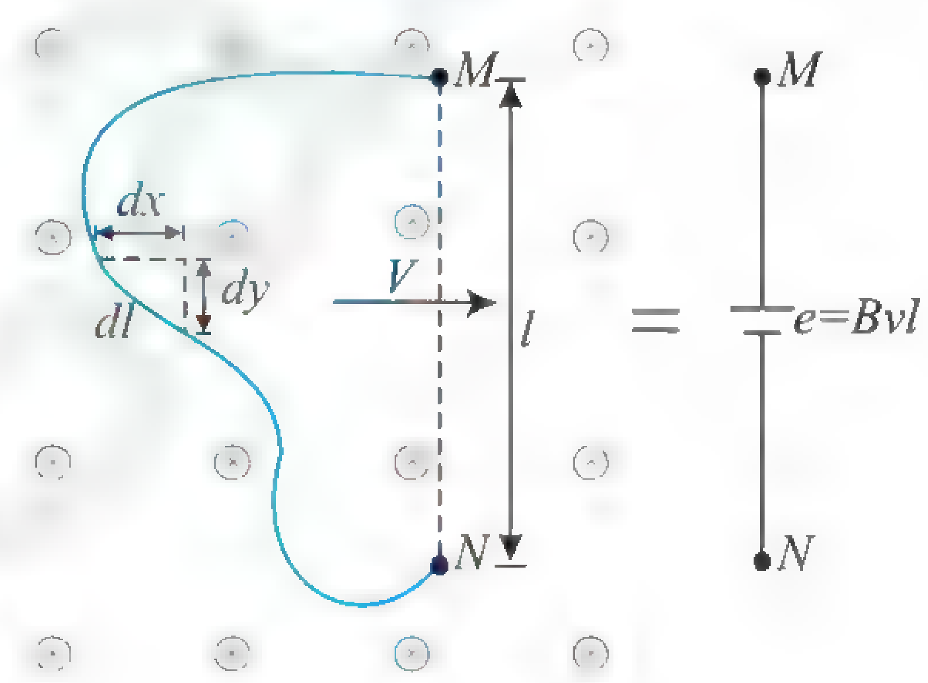
Let us consider one more situation, shown in Fig. (a) in which two sliding wires ab and cd are sliding on three conducting rails which are connected to some resistances at their ends as shown. In this case we can determine the currents through resistances and different sections of the setup by making an equivalent circuit of this setup as shown in Fig. (b).



(a) (b)

Motional EMF in a Random Shaped Wire Moving in Magnetic Field

Consider a random shaped conducting wire MN in which the distance between its ends M and N is l is moving with a velocity v perpendicular to line MN in a uniform magnetic field B as shown in figure.



Let us consider a small element dl in the wire as shown in figure which will have its two components, one along the line MN and other perpendicular to the line MN. There will not be no induced EMF in component dx, as this component is parallel to the velocity vector $\vec{v}$ .The EMF will be induced only in the component of dy, thus EMF induced in this component

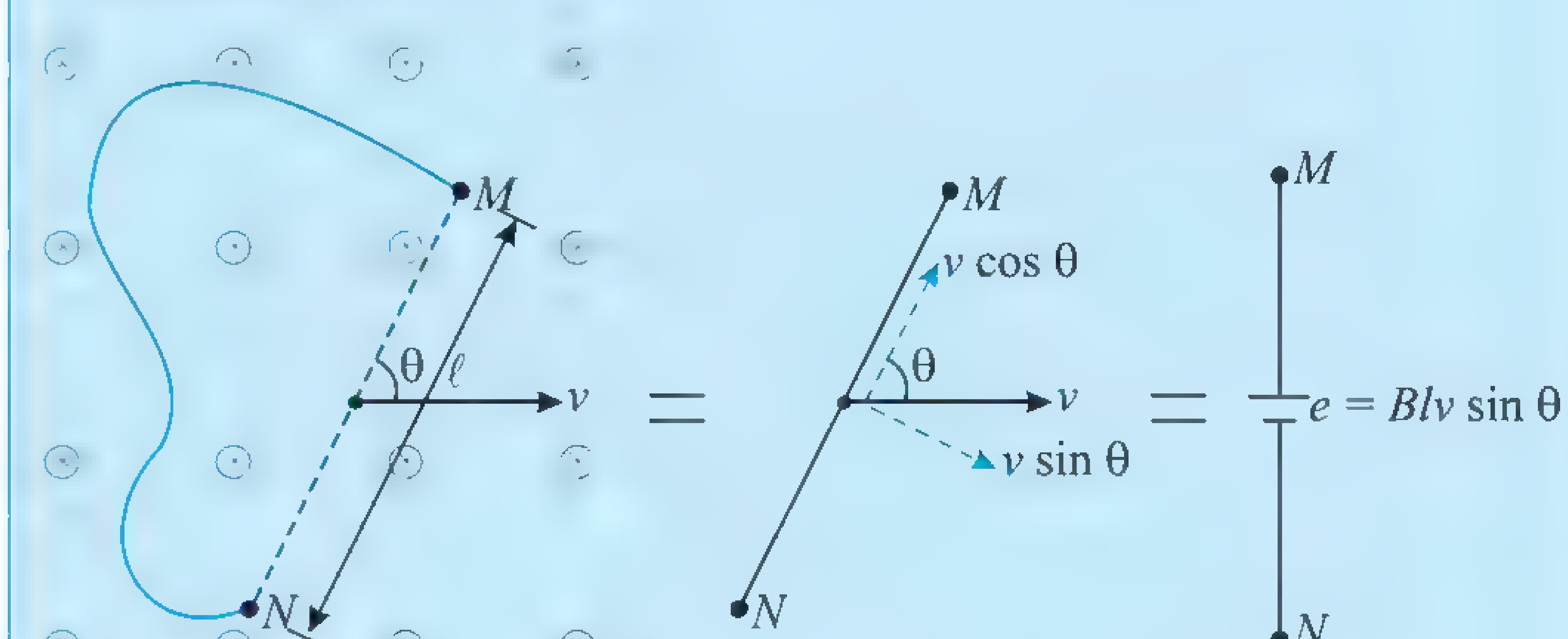
$$de = Bvdy \qquad \text{...(i)}$$

Hence total EMF induced between ends MN of the wire is given as

$$e_{MN} = \int de = Bv \int dy = Bvl$$

Important Point:

We can find the EMF induced in case of different random shaped wires are moving in different directions by using the component of velocity perpendicular to the line MN as shown in given figures.



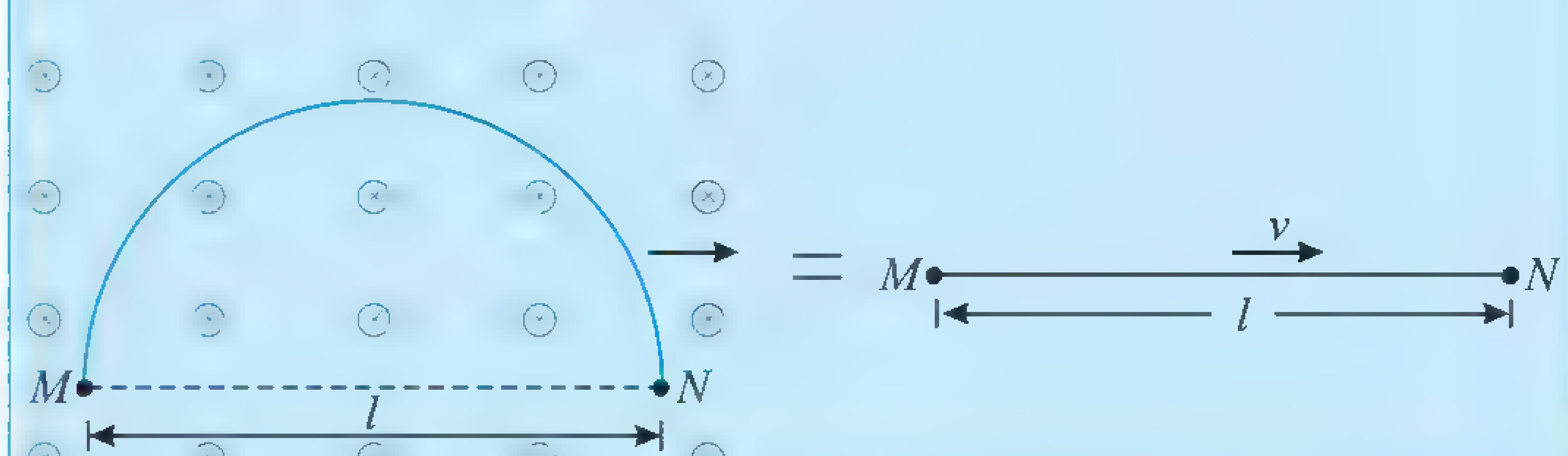
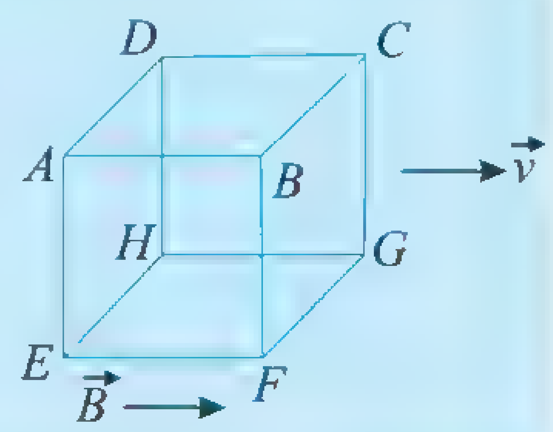


ILLUSTRATION 4.23

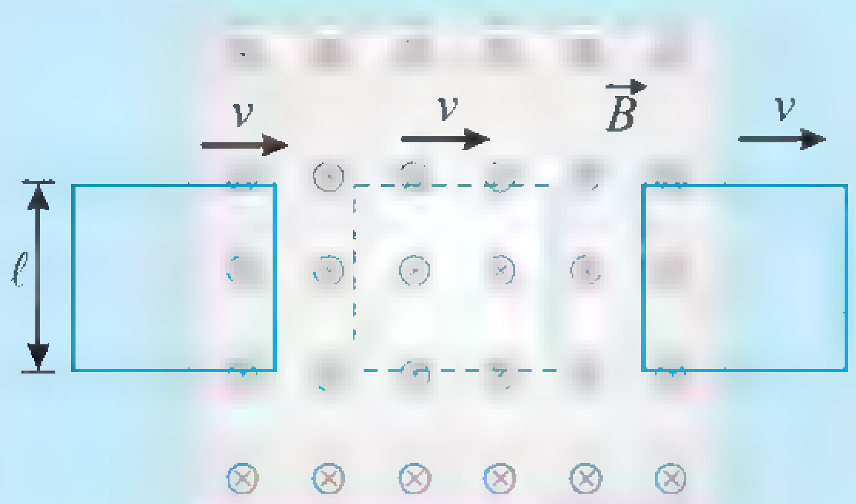
Twelve wires of equal lengths are connected in the form of a skeleton cube which is moving with a velocity $\vec{v}$ in the direction of magnetic field $\vec{B}$. Find the e.m.f. in each arm of cube.



Sol. No e.m.f. is induced in any arm because $\vec{v}$ is parallel to $\vec{B}$. ($\because$ $e_d = \vec{l}.(\vec{B}\times\vec{v})$)

ILLUSTRATION 4.24

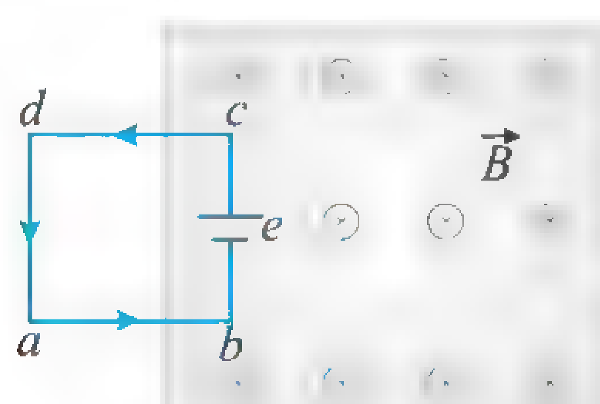
A square loop of side l having resistance R moves with constant velocity v, through a constant magnetic field as shown in figure. Find the magnitude and direction of induced current in the situation when



(i) the loop enters into magnetic field
(ii) it is moving in magnetic field.
(iii) it is coming out of magnetic field.

Sol.

(i) When loop starts entering into magnetic field. The side ab is outside the magnetic field, hence no EMF will develop across this side.

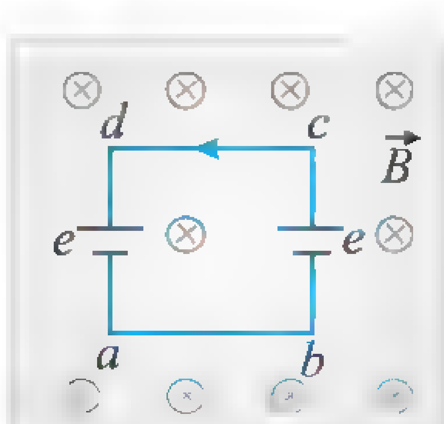


The length of the sides ab and cd are parallel to the velocity hence no EMF will be induced in these sides. The EMF will be induced across the side bc.

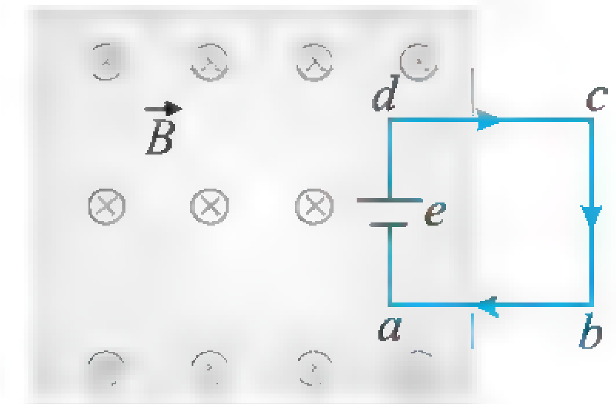
Hence net induced emf $\varepsilon = Blv$

The current in the loop $I = \dfrac{\varepsilon}{R} = \dfrac{Blv}{R}$. The current will be in counter clockwise sense.

(ii) When loop is completely submerged in magnetic field. The EMF will be induced insides bc and ad as shown in figure. In this case net EMF in the loop will be zero. Hence no induced current.



(iii) When the loop starts coming out of magnetic field. The EMF will be induced in side ad only. The direction of induced EMF or induced current can be found out by Lenz's law or by right hand rule.

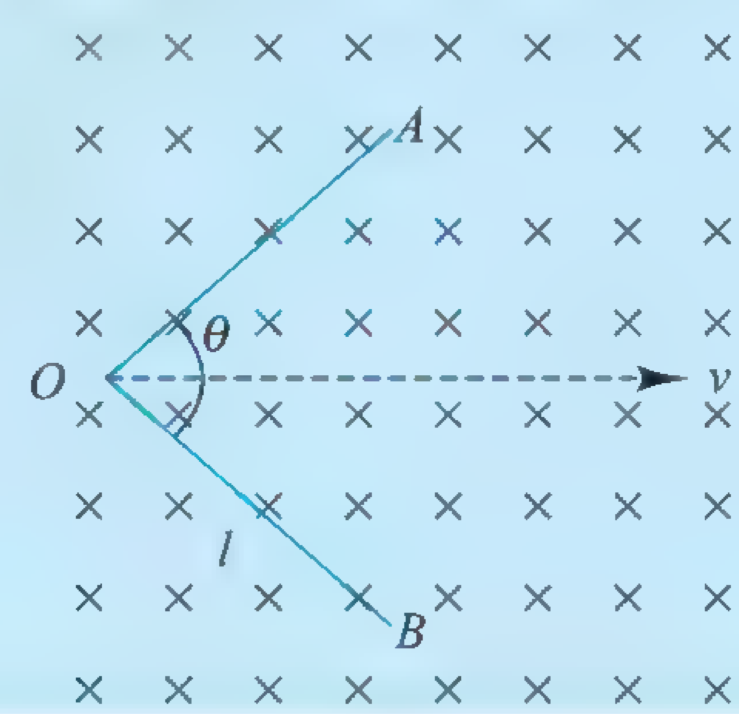


Net induced emf $\varepsilon = Blv$

The current in the loop $I = \dfrac{\varepsilon}{R} = \dfrac{Blv}{R}$

ILLUSTRATION 4.25

An angle $\angle AOB$ made of a conducting wire moves along its bisector through a magnetic field B as suggested by figure. Find the emf induced between the two free ends if the magnetic field is perpendicular to the plane at the angle.



Sol. The rod OA is equivalent to a battery of emf $vBl \sin \theta/2$. The positive charges of OA shift toward A due to the force. The positive terminal of the battery appears toward A. Similarly, the rod OB is equivalent to a battery of emf $vBl \sin \theta/2$ with the positive terminal toward O. The equivalent circuit is shown in figure. Clearly, the emf between points A and B is $2Blv \sin \theta/2$.

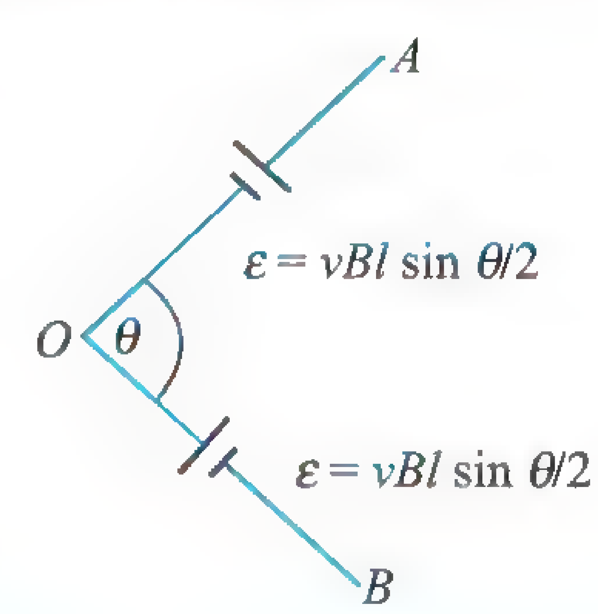
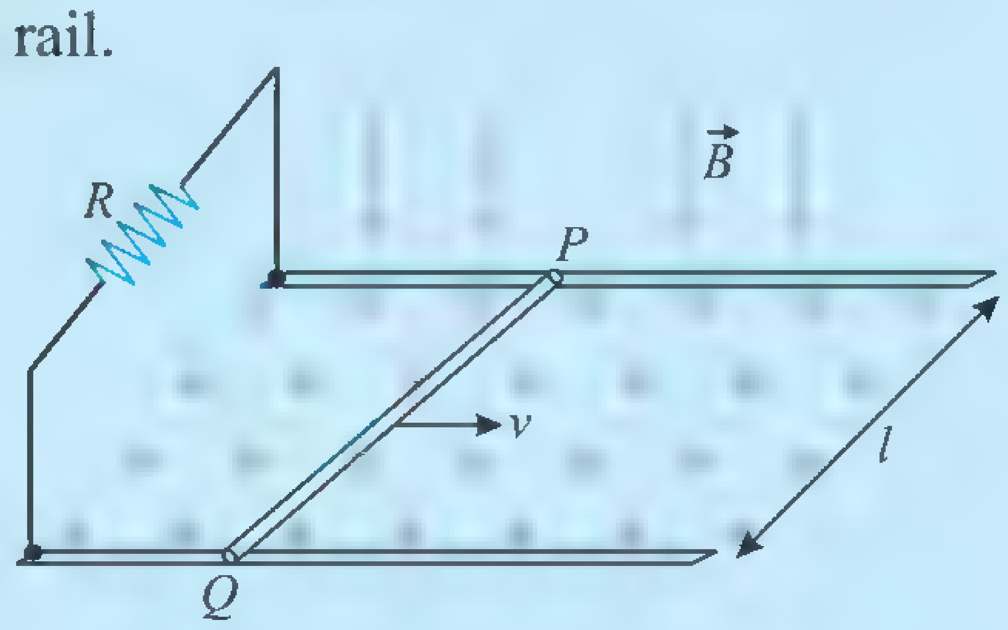


ILLUSTRATION 4.26

A conducting rod of length l slides at constant velocity v on two parallel conducting rails, placed in a uniform and constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail.

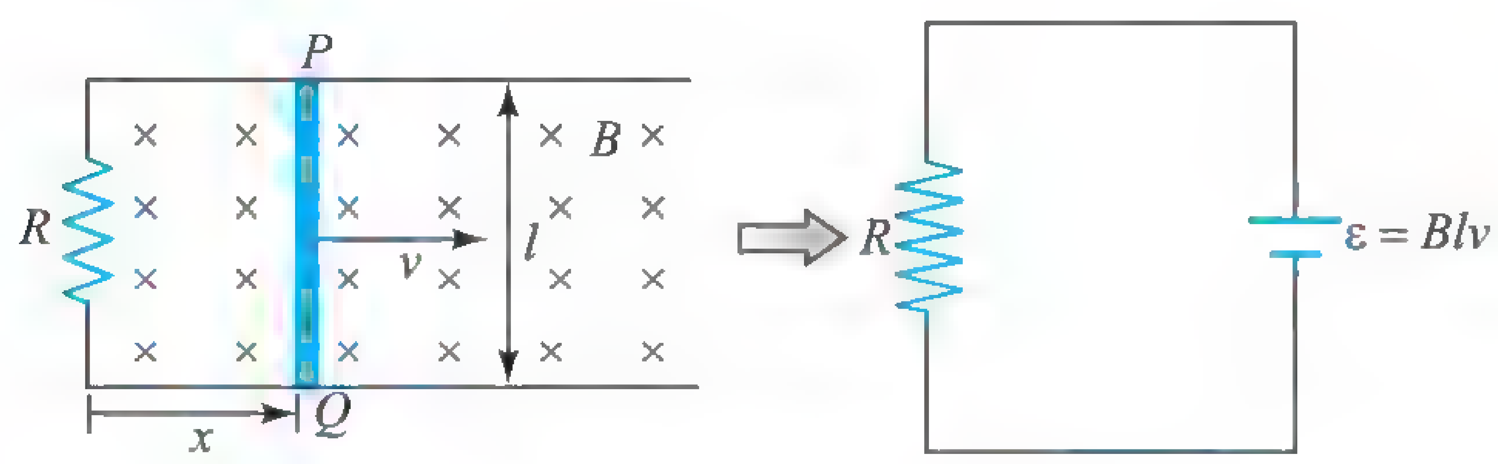


(a) Identify the cause which produces change in magnetic flux.
(b) Identify the direction of current in the loop.
(c) Determine the emf induced in the loop.

(d) Compute the electric power dissipated in the resistor.
(e) Calculate the mechanical power required to pull the rod at a constant velocity.

Sol.

(a) The change in area produces the change in magnetic flux.
(b) The direction of current in the loop is anticlockwise. As the rod moves toward right, the number of crosses in the loop increases with time and to oppose the increasing number of crosses in the loop, the current in the loop must be anticlockwise.



(c) According to Faraday's law,

$$|\varepsilon| = \frac{d\phi_B}{dt} = \frac{d}{dt}(Blx) = Bl\frac{dx}{dt} = Blv.$$

The emf across rod PQ can also be calculated by using the concept of motional emf.

(d) The magnitude of current is $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$.

The electric power dissipated in the resistor is

$$P_{\text{ele}} = I^2R = \frac{B^2l^2v^2}{R}$$

(e) The mechanical power is $P_{\text{mech}} = F_{\text{ext}}v$.
The external force is equal and opposite to Ampere's force:

$$F_{\text{ext}} = -F_{\text{Ampere}}$$

Ampere's force is given by

$$F_{\text{Ampere}} = \int I d\vec{l} \times \vec{B} \text{ or } F_{\text{Ampere}} = BIl$$

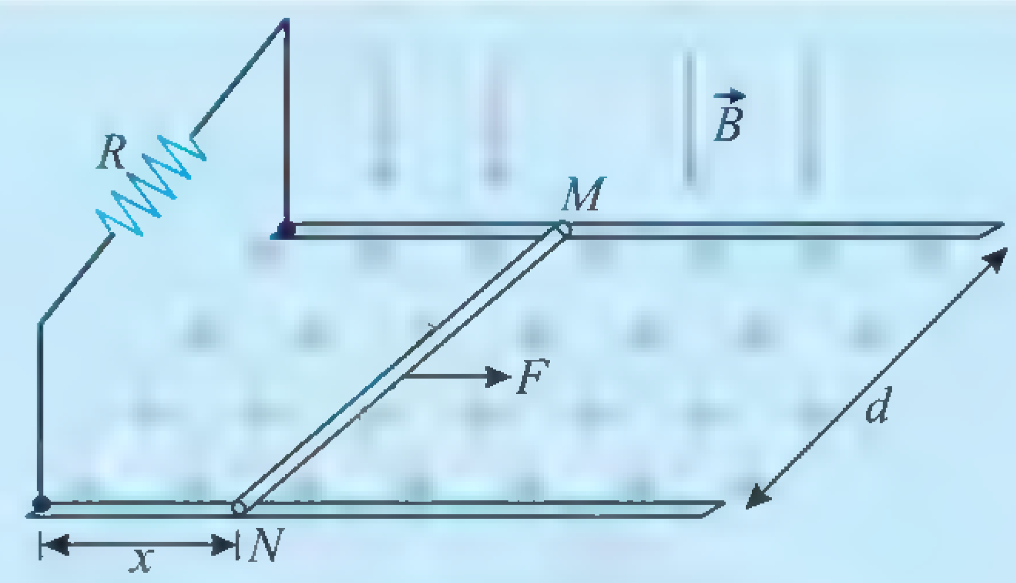
Now, $P_{\text{mech}} = (BIl)v$.

Substituting the value of I, we get $P_{\text{mech}} = \frac{B^2l^2v^2}{R}$.

We see that mechanical power is same as electric power. It means mechanical power supplied to the rod is dissipated in the form of heat through resistor.

ILLUSTRATION 4.27

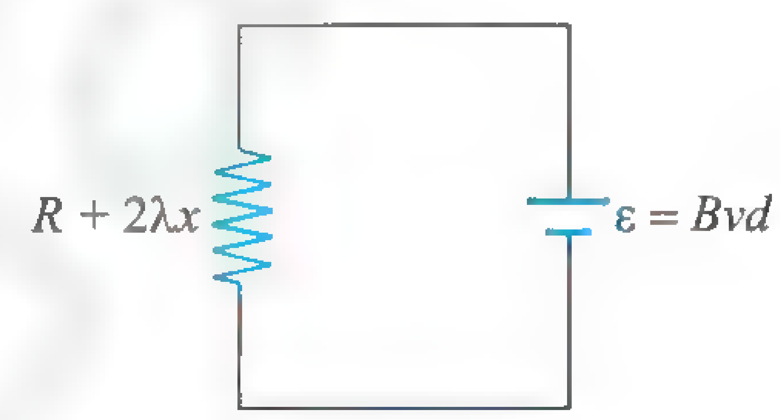
Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to rod MN such that as the rod moves, a constant current i flows through R.



(a) Find the velocity of the rod and the applied force F as a function of the distance x of the rod from R.
(b) What fraction of the work done per second by F is converted into heat?

Sol.

(a) If the rod has instantaneous velocity v at a distance x from R, the induced emf is Bvd.



Instantaneous resistance of current $= R + 2\lambda x$

$\therefore$ Induced current $i = \frac{Bvd}{R+2\lambda x}$ = constant

$\therefore$ Velocity $v = \frac{(R+2\lambda x)i}{Bd}$

Magnetic force on the rod $= Bid$.
This force will be opposite to F. Hence, net force acting on rod,

$$F - Bid = m\frac{dv}{dt}$$

or $$F - Bid = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{d}{dx}\left\{\frac{(R+2\lambda x)i}{Bd}\right\} = mv\frac{2\lambda i}{Bd}$$

$$\therefore \quad F = Bid + 2m\lambda\frac{(R+2\lambda x)}{B^2d^2}i^2$$

(b) Work done per second $= Fv$
Heat produced per second $= i^2(R + 2\lambda x)$

$$\text{Required ratio} = \frac{i^2(R+2\lambda x)}{Fv} = \frac{i^2(R+2\lambda x)Bd}{F(R+2\lambda x)i} = \frac{iBd}{F}$$

$$= \frac{Bid}{Bid + \frac{2m\lambda(R+2\lambda x)i^2}{B^2d^2}}$$

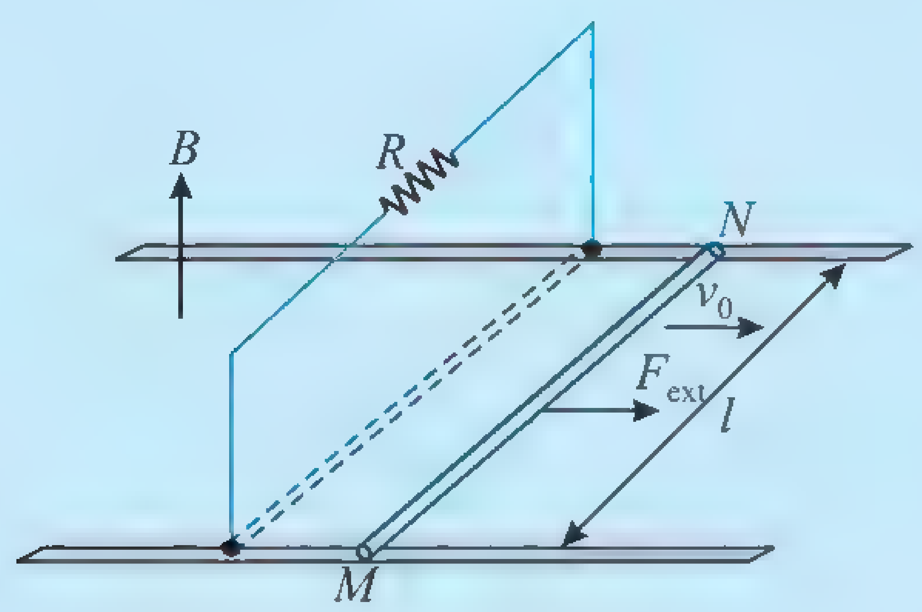
$$= \frac{1}{1+\frac{2m\lambda(R+2\lambda x)i}{B^3d^3}}$$

ILLUSTRATION 4.28

Figure shows a setup with two long conducting rails with separation l in a plane perpendicular to a uniform magnetic field of magnetic induction B. A sliding rod MN of mass m is placed on rails as shown. The rails are connected with a

resistance R and the sliding rod is pulled with constant velocity v_0 as shown in the figure.

Then find



(a) ampere force acting on the rod

(b) power developed by the external force

(c) electrical power delivered by the external agent

(d) thermal power generated in the resistor

Sol.

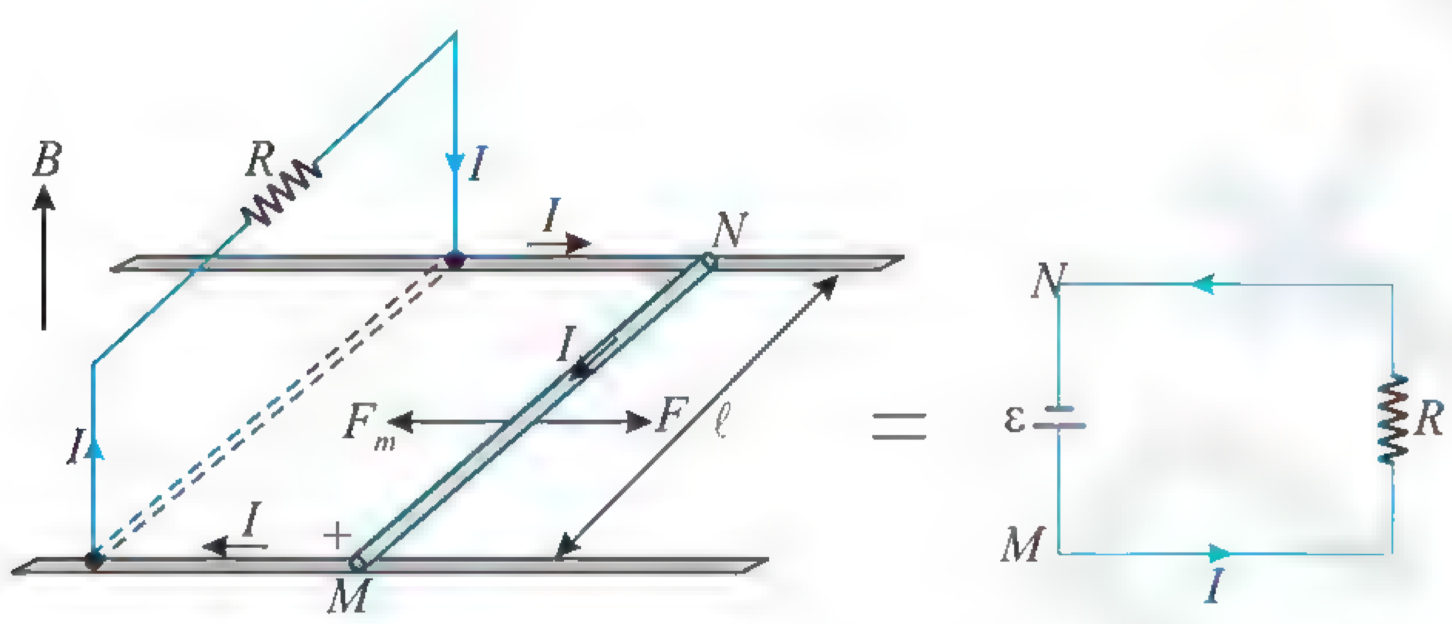
(a) In this case the rod moves with a velocity v_0, normal tothe magnetic field of induction B .

The emf induced in rod $\varepsilon_i = Blv_0$.

The induced current in rod, $i = \dfrac{Blv_0}{R}$

The magnetic field experiences a magnetic force

$$F_m = ilB = \frac{B^2l^2v_0}{R}$$



(b) Power developed by the external agent

$$P_{ext} = \vec{F}_{ext} \cdot \vec{v} = F_{ext}.v_0 \cos 0°$$

As the bar moves with constant velocity v, it experiences no net force, $F_{ext} = F_m = \dfrac{B^2l^2v_0}{R}$.

Thus, $P_{ext} = F_{ext}v_0 = \dfrac{B^2l^2v_0^2}{R}$

(c) Thermal power generated in the resistor, $P_{ther.} = i^2R$

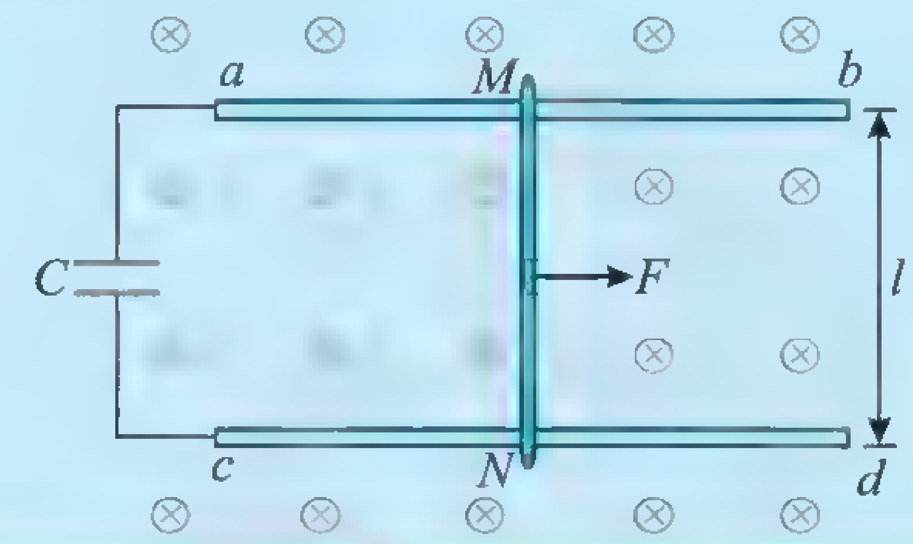
Substitution of i in this equation yields $P_{ther.} = \dfrac{B^2l^2v_0^2}{R}$

(d) Electrical power generated

$$P_{el} = \varepsilon_i i = (Blv_0)\left(\frac{Blv_0}{R}\right) = \frac{B^2l^2v_0^2}{R}$$

ILLUSTRATION 4.29

Figure shows two long smooth conducting rails ab and cd separated by a distance l. Ends a and c are connected with capacitor of capacitance C. A rod MN of mass m is horizontally kept in touch with both rails as shown and it can slide along the rails without friction.The entire system is placed in a downward magnetic field of induction B. At $t = 0$, a force F is applied on the rod as shown in figure. Find the velocity of the rod at any time t.



Sol. When the rod moves then at any instant when its speed is v, the motional emf across the rod is given as

$$e = Blv$$

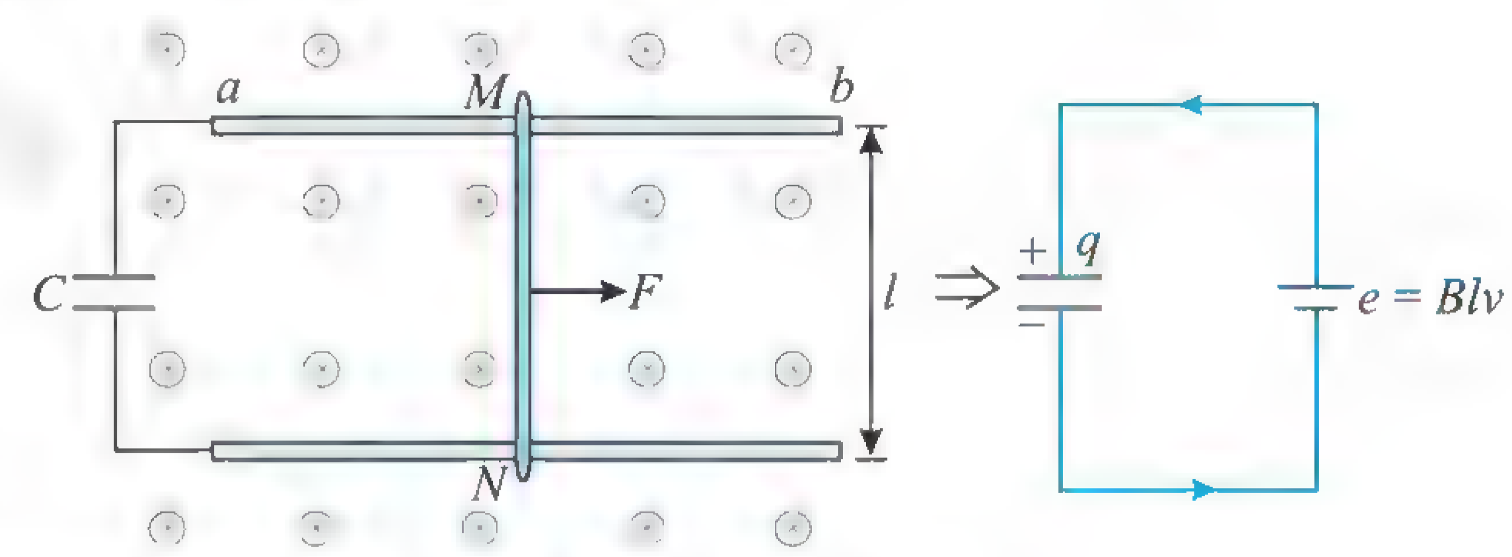
As we can neglect the resistance of all connecting wires and rails the charge on capacitor is given as

$$q = Ce = CBlv \qquad ...(i)$$

Current through the capacitor is given as

$$i = \frac{dq}{dt} = CBla$$

The rod experiences an leftward magnetic force rightward force as shown in figure.



The acceleration a, of the rod is given as

$$a = \frac{F - Bil}{m} = \frac{F - B^2l^2Ca}{m}$$

$$\Rightarrow \quad a = \frac{F}{m + B^2l^2C}$$

As acceleration is constant, after time t its velocity is given as

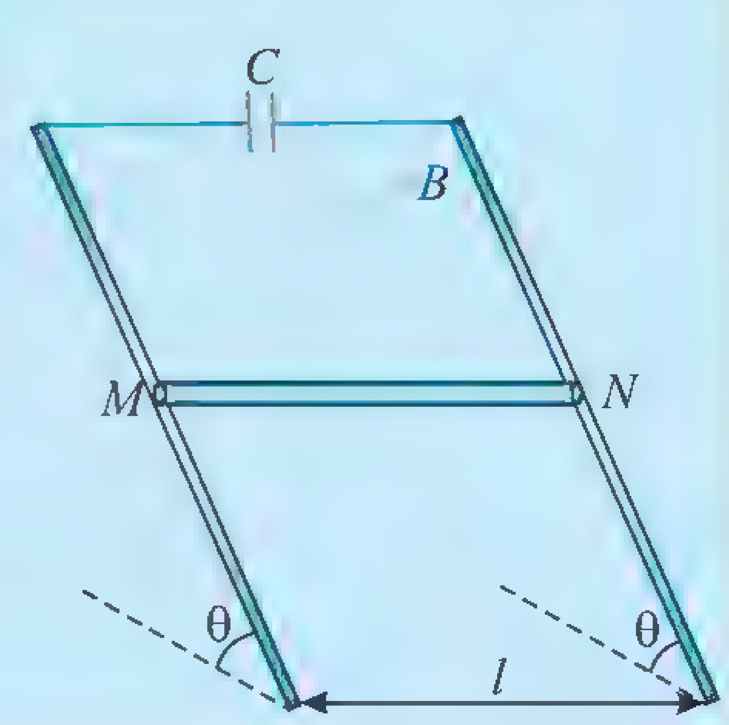
$$v = at = \frac{Ft}{m + B^2l^2C}$$

Thus charge on capacitor after time t is given by equation as

$$q = CBl\left(\frac{Ft}{m + B^2l^2C}\right) = \frac{CBlFt}{m + B^2l^2C}$$

ILLUSTRATION 4.30

Two smooth conducting fixed parallel rails are inclined at an angle θ to the horizontal. The top end of the rails are connected with a capacitor of capacitance C. A straight horizontal conducting rod MN of length l, and mass m slides down on these rails as shown in figure.



The system is placed in a uniform magnetic field, in the direction perpendicular to the inclined plane formed by the rails. If the resistance of the bars and the sliding conductor are negligible, calculate the acceleration of sliding conductor as a function of time if it is released from rest at $t = 0$.

Sol. Let speed of the rod at any time be v, then EMF induced in the rod is given as

$$e_{MN} = Blv$$

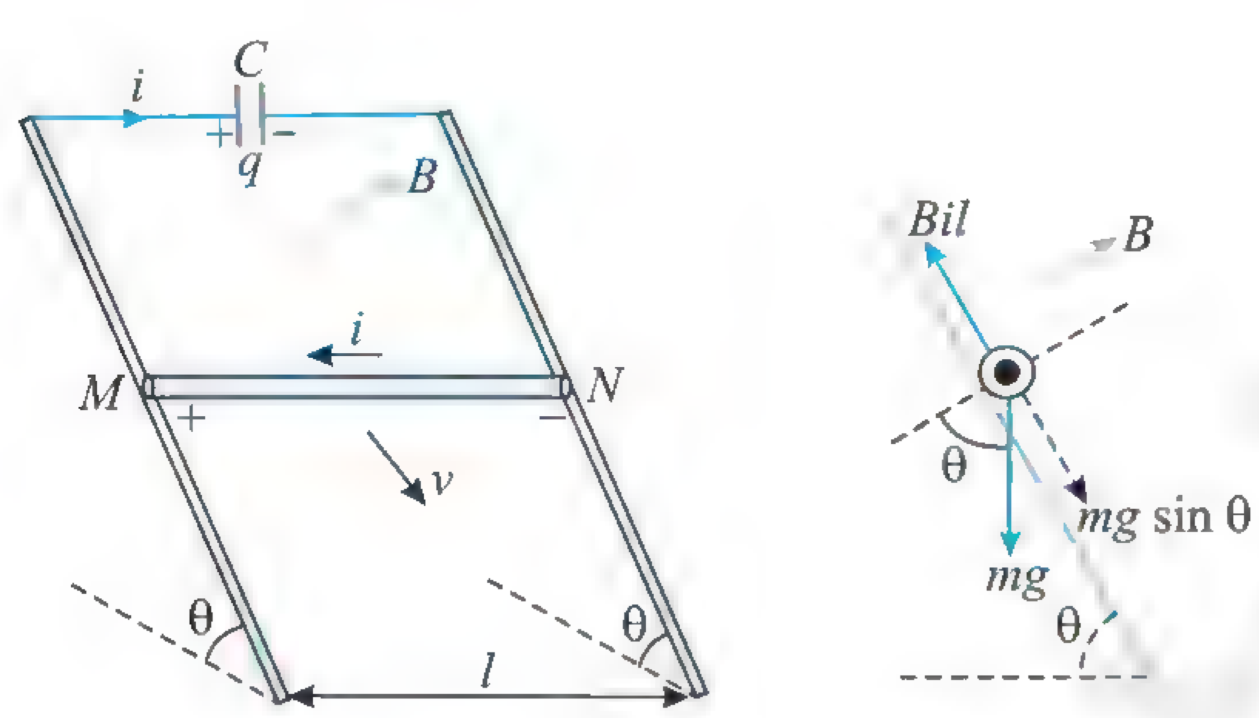
Hence potential difference across the capacitor should be equal to the EMF induced in the rod.

The instantaneous charge on capacitor, $q = CV = CBlv$

Current through the rod, $i = \dfrac{dq}{dt} = CBi\dfrac{dv}{dt} = CBla$

In magnetic field the current carrying conductor experiences a magnetic force in upward direction by right hand palm rule as shown in Fig. (b) which shows the side view of sliding conductor and its free body diagram. This gives

$$F_{\text{mag}} = Bil = B^2l^2Ca$$



(a) (b)

If a is the acceleration of conductor, then its equation of motion is given as

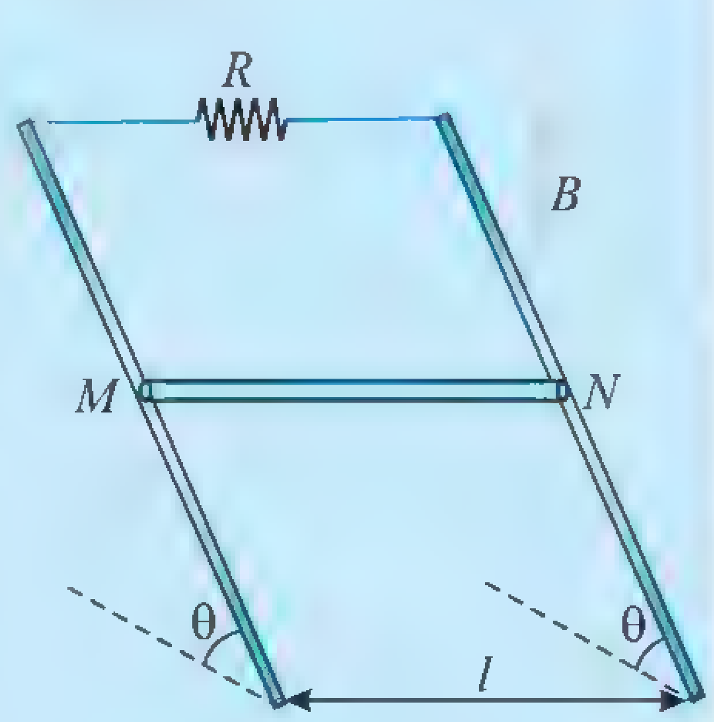
$$ma = mg\sin\theta - B^2l^2Ca$$

$$\Rightarrow \quad a[m + B^2l^2C] = mg\sin\theta$$

$\Rightarrow \quad a = \dfrac{mg\sin\theta}{m + B^2l^2C}$, Here we can see that a is constant in time.

ILLUSTRATION 4.31

Two smooth conducting fixed parallel rails are inclined at an angle θ to the horizontal. The top end of the rails are connected with a resistance R. A straight horizontal conducting rod MN of length l, and mass m slides down on these rails as shown in figure. The system is located in a uniform magnetic field of induction B in vertically upward direction as shown in figure. The resistances of the bars, the rod and the sliding contacts are considered to be negligible. If the rod is released from rest, find the steady state velocity of the rod.

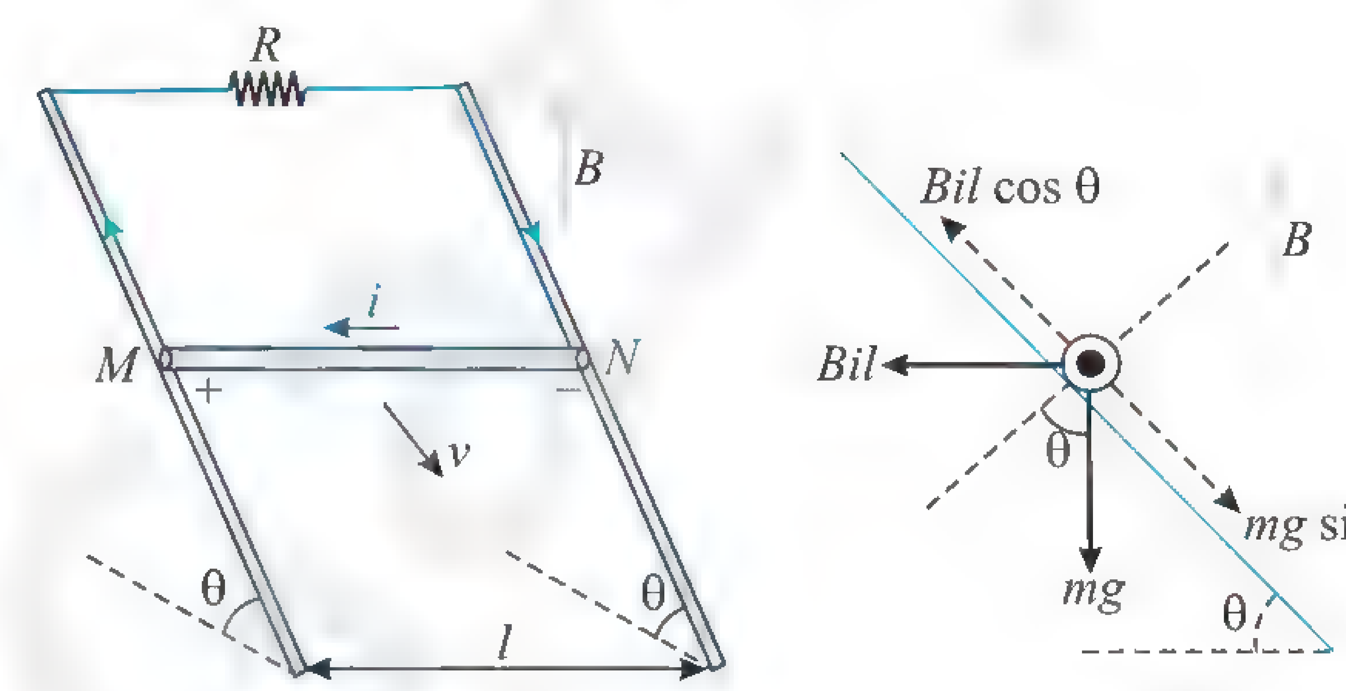


Sol. The rod starts moving from rest, it cuts the magnetic flux and an EMF is induced in it with polarity given by right hand palm rule as shown in figure. Let the steady state velocity of the rod be v and as it is moving perpendicular to the magnetic field component $B\cos\theta$, the induced EMF in the rod is given as

$$e = (B\cos\theta)vL = BvL\cos\theta$$

Thus current induced due to induced EMF in the loop containing resistance as shown is given as

$$i = \frac{BvL\cos\theta}{R}$$



(a) (b)

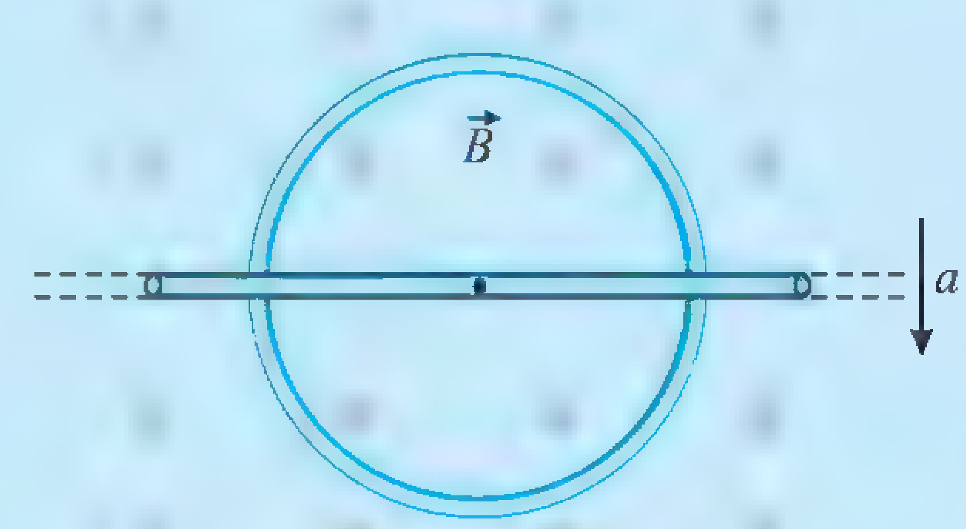
In Fig. (b), we can observe the side view of the sliding rod. By right hand rule, we can observe that magnetic force will act along leftward direction and if rod slides with constant velocity. At this time

$$mg\sin\theta = BiL\cos\theta \Rightarrow mg\sin\theta = \frac{B^2L^2v\cos^2\theta}{R}$$

Hence steady state velocity, $v = \dfrac{mgR\sin\theta}{B^2L^2\cos^2\theta}$

ILLUSTRATION 4.32

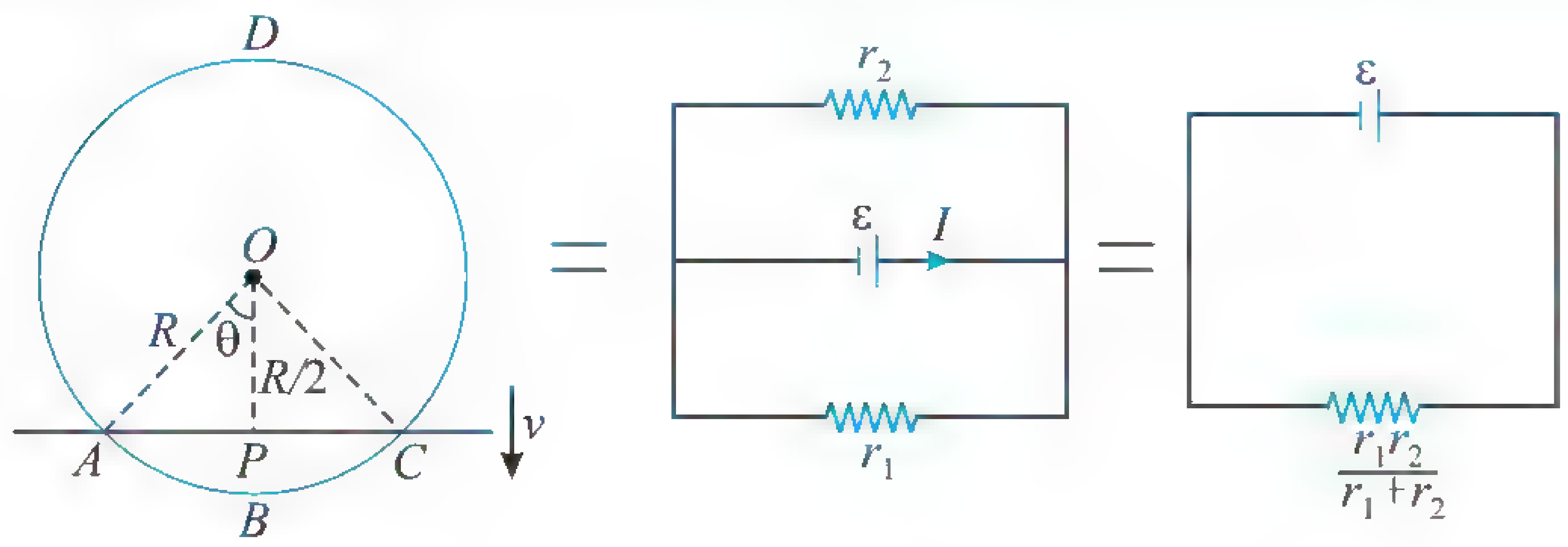
A circular conducting wire ring of radius R is fixed in a horizontal plane. The resistance per unit length of wire of the ring has a resistance of $\lambda\Omega m^{-1}$. There is a uniform vertical downward magnetic field B in entire space. A perfectly conducting rod (l) is kept along the diameter of the ring. The rod is made to move with a constant acceleration a in a direction perpendicular to its own length. Find the current through the rod at the instant it has travelled through a distance $x = \dfrac{R}{2}$.



Sol. The velocity of the rod when it has travelled a distance

$$x = \frac{R}{2} \text{ is } v = \sqrt{2ax} = \sqrt{2a\frac{R}{2}} = \sqrt{aR}$$

Length of rod inside the ring, $L = AC = 2\sqrt{R^2 - \dfrac{R^2}{4}} = \sqrt{3}\,R$



Emf induced in the rod at this instant is

$$\varepsilon = BvL = B\cdot\sqrt{aR}\sqrt{3}R = \sqrt{3a}\,BR^{3/2}$$

$$\Delta AOP,\ \cos\theta = \frac{R/2}{R} = \frac{1}{2}$$

It means $\theta = 60^\circ = \dfrac{\pi}{3}$ radian

$\therefore$ Length of lower arc ABC, $l_1 = 2\pi R\left(\dfrac{\pi/3}{2\pi}\right) = \dfrac{\pi R}{3}$

Length of lower arc ADC, $l_2 = 2\pi - \dfrac{\pi R}{3} = \dfrac{5\pi R}{3}$

Resistance of arc ABC; $r_1 = l_1\lambda = \dfrac{\pi R}{3}\lambda$

Resistance of arc ADC; $r_2 = l_2\lambda = \dfrac{5\pi R}{3}\lambda$

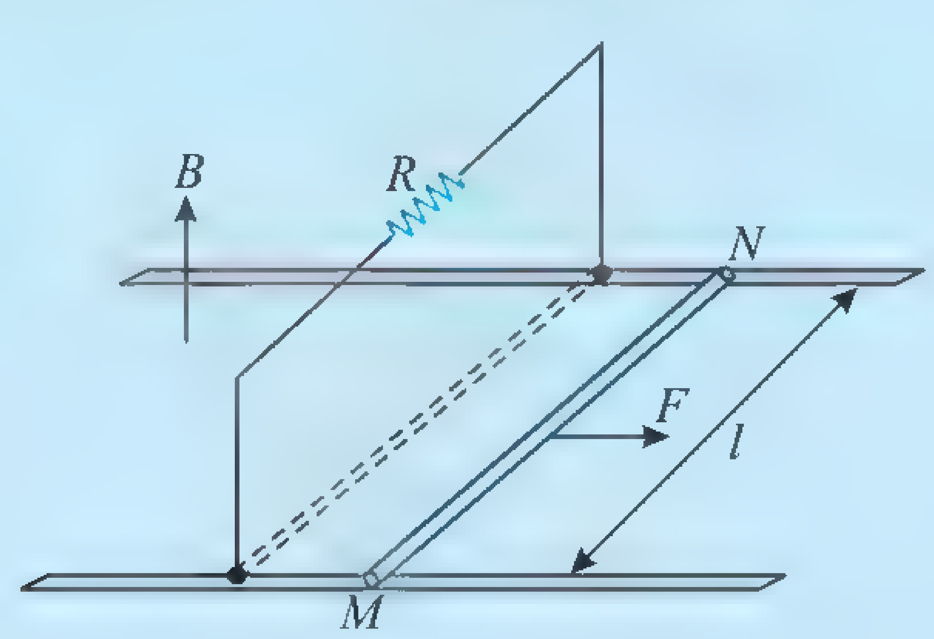
Equivalent resistance connected across equivalent battery,

$$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{5\pi}{18}\lambda R$$

$\therefore$ Current through the rod $I = \dfrac{\varepsilon}{r_{eq}} = \dfrac{18B}{5\pi}\dfrac{\sqrt{3aR}}{\lambda}$

ILLUSTRATION 4.33

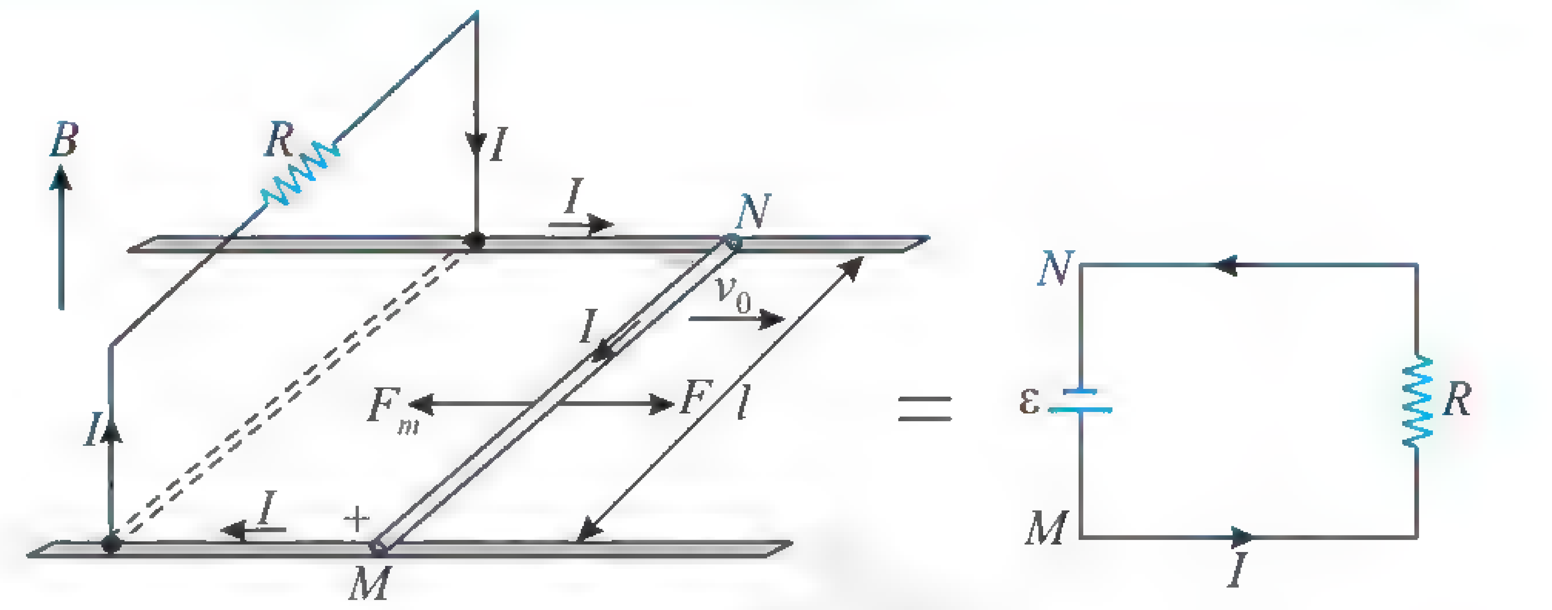
Figure shows a setup with two long conducting rails with separation l in a plane perpendicular to a uniform magnetic field of magnetic induction B. A sliding rod MN of mass m is placed on rails as shown. The rails are connected with a resistance R and the sliding rod is pulled with a constant external force F. If initially the sliding rod is at rest at $t = 0$,



Find the velocity of the rod at any time t and terminal velocity of the rod.

Sol. At a general instant of time t, let the rod is moving with velocity v then at this instant motional EMF induced in this wire is given as

$e = Bvl$ (with end M of the rod to be positive)



The current through the resistance due to induced EMF is given as

$$i = \frac{e}{R} = \frac{Bvl}{R} \qquad \text{...(i)}$$

Due to the direction of induced EMF current flows from N to M in the sliding wire because of which magnetic field exerts a magnetic force on the sliding wire which is given as

$$F_m = Bil = \frac{B^2 l^2 v}{R} \qquad \text{...(ii)}$$

Using the right-hand palm rule we can find the direction of this magnetic force on sliding wire MN is toward left which is in opposition to the external force. This is also validating Lenz's law that the effects produced by induction oppose the causes of induction.

If at a general instant of time t acceleration of wire MN is a then we have

$$F - F_m = ma \qquad \text{...(iii)}$$

$$\Rightarrow \quad a = \frac{F - F_m}{m}$$

$$\Rightarrow \quad a = \frac{F - \dfrac{B^2 l^2}{R}v}{m} \qquad \text{...(iv)}$$

$$\Rightarrow \quad \frac{dv}{dt} = a = \frac{FR - B^2 l^2 v}{mR}$$

$$\Rightarrow \quad \frac{dv}{dt} = a = \frac{FR - B^2 l^2 v}{mR}$$

To find the velocity of wire MN as a function of time we integrate the above expression from initial instant $t = 0$ to a general time instant $t = t$ given as

$$\int_0^v \frac{dv}{FR - B^2 l^2 v} = \int_0^t \frac{dt}{mR}$$

$$\Rightarrow \quad -\frac{1}{B^2 l^2}[\ln(FR - B^2 l^2 v)]_0^v = \frac{1}{mR}[t]_0^t$$

$$\Rightarrow \quad -\frac{1}{B^2 l^2}[\ln(FR - B^2 l^2 v) - \ln(FR)] = \frac{1}{mR}[t - 0]$$

$$\Rightarrow \quad \ln\left(\frac{FR - B^2 l^2 v}{FR}\right) = -\frac{B^2 l^2 t}{mR}$$

$$\Rightarrow \quad \frac{FR - B^2 l^2 v}{FR} = e^{-\frac{B^2 l^2 t}{mR}}$$

$$\Rightarrow \quad FR - B^2l^2v = FRe^{-\frac{B^2l^2t}{mR}}$$

$$\Rightarrow \quad B^2l^2v = FR\left(1 - e^{-\frac{B^2l^2t}{mR}}\right)$$

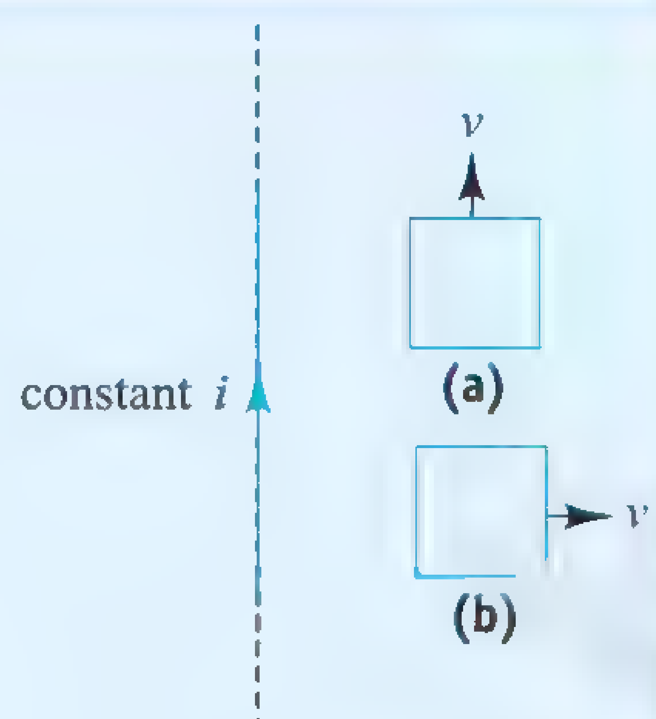
$$\Rightarrow \quad v = \frac{FR}{B^2l^2}\left(1 - e^{-\frac{B^2l^2t}{mR}}\right) \quad \text{...(v)}$$

With the above expression given in Eq. (v) it can be seen that with time velocity of sliding wire approaches to a steady value and at $t \to \infty$ velocity approaches to the steady velocity called terminal velocity and it is given by Eq. (v) as

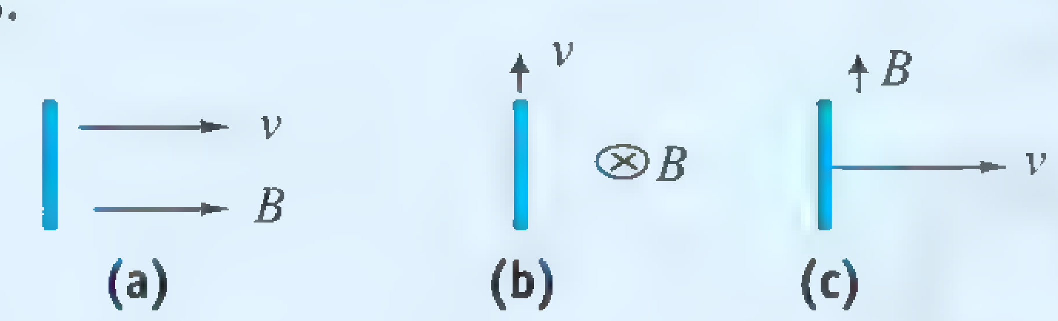
$$v_{\text{terminal}} = \frac{RF}{B^2l^2} \quad \text{...(vi)}$$

Concept Application Exercise 4.2

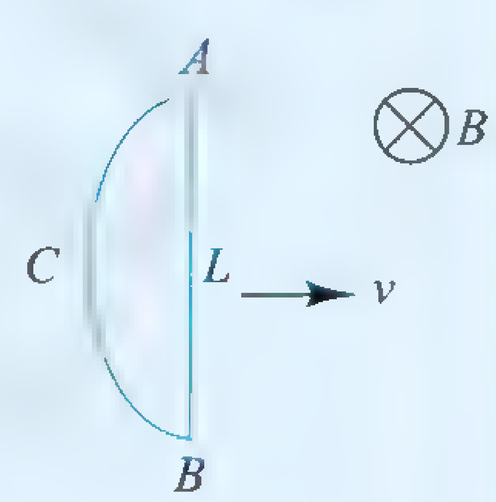
1. Figure shows a long current-carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



2. A rod of length l is moving with velocity v in magnetic field B as seen in figure. Find the emf induced in all three cases.



3. Figure shows a closed coil $ABCL$ moving in a uniform magnetic field B with a velocity v.

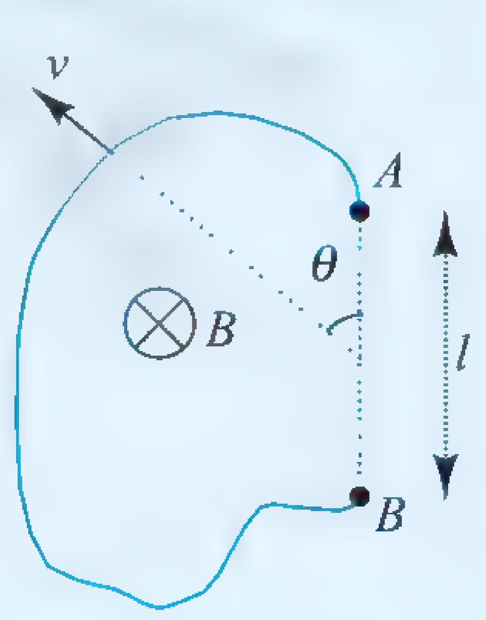


Find

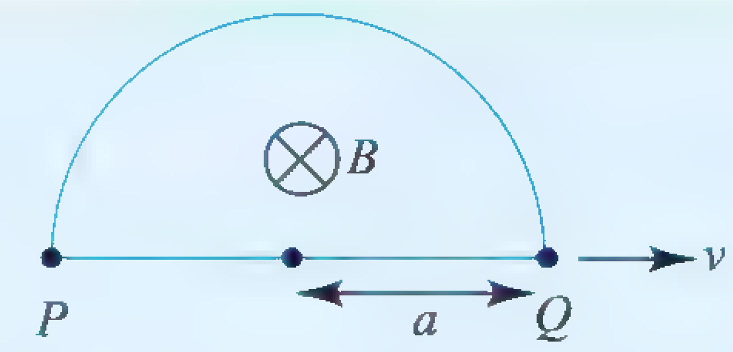
(a) emf induced in the coil.

(b) emf induced in curved part ACB and straight part AB

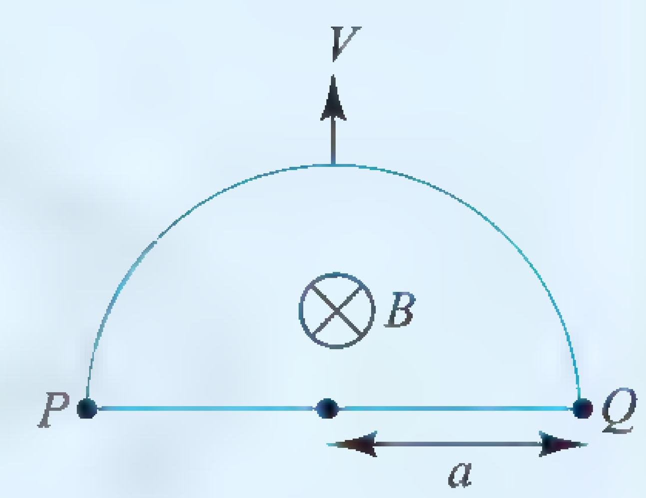
4. Figure shows an irregular shaped wire AB moving with velocity v. Find the emf induced in the wire.



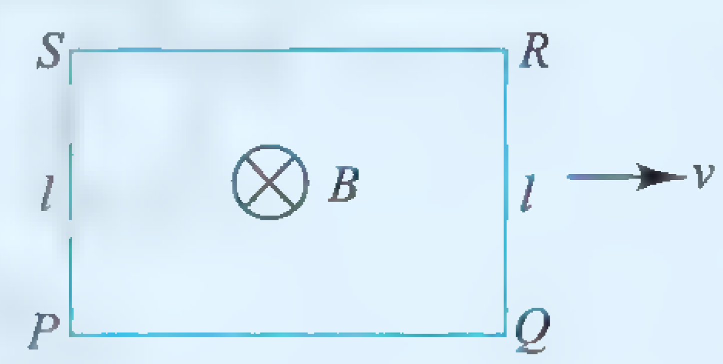
5. Find the emf across points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in figure.



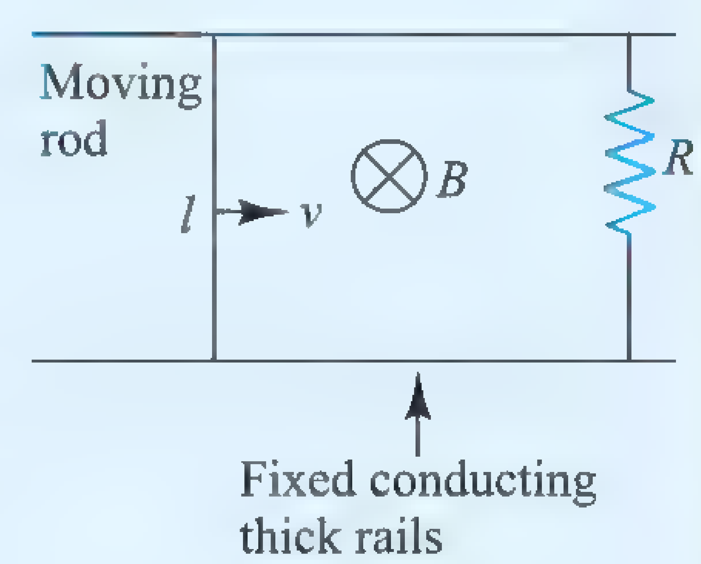
6. Find the emf across points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in figure. Also draw the electrical equivalence of each branch.



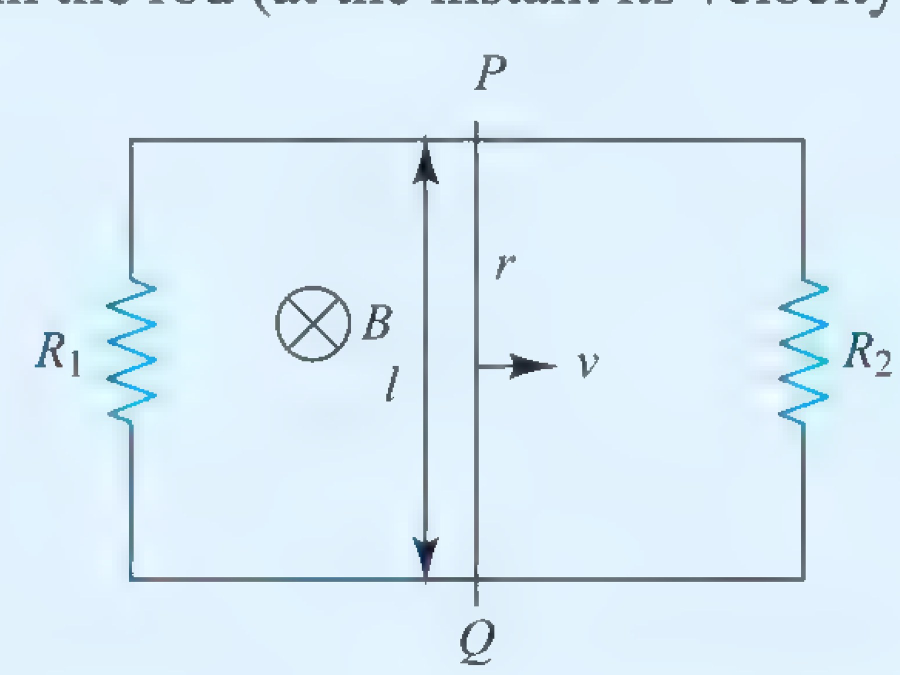
7. Figure shows a rectangular loop moving in a uniform magnetic field. Show the electrical equivalence of each branch. What is the net induced emf?



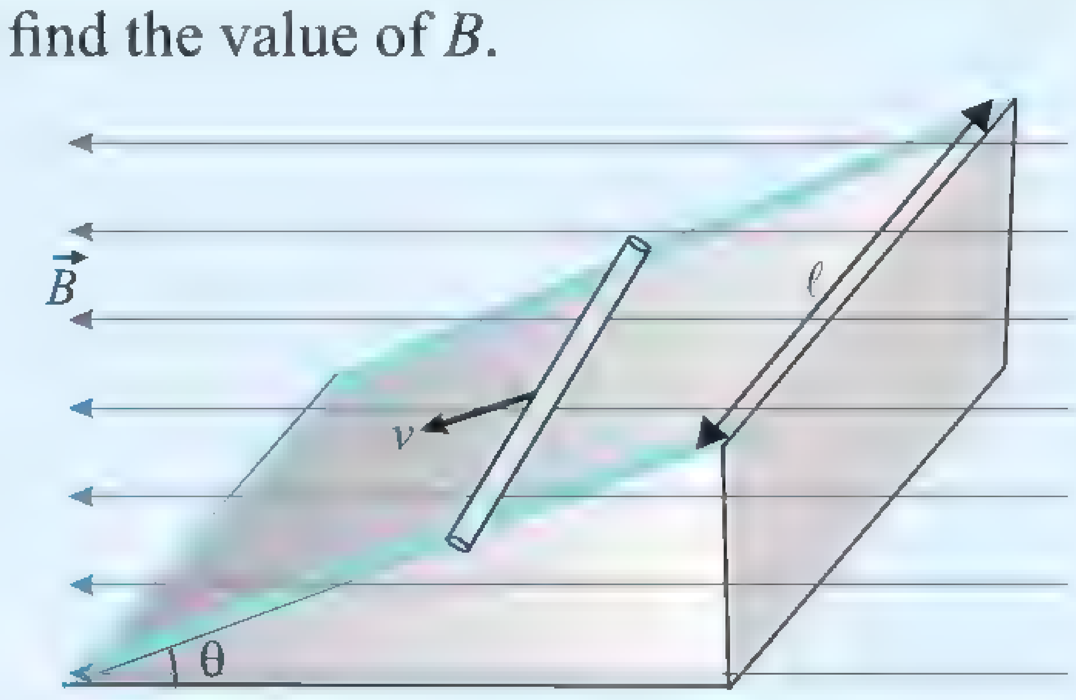
8. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R. A uniform magnetic field B is present normal to the plane of rod and rails. Show the electrical equivalence of each branch.



9. Figure shows rod PQ of mass m and resistance r moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2). Find the current in the rod (at the instant its velocity is v).

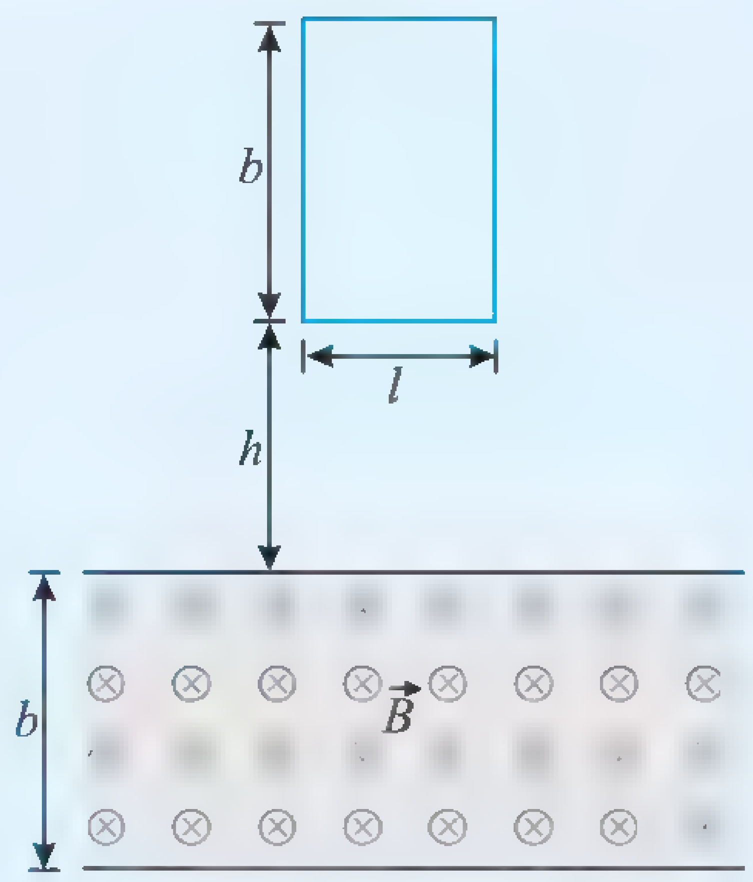


10. A rod of mass m, length l and resistance R is sliding down on a smooth inclined parallel rails with a constant velocity v. If a uniform horizontal magnetic field B exists, then find the value of B.

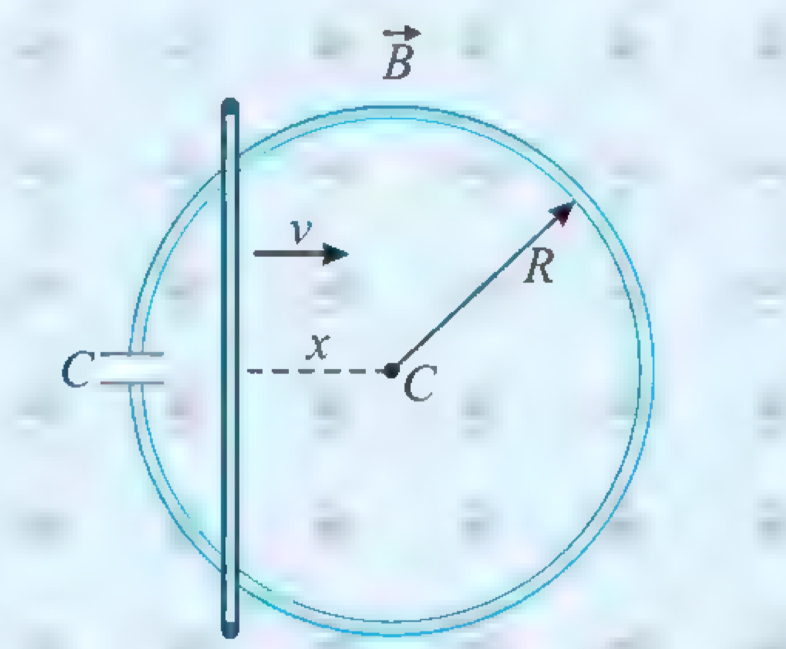


11. A rectangular loop of dimensions $b \times l$ having resistance R, is released from a certain height h into a region of

horizontal magnetic field of induction B. What must be the height of fall 'h' so that the loop will move with a constant velocity in the magnetic field?



12. The conducting rod slides on a conducting ring of radius R which is placed in horizontal plane . A capacitor having capacity C is connected with ring as shown in figure. The rod moves towards right with a constant velocity v in an outward magnetic field B. **(a)** When the rod is at a distance $x = \dfrac{R}{2}$ from the centre of the circular wire, find the current through the capacitor. **(b)** If we substitute the capacitor by a resistor R_0, find the current as the function of time.



ANSWERS

1. loop (a) —zero, loop (b) —clockwise.

2. (a) 0 as $\vec{B} \parallel \vec{v}$ (b) 0 as $\vec{l} \parallel \vec{v}$ (c) 0 as $\vec{l} \parallel \vec{B}$

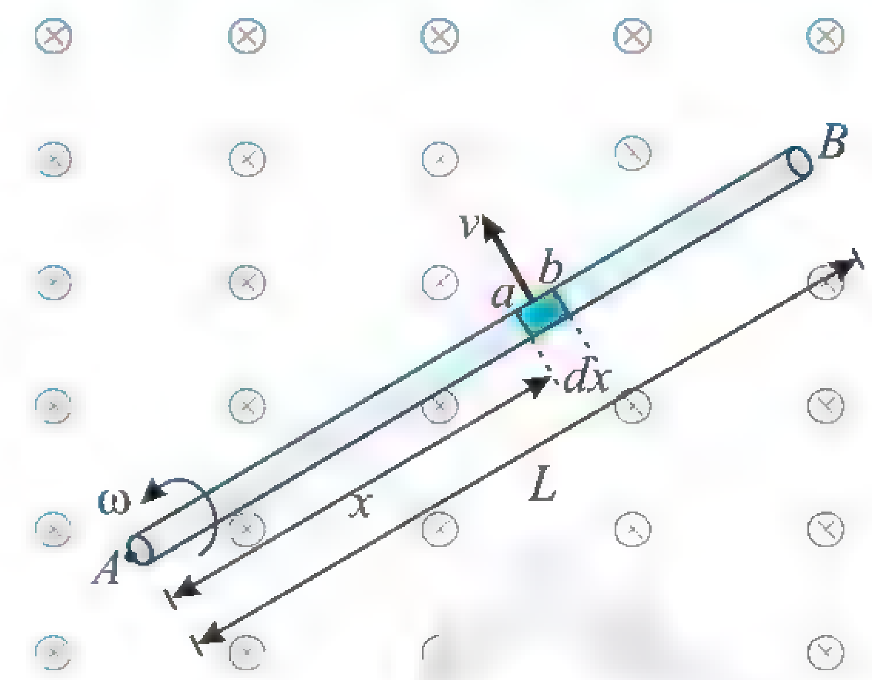
3. (a) 0 (b) vBL **4.** $Bvl \sin\theta$ **5.** 0 **6.** $2Bav$

7. 0 **9.** $\dfrac{Blv}{r + \dfrac{R_1R_2}{R_1+R_2}}$ **10.** $\sqrt{\dfrac{mgR}{l^2 v \sin\theta}}$

11. $\dfrac{m^2gR^2}{2B^4l^4}$ **12.** (a) $\dfrac{2BCv^2}{\sqrt{3}R}$ (b) $\dfrac{2Bv}{R_0}\sqrt{R^2-(R-vt)^2}$

EMF ACROSS ROTATING STRAIGHT CONDUCTOR

Consider a straight conductor AB of length L rotating about end A with angular velocity ω. Magnetic field B is perpendicular to the plane of rotation as shown in figure. We want to find emf induced in the conductor.



Take a very small element ab of length dx as shown.

Velocity of this element: $v = \omega x$

Small emf induced in this element: $d\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{dx}$

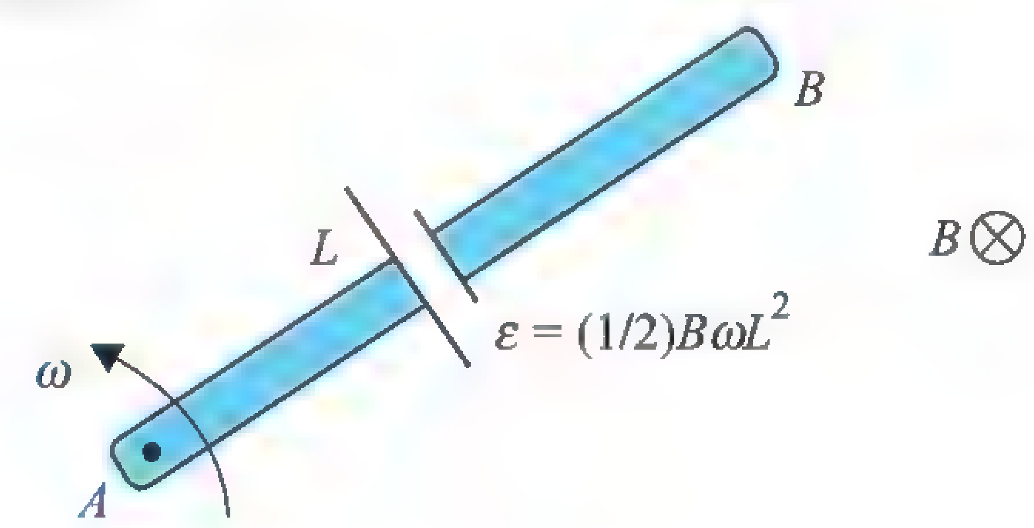
$\Rightarrow \quad d\varepsilon = -vBdx$

(because $\vec{v} \times \vec{B}$ will be opposite to $\vec{dx}$)

Negative sign indicates that a will be positive w.r.t. b.

Integrating: $\varepsilon = -\int_0^L Bv\,dx = -B\omega \int_0^L x dx = -\dfrac{1}{2}B\omega L^2$

Negative sign indicates that end A will be at higher potential than B. Hence, magnitude of induced emf in the rod is $\varepsilon = \dfrac{1}{2}B\omega L^2$ with the polarities induced as shown in figure.



Electric Field at Any Point in the Rod

PD between b and a: $dV_{ba} = -vBdx$

Electric field in this element ab:

$$E = \frac{dV_{ba}}{dx} \quad \rightarrow \text{ directed from } b \text{ to } a$$

$$= \frac{-vB\,dx}{dx} \quad \rightarrow \text{ directed from } b \text{ to } a$$

$$= -B\omega x \quad \rightarrow \text{ directed from } b \text{ to } a$$

$$= B\omega x \quad \rightarrow \text{ directed from } a \text{ to } b$$

At end A: $x = 0 \quad \Rightarrow \quad E = 0$

At end B: $x = L \quad \Rightarrow \quad E = B\omega L$ (max)

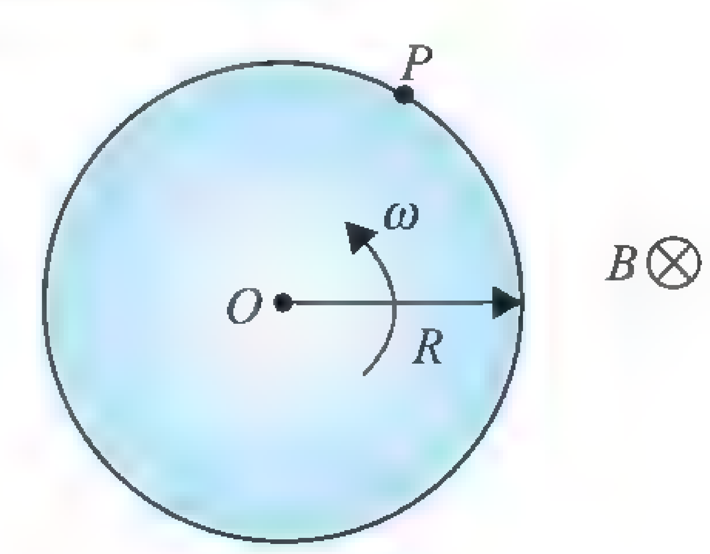
Electric field in the rod is directed from A to B.

A Rotating Conducting Disc

Consider a conducting disc of radius R rotating about its axis in a uniform magnetic field B. Magnetic field is perpendicular to the plane of disc.

Due to the rotation of the disc, electrons inside it experience magnetic force due to which emf is induced in the disc.

In the present situation electrons experience force away from the center. So center acquires positive potential w.r.t. circumference. Induced emf between center O and circumference P is



$$\varepsilon = \frac{1}{2}B\omega R^2$$

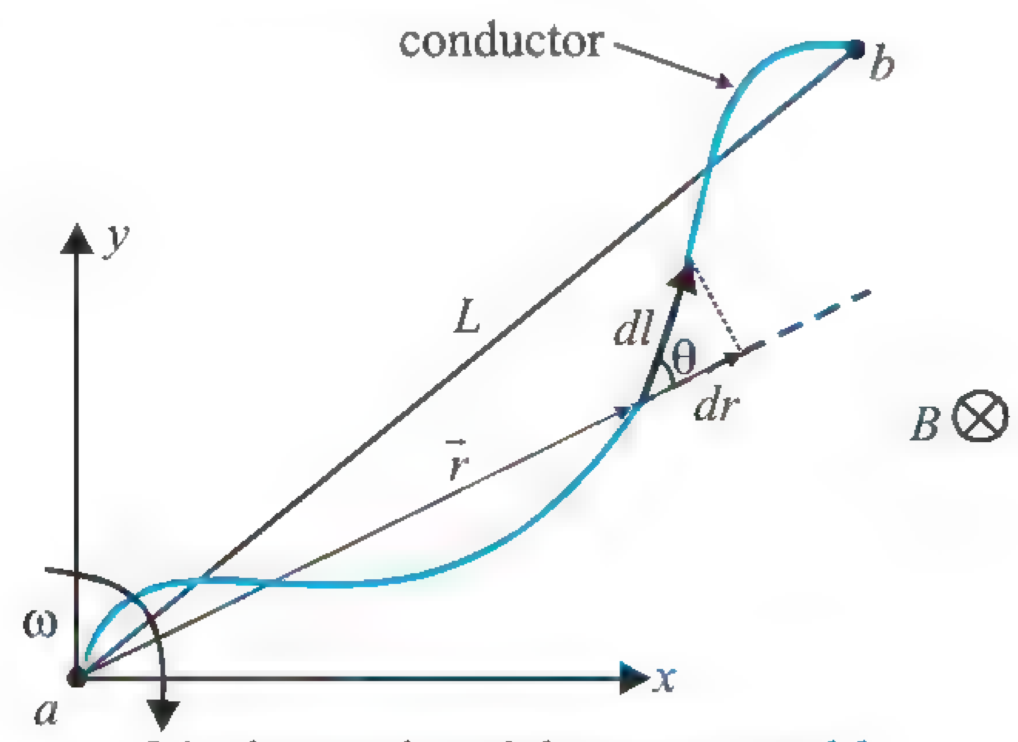
This expression is similar to the emf induced in a rotating rod with replacing L with R.

A Rotating Conductor of Arbitrary Shape

Consider an arbitrary shaped conductor ab which is rotating about end a with angular velocity ω as shown in figure. We want to find induced emf between ends a and b.

Let us consider an element 'dl' of the conductor. The emf induced across the element $d\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{dl}$

$\vec{v}$ is the velocity of the element.



L is shortest length between a and b

Induced emf developed between ends a and b is given by:

$$\varepsilon = \int d\varepsilon = \int (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

$$\Rightarrow \quad \varepsilon = \int [(\vec{\omega} \times \vec{r}) \times \vec{B}] \cdot \vec{dl} \qquad \text{Here } \vec{r} = x\hat{i} + y\hat{j}$$

$$\varepsilon = \int [\{-\omega\hat{k} \times (x\hat{i} + y\hat{j})\} \times (-B\hat{k})] \cdot \vec{dl}$$

$$= \int [(-\omega x\hat{j} + \omega y\hat{i}) \times (-B\hat{k})] \cdot \vec{dl}$$

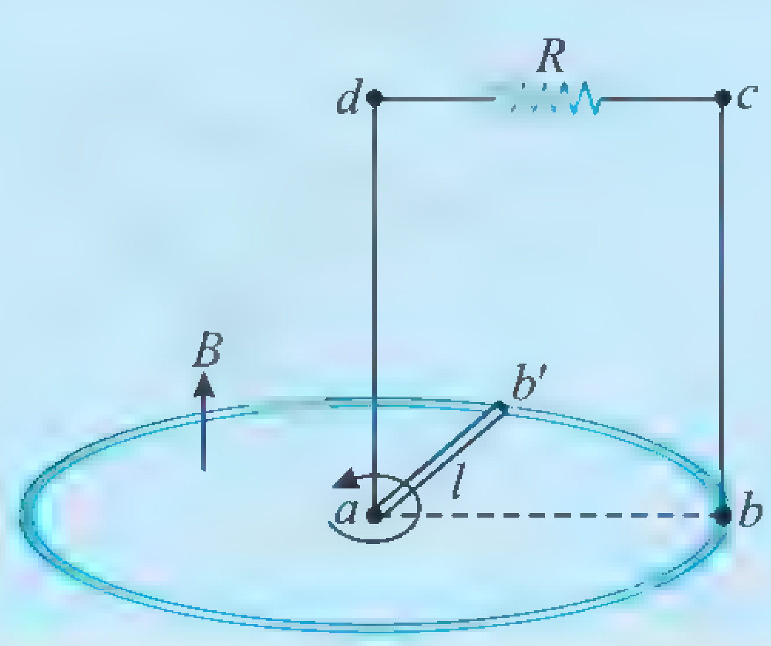
$$= \int (\omega x B\hat{i} + \omega B y\hat{j}) \cdot \vec{dl} = B\omega \int (x\hat{i} + y\hat{j}) \cdot \vec{dl}$$

$$= B\omega \int \vec{r} \cdot \vec{dl} = B\omega \int r\, dl \cos\theta = B\omega \int_0^L r dr$$

$$= \frac{1}{2} B\omega L^2$$

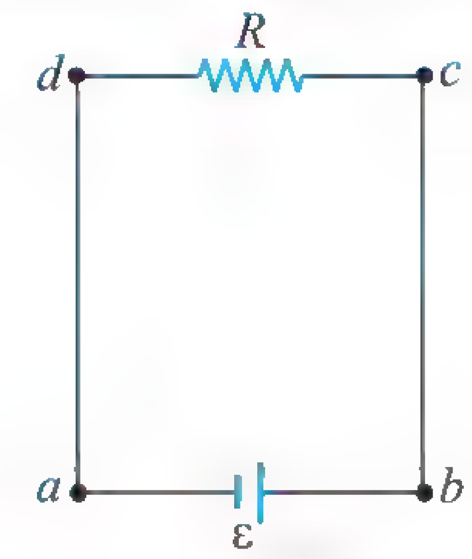
ILLUSTRATION 4.34

A metal rod ab of length l, rotates with constant angular velocity ω about one of its end in a vertically upward magnetic field B. Other end of the rod slides on a circular conducting rail as shown in figure. A resistance R is connected between the fixed points a and b. Find the induced current in the resistor.



Sol. The EMF induced across the rod $\varepsilon = \frac{1}{2} B\omega l^2$

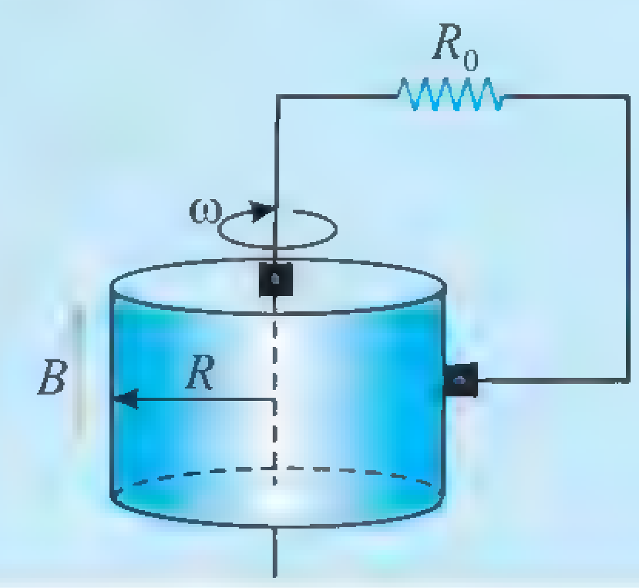
From right hand law we can verify that end b of the rod will be at higher potential. We can draw an equivalent circuit diagram of this situation as shown in figure.



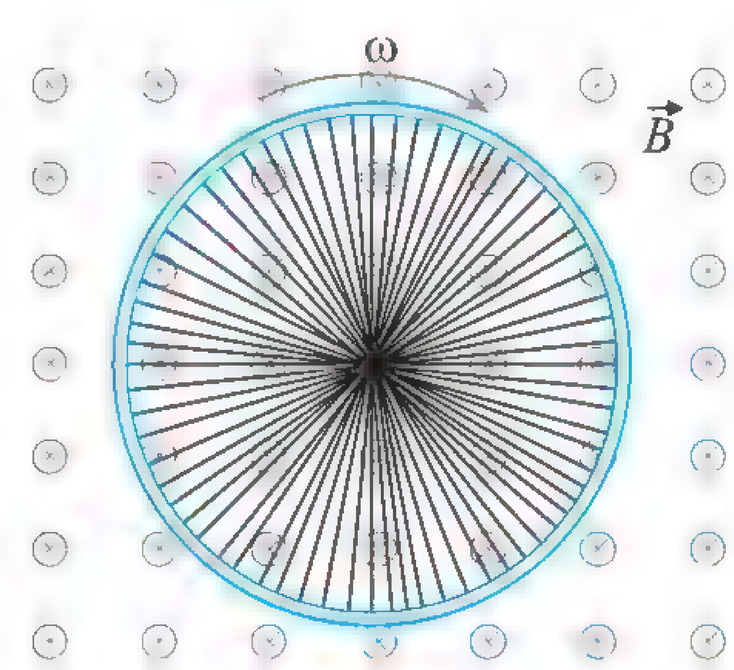
The induced current $i = \frac{\varepsilon}{R} = \frac{Bl^2\omega}{2R}$.

ILLUSTRATION 4.35

A cylinder of radius R is rotating about its axis with constant angular velocity ω in a vertically upward magnetic field B. A resistance R_0 is connected between the axis and the periphery of the cylinder as shown in figure. Find the induced current in the resistor.



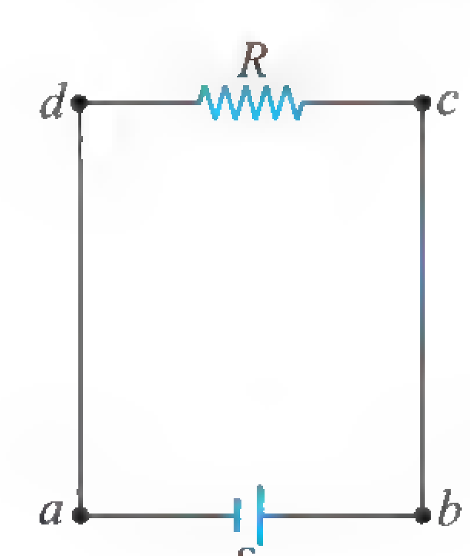
Sol. This case of rotating metallic cylinder (or disc) can considered as a ring connected with a number of spokes connected between center and circumference of ring.



The EMF induced across any of spoke, $\varepsilon = \frac{1}{2} B\omega l^2$

The potential difference across each spoke is same as they are connected in parallel. Hence the potential difference across the axis and the periphery of the cylinder should be $\frac{1}{2} B\omega l^2$.

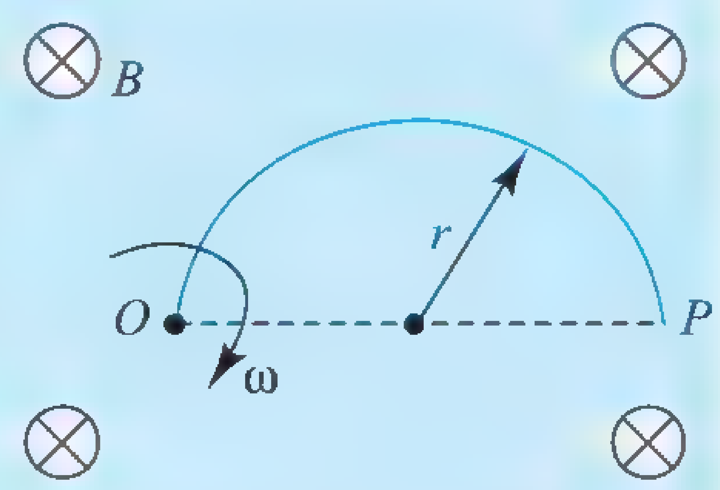
From right hand rule we can verify that the periphery of the cylinder will be at higher potential. We can draw an equivalent circuit diagram of this situation as shown in figure.



The induced current $i = \frac{\varepsilon}{R} = \frac{Bl^2\omega}{2R}$.

ILLUSTRATION 4.36

A wire is in the form of a semicircle of radius r. One end is attached to an axis about which it rotates with an angular speed ω. The axis is normal to the plane of the semicircle. The wire is immersed in a uniform magnetic field B parallel to the axis. Find the induced emf between points O and P of the semicircle.



Sol. It is given that the magnetic field is uniform.

Join the end points O and P and replace the semicircle by a straight rod of length $2r$.

We now have a straight rod rotating in a uniform magnetic field in a plane perpendicular to the magnetic field.

Therefore, the induced emf between O and P will be

$$\xi_{\text{ind}} = \frac{1}{2} B\omega(2r)^2 = 2B\omega r^2$$

From the right hand rule, we see that electrons will accumulate at end O. Therefore, end P is at a higher potential than O.

ILLUSTRATION 4.37

A metal disc of radius $R = 25$ cm rotates with a constant angular velocity $\omega = 130$ rad s^{-1} about its axis. Find the potential difference between the center and rim of the disc if

(a) the external magnetic field is absent,

(b) the external uniform magnetic field $B = 5.0$ mT is directed perpendicular to the disc.

Sol.

(a) Centripetal force required for circular motion of electron is generated by a radial electric field caused by the redistribution of the electrons in the disc.

$$F = eE = mr\omega^2 \quad \Rightarrow \quad E = \frac{mr\omega^2}{e}$$

From $dV = -E\,dr$, we have

$$dV = -\frac{m\omega^2}{e} r\,dr$$

$$\Rightarrow \int_{v_1}^{v_2} dV = -\frac{m\omega^2}{e}\int_0^R r\,dr$$

$$V_1 - V_2 = \frac{m\omega^2 R^2}{2e} = \frac{(9.1\times10^{-31})(130)^2(0.25)^2}{(2)(1.6\times10^{-19})}$$

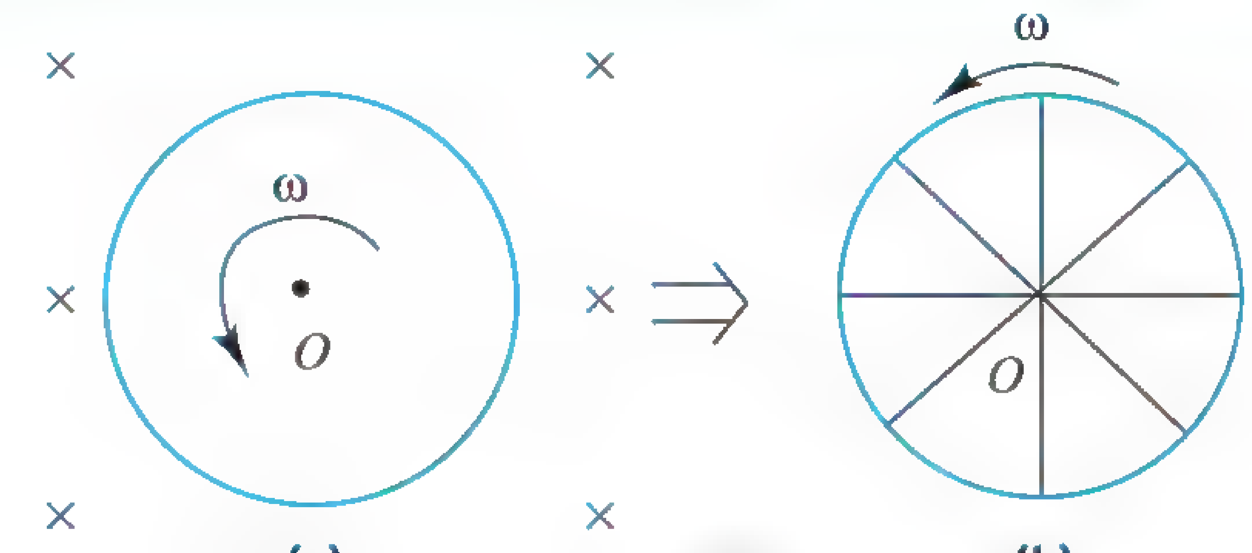
$$= 3.0 \times 10^{-9}\text{ V} = 3.0\text{ nV}$$

$V_1 > V_2$, i.e., potential at center is more than the potential at edge.

(b) A disc may be assumed to be made up of a large number of radial, conducting, differential elements rotating with angular velocity ω about the center of disc O. Thus,

$$V_{\text{center}} - V_{\text{edge}} = \frac{1}{2} BR^2\omega$$

Now, $V_{\text{center}} > V_{\text{edge}}$ for anticlockwise rotation and $V_{\text{edge}} > V_{\text{center}}$ for clockwise rotation.



(a) (b)

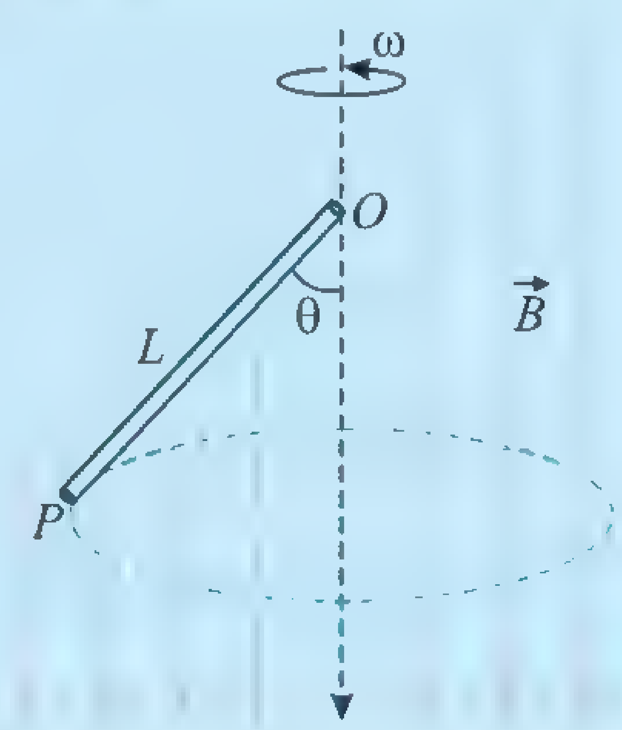
Substituting the values, we have

$$V_{\text{center}} - V_{\text{edge}} = \frac{1}{2} \times 5.0 \times 10^{-3} \times 0.25 \times 0.25 \times 130$$

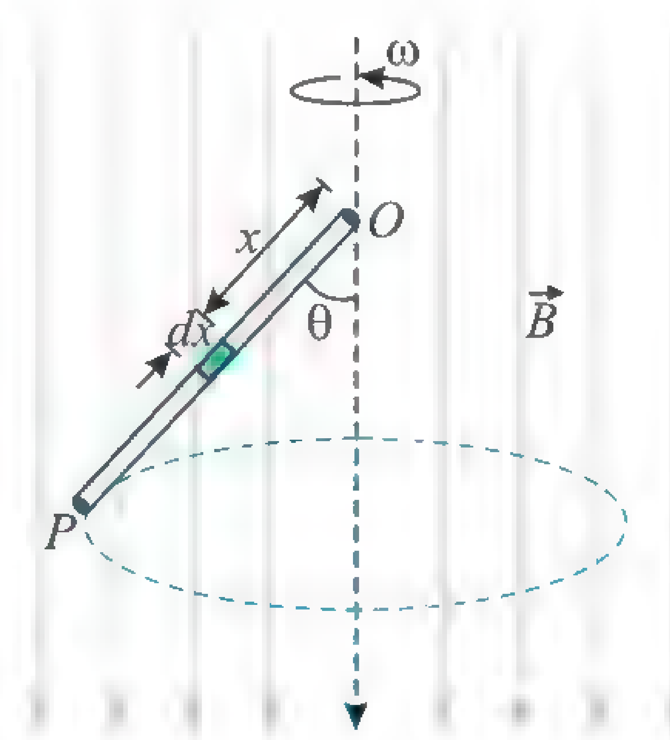
$$= 0.02\text{ V} = 20\text{ mV}$$

ILLUSTRATION 4.38

A metallic rod of length L rotates in the form of a conical pendulum with an angular velocity ω about a fixed vertical axis passing through its end O as shown in figure. A uniform magnetic field B present in vertical downward direction. The rod makes an angle θ with the direction of the magnetic field. Find the emf induced across the ends of the rod.



Sol. Let us consider an elemental length dx at a distance x from O. The radius of rotation of the element is $x \sin \theta$. The speed of the element is perpendicular to the plane of the figure and its magnitude is $v = \omega(x\sin\theta)$



Emf induced in the element is

$d\varepsilon = Bv(dx)\sin\theta$ [θ is angle between dx and B]

$d\varepsilon = B\omega\sin^2\theta\, xdx$ ...(i)

For total emf can be calculated by integrating Eq. (i)

$$\therefore \quad \varepsilon = \int d\varepsilon = B\omega\sin^2\theta\int_0^L xdx = \frac{1}{2}B\omega\sin^2\theta L^2$$

From right hand rule we can verify that the free electrons experience force towards P. There will be deficiency of electron at O, hence O is positive.

Motional EMF When the Magnetic Field is Non-Uniform

In some of the cases, motion of a conductor may be in a non-uniform magnetic field. Take the following steps while calculating motional emf.

Step 1: Determine the magnetic field at all points on the rod.

Step 2: Consider a small element at some distance from one end of the rod.

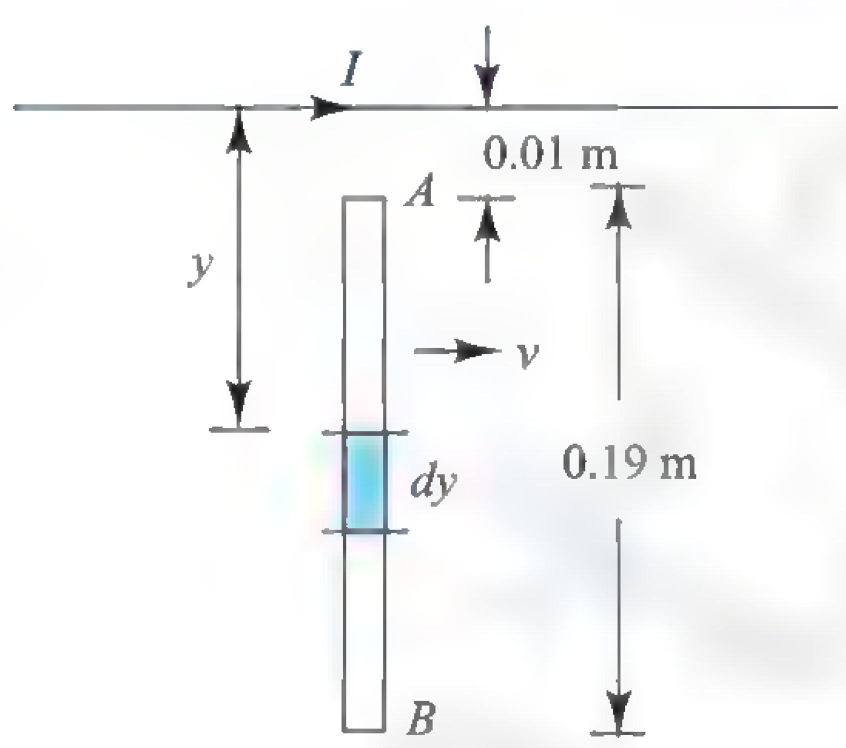
Step 3: Assuming B to be uniform over this element, calculate the potential difference across this element using the procedures outlined earlier.

Step 4: Integrate over the entrie rod to calculate the total induced emf.

Let us learn to calculate the induced emf through the illustrations given below.

Illustration 4.39

A copper rod of length 0.19 m is moving with uniform velocity 10 m s^{-1} parallel to a long straight wire carrying a current of 5.0 A. The rod is perpendicular to the wire with its ends at distances 0.01 and 0.2 m from it. Calculate the emf induced in the rod.



Sol. As shown in figure, consider an element of length dy at a distance y from the wire, then at this position of the element, the magnetic field due to the current-carrying wire PQ will be $B = \frac{\mu_0}{4\pi}\frac{2I}{y}$ into the plane of the paper.

So, the emf induced in the element $d\varepsilon = Bv\,dy = \frac{\mu_0}{4\pi}\frac{2I}{y}v\,dy$

and hence the emf induced across the ends of the rod due to its motion in the field of the wire,

$$\varepsilon = \int_a^b d\varepsilon = \frac{\mu_0}{4\pi}2Iv\int_a^b \frac{dy}{y}, \text{ i.e., } \varepsilon = \frac{\mu_0}{4\pi}2Iv\log_e\left(\frac{b}{a}\right)$$

Substituting the given data with $b = (a + l)$,

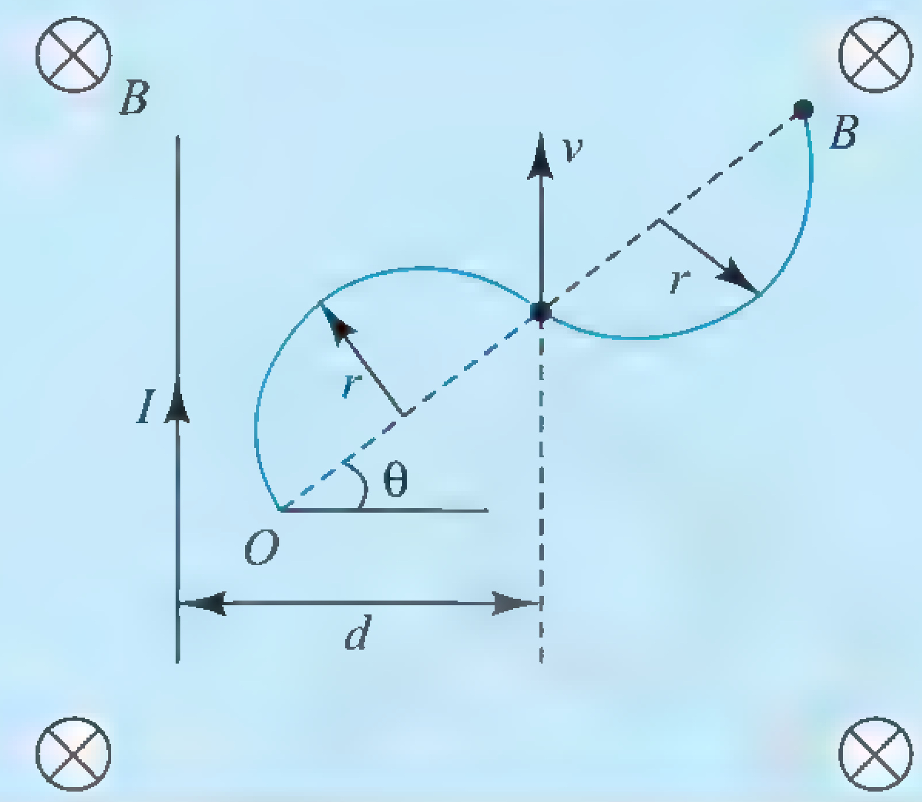
$$\varepsilon = 10^{-7}\times 2\times 5\times 10\log_e\frac{0.20}{0.01}$$

$$= 10^{-5}\times \log_e 20$$

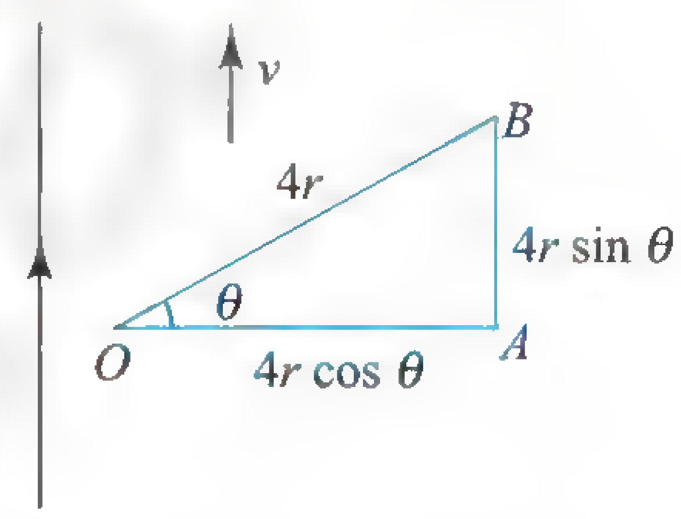
$$\simeq 30\ \mu\text{V}$$

Illustration 4.40

An infinite wire carries a current I. An S-shaped conducting rod of two semicircles each of radius r is placed at an angle θ to the wire. The center of the conductor is at a distance d from the wire. If the rod translates parallel to the wire with a velocity v as shown in figure, calculate the emf induced across the ends OB of the rod.



Sol. If we join the end points O and B and replace the two semicircles by a straight rod of length $4r$ (figure).



Induced emf in straight rod OB will be same as in the actual conductor. Now this straight rod can be replaced by a combination of two rods OA and AB. Induced emf in OB will be same as the sum of induced emf in OA and induced emf in AB. But induced emf in AB will be zero because its velocity will be parallel to its length. Hence, induced emf in OA will be the net induced emf in actual conductor.

We now have a rod of effective length $4r\cos\theta$ translating in a non-uniform magnetic field. Consider a small element of the wire of length dx located at a distance x from the wire. The magnetic field at this element is $B = \frac{\mu_0 I}{2\pi x}$

The potential difference across this element is $dV = \frac{\mu_0 I}{2\pi x}v\,dx$.

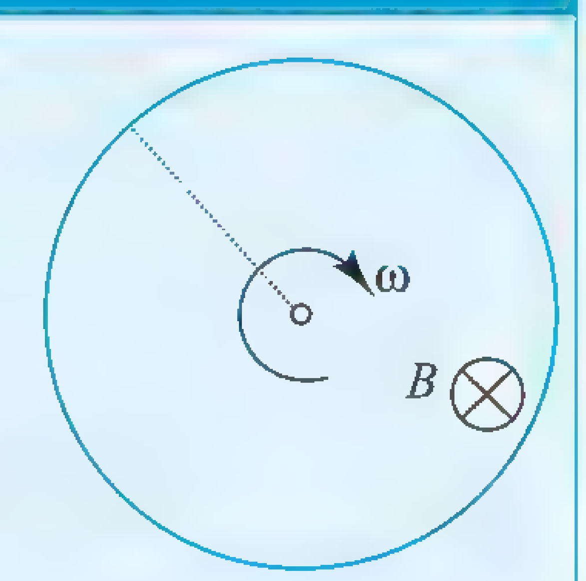
The potential difference across the ends of the rod can be calculated by integrating over the whole end. Therefore,

$$V = \int dV = \int_{d-2r\cos\theta}^{d+2r\cos\theta}\frac{\mu_0 I}{2\pi x}v\,dx = \frac{\mu_0 Iv}{2\pi}\ln\left[\frac{d+2r\cos\theta}{d-2r\cos\theta}\right]$$

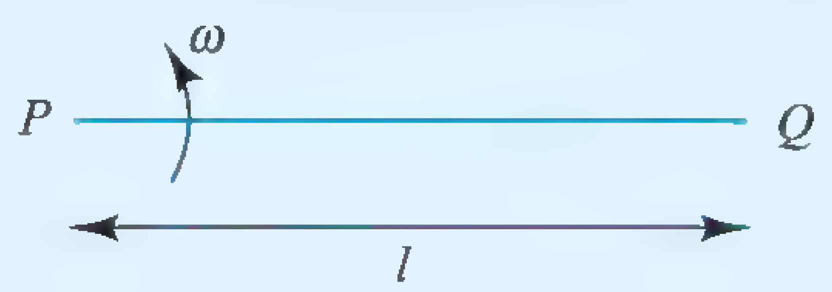
From the right hand rule, we see that electrons will accumulate at end B. Therefore, end O is at a higher potential than B.

Concept Application Exercise 4.3

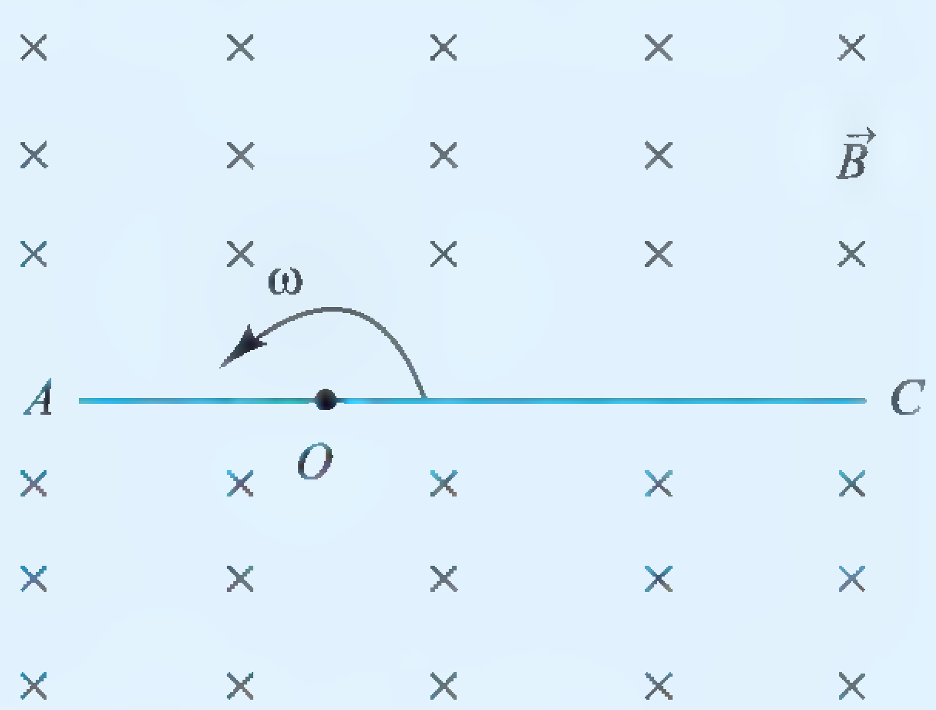
1. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring (figure). A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring.



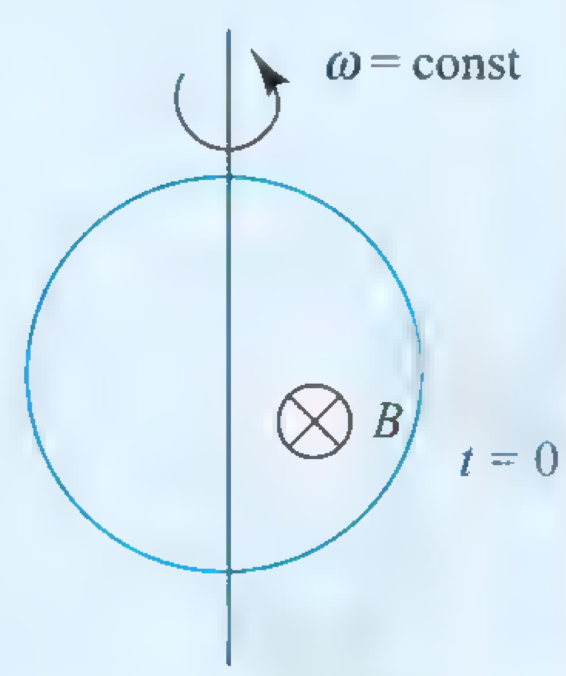
2. A rod PQ of length l is rotating about end P, with an angular velocity ω (figure). Due to centrifugal forces the free electrons in the rod move toward the end Q and an emf is created. Find the induced emf.



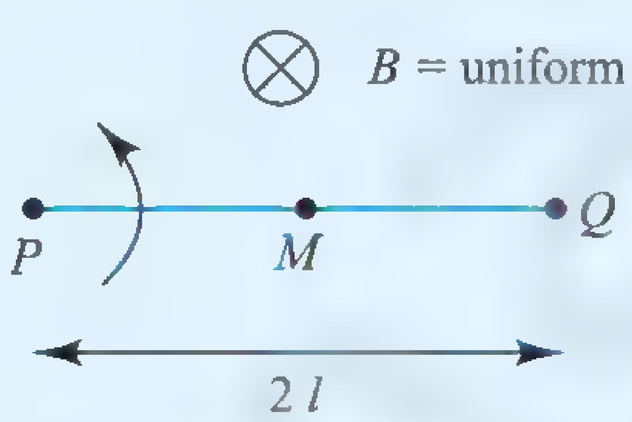
3. A conducting rod AC of length $4l$ is rotated about point O in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. $AO = l$ and $OC = 3l$. Find $V_A - V_C$.



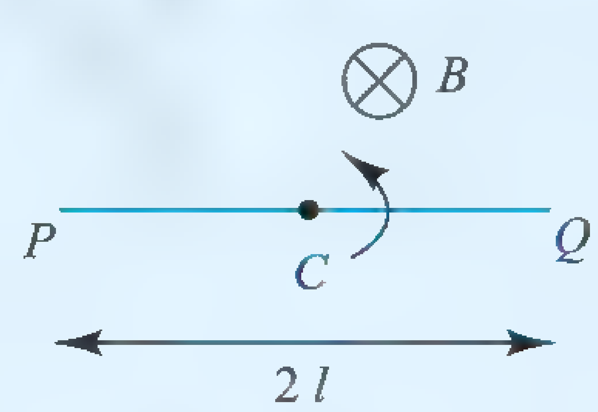
4. A ring rotates with angular velocity ω about an axis in the plane of the ring which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring (figure). Find the emf induced in the ring as a function of time.



5. Rod PQ of length $2l$ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod (figure). Point M is the mid-point of the rod. Find the induced emf between M and Q if the potential between P and Q is 100 V.

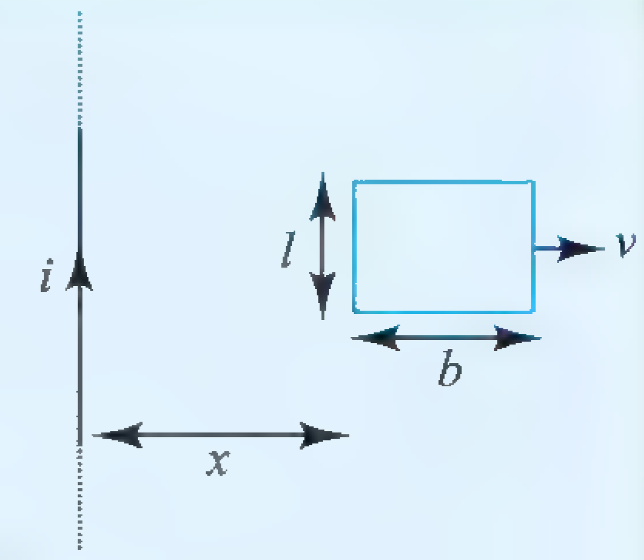


6. Rod PQ of length $2l$ is rotating about its midpoint C in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod (figure). Find the induced emf between PQ and PC. Draw the circuit diagram of parts PC and CQ.

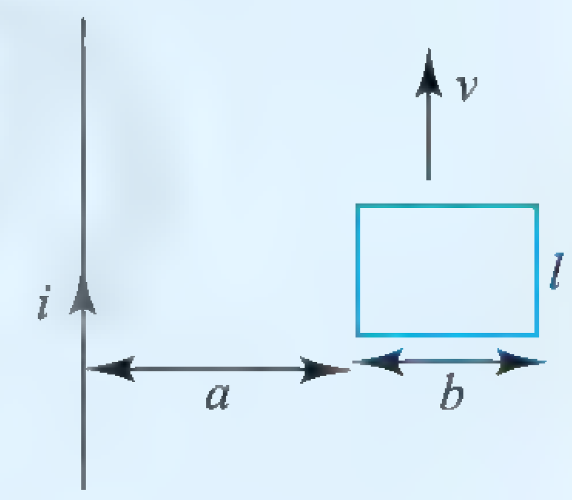


7. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

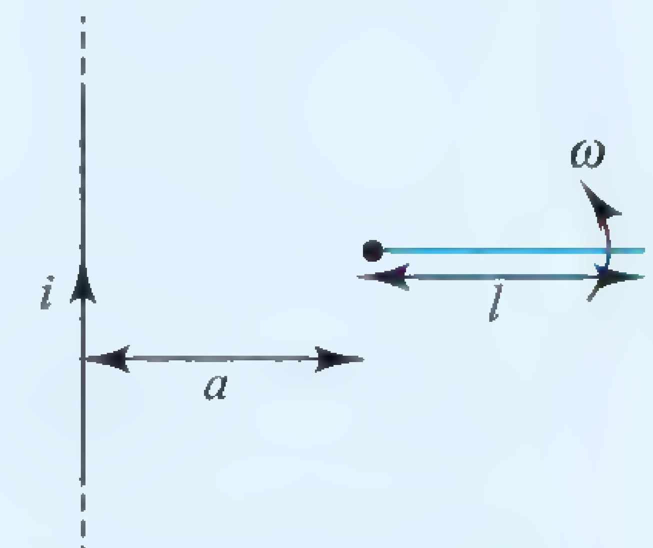
8. A rectangular loop, as shown in figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



9. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop (figure) if its nearest end is at a distance a from the wire. Draw equivalent electrical diagram.



10. A rod of length l is rotating with an angular speed ω about one of its ends which is at a distance a from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in figure.



ANSWERS

1. 0 **2.** $\dfrac{m_e\omega^2 l^2}{2e}$ **3.** $4B\omega l^2$ **4.** $BA\omega \sin \omega t$

5. 75 V **6.** $\varepsilon_{PQ} = 0, \varepsilon_{PC} = \dfrac{B\omega l^2}{2}$ **7.** $\dfrac{\mu_0 ilv}{2\pi x}$

8. $\dfrac{\mu_0 iblv}{2\pi x(l+b)}$ **9.** 0 **10.** $\dfrac{\mu_0 i\omega}{2\pi}\left[l - a\ln\left(\dfrac{l+a}{a}\right)\right]$

INDUCED ELECTRIC FIELD

INDUCED EMF IN A STATIONARY CONDUCTOR

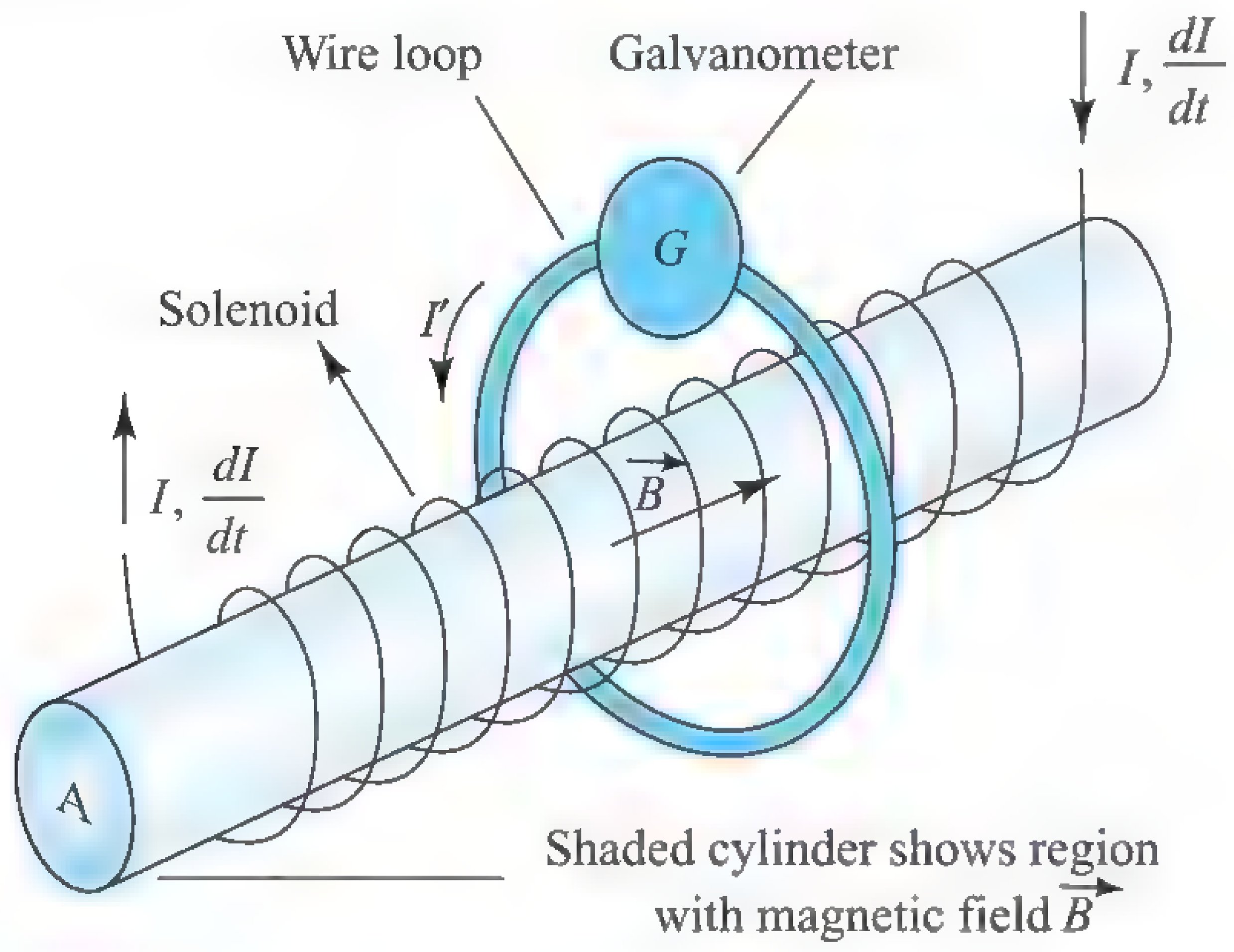
According to Faraday's law, it is the relative motion between the loop and the magnet which produces the induced emf; it does not matter whether the loop moves toward the magnet or the magnet moves toward the loop. When the loop moves toward the magnet, it is the magnetic force which drives the charge to flow. But what causes the induced current in a stationary loop when magnet moves toward it? A magnetic field cannot exert a force on a stationary conductor. Whenever a magnetic field is varying with time, an induced electric field E is produced in any closed path whether in matter or in empty space.

$$\oint \vec{E}\cdot d\vec{l} = -A\frac{dB}{dt}$$

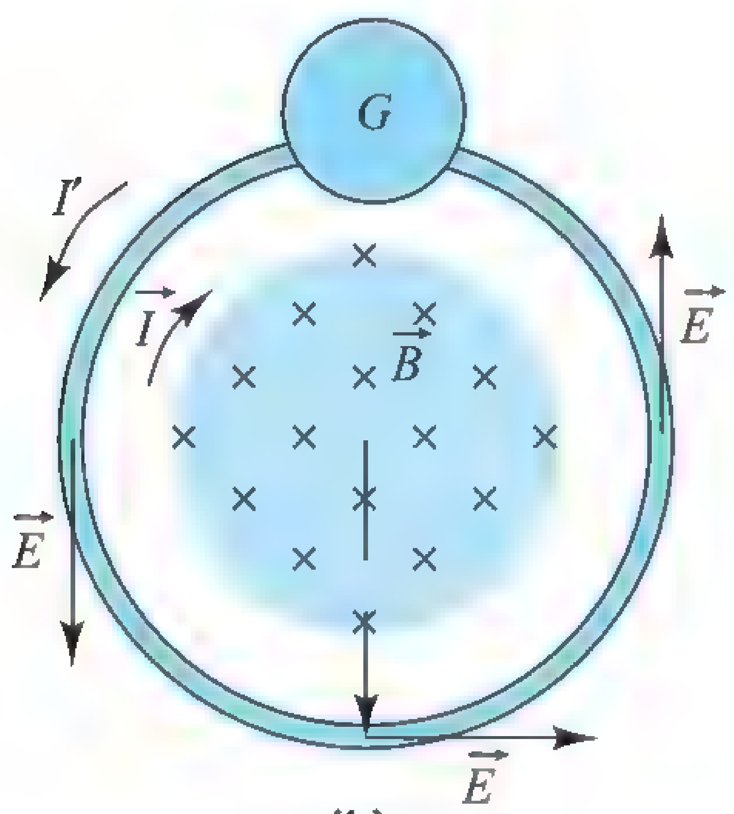
The induced electric field is different from the electrostatic field of the Coulomb field.

- Unlike electrostatic field, the lines of induced field form closed loops. It is also called a circuital field or vortex field.
- It is not a conservative field, i.e., $\oint \vec{E}\cdot d\vec{l} \neq 0$.

- The line integral of the electrostatic field between any two points is the potential difference while the line integral of the induced electric field between any two points is the electromotive force.



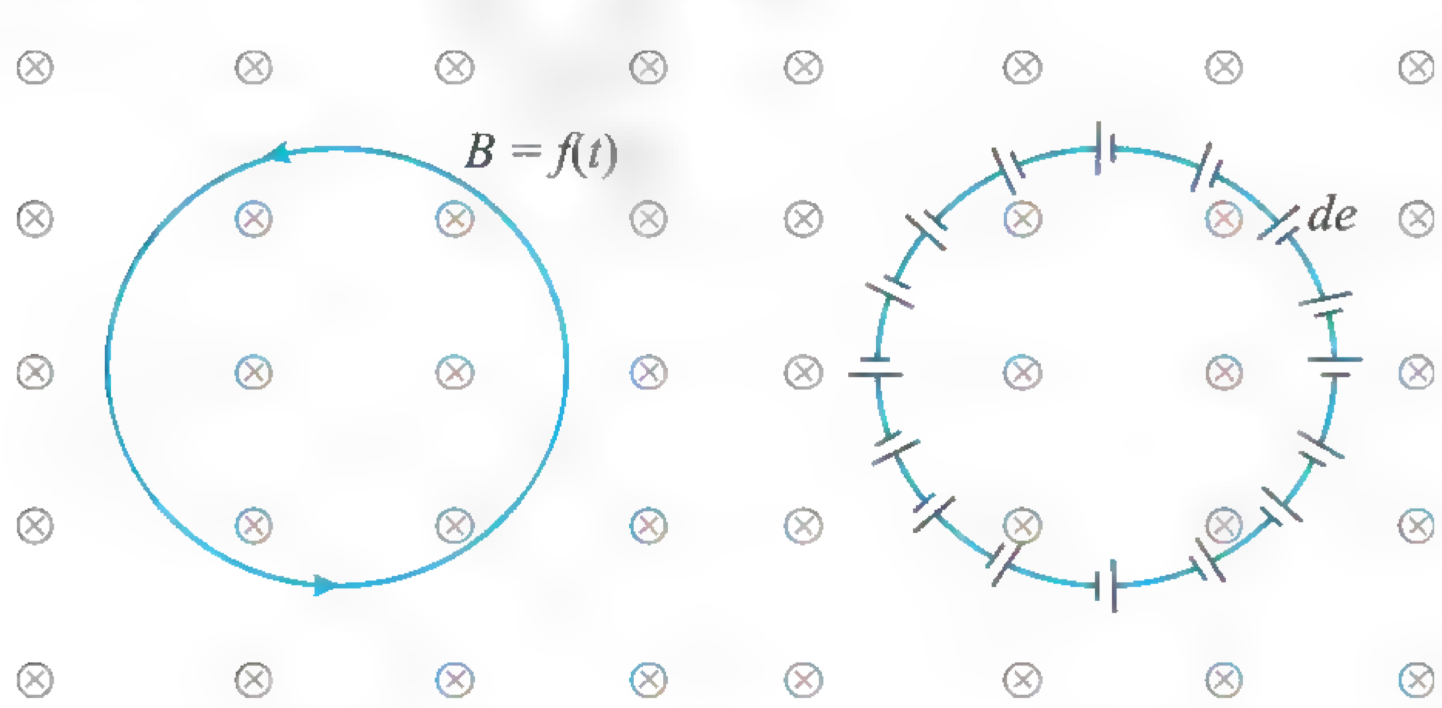
(a)



(b)

Figure (a) shows the windings of a long solenoid carrying current I that is increasing at a rate of dI/dt. The magnetic flux in the solenoid is increasing at a rate of $d\Phi_B/dt$, and this changing flux passes through a wire loop. An emf $\varepsilon = -d\Phi_B/dt$ is induced in loop, inducing a current I' that is measured by the galvanometer G. Figure (b) shows the cross-sectional view.

Figure (a) shows a single turn circular coil placed in time varying magnetic field in inward direction of which magnitude is increasing with the function $B = f(t)$. According to Lenz's law due to increase in magnetic flux through the coil an anticlockwise current is induced in it to oppose the increasing flux. Actually, this induced current is caused by the EMF induced in every element of the coil as shown in Fig. (b). Every small element of the coil behaves like an elemental EMF de and all such elemental EMFs are in series. Sum of all these EMFs in the coil is called loop EMF .



(a) (b)

Note:

There are two basic mechanisms of induced emf generation.

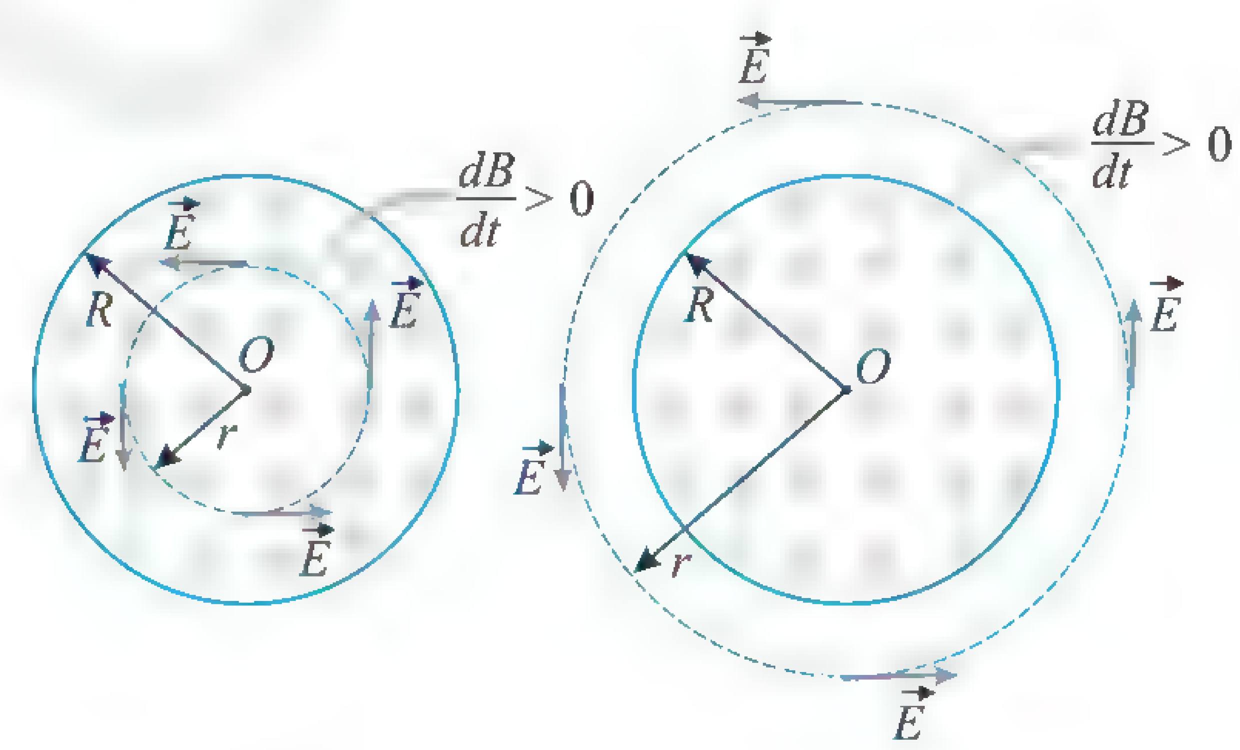
1. The first one involves the motion of a conductor relative to magnetic field lines, called the motional emf.
2. The second one involves the generation of an electric field associated with a time-varying magnetic field. In the modified form, Faraday's law may be stated as:

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} - \vec{A} \cdot \frac{d\vec{B}}{dt}$$

Time-Varying Magnetic Field

Consider a conducting loop of area A in a uniform but time-varying magnetic field. Rate of change of magnitude of magnetic field = dB/dt for the loop, flux linked with it = $BA = \phi$ (say) (take area vector directed along $\vec{B}$).

Hence, rate of change of flux ϕ is $A\dfrac{dB}{dt}$ and hence induced emf $= -A\dfrac{dB}{dt}$.



$r < R$ (a) $r > R$ (b)

For non-zero values of dB/dt, there could be a definite current in the loop, whose direction can be obtained using Lenz's rule. For example, if $\dfrac{dB}{dt} > 0$, i.e., B is increasing with time, magnetic field produced by the induced current would oppose the existing magnetic field. Hence, the induced current would be anticlockwise.

The current in the loop can be easily known if the resistance of the loop is known as $I = \varepsilon/R$.

The field associated with the induced emf in case of time-varying magnetic field is non-conservative as, then only, we would have non-zero value for $\oint \vec{E}_n \cdot d\vec{l}$. Here $\vec{E}_n$ denotes the induced field caused by the time-varying magnetic field.

For the path described by the loop, $\varepsilon_{\text{induced}} = -\dfrac{d\phi}{dt} = \oint \vec{E}_n \cdot d\vec{l}$.

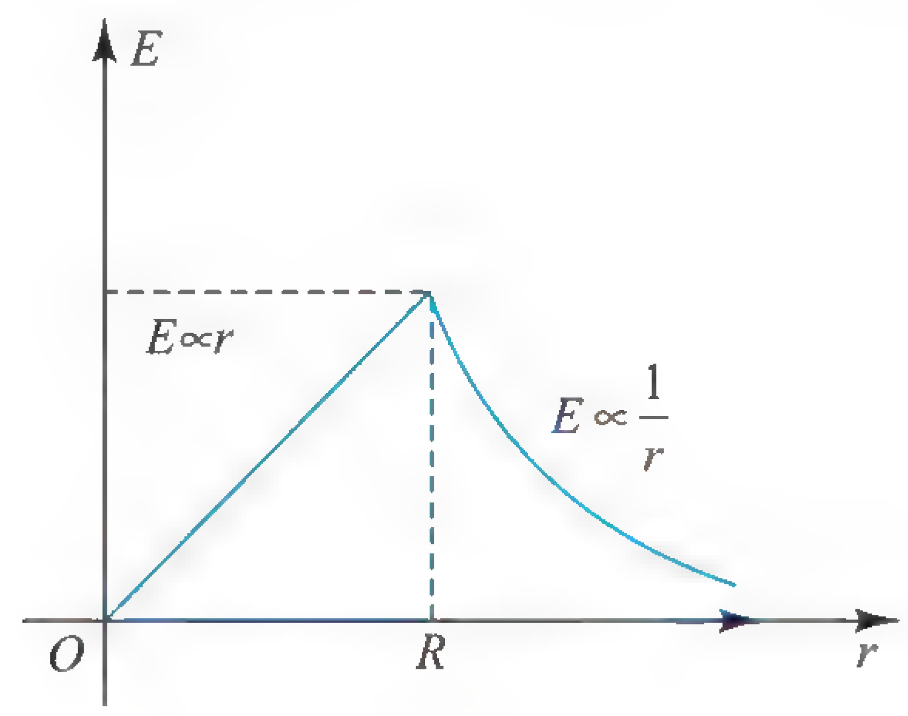
Consider a magnetic field where B (magnitude of the magnetic field) is a function of $r (r < R)$. The distance of the point from O is shown in Fig. (a). For the circular path.

For $r < R$, $E_n(2\pi r) = \pi r^2 \left|\dfrac{dB}{dt}\right| \Rightarrow E_n = \dfrac{r}{2}\left|\dfrac{dB}{dt}\right|$

Now follow Fig. (b).

For $r > R$, $E_n 2\pi r = \pi R^2 \left|\frac{dB}{dt}\right| \Rightarrow E_n = \frac{R^2}{2r}\left|\frac{dB}{dt}\right|$

Direction of $\vec{E}_n$ can be easily obtained as it would be responsible for the induced current when a conducting loop is placed on the given path. For example, in the present case, for $\frac{dB}{dt} > 0$, path is in anticlockwise direction. Variation of E with r is shown in figure.



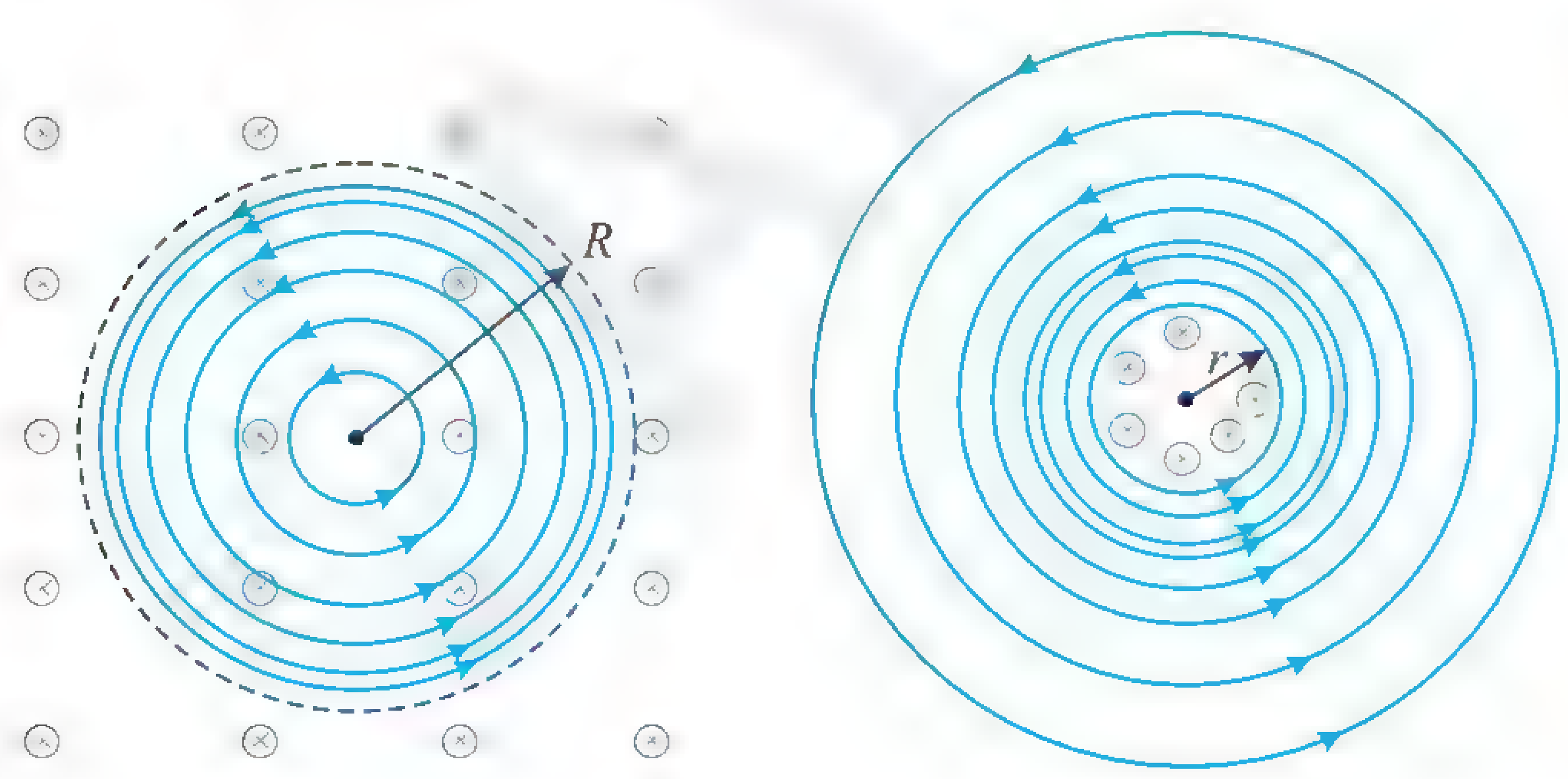
Electric Field Lines of Forces

The induced electric field in time varying magnetic field have closed loop electric lines of forces as there are no static charges or source of electric field is present in this situation for which the field is non-conservative in nature as for a charge going round the loop electric force acts in same direction and hence work done is non zero. Thus for the induced electric field in time varying magnetic field we have for a general closed path

$$\oint \vec{E} \cdot \vec{dl} \neq 0$$

Thus in regions of time varying magnetic fields at any point in space we cannot define potential as electric field in this region is non-conservative in nature.

Inside the magnetic field region the electric field increases with distance from axis of region thus the density of electric lines of forces increases as we move away from the central axis. The configuration of electric lines is shown in Fig. (a). But outside the magnetic field region the electric field decreases with distance from axis of region thus the density of electric lines of forces decreases as we move away from the central axis. The configuration of electric lines is shown in Fig. (b).



For $r < R$
$E \propto r$
(a)

For $r > R$
$E \propto \frac{1}{r}$
(b)

Illustration 4.41

A thin non-conducting ring of mass m carrying a charge q can freely rotate about its axis. At the initial moment, the ring was at rest and no magnetic field was present. Then a uniform magnetic field was switched on, which was perpendicular to the plane of the ring and increased with time according to a certain law: $\frac{dB}{dt} = k$.

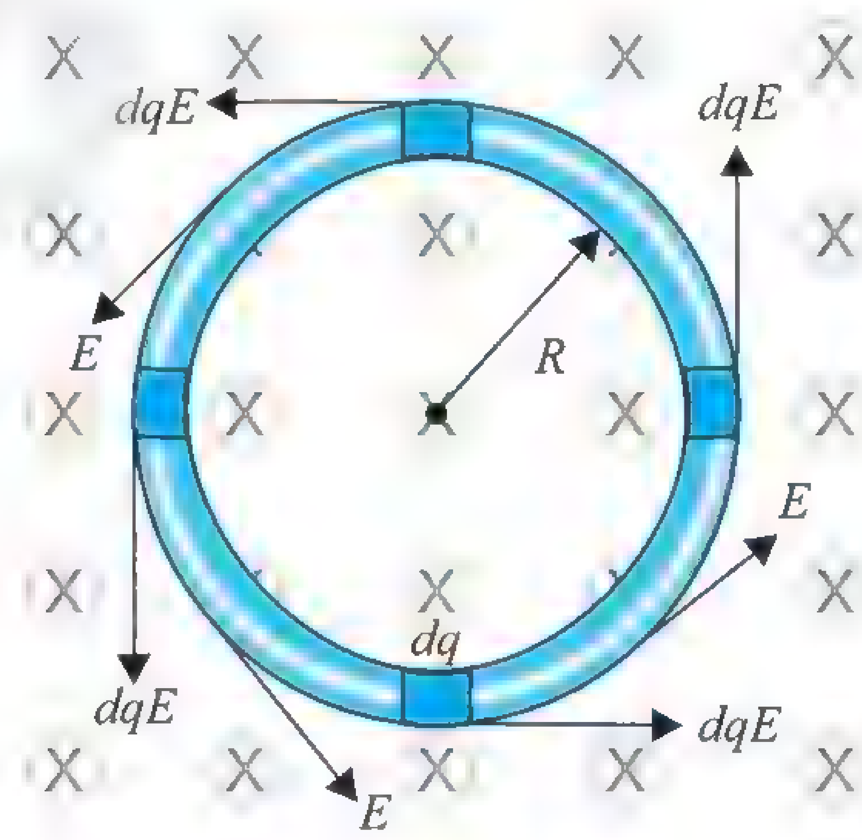
Find the angular velocity ω of the ring as a function of k.

Sol. $E = \frac{1}{2}R\frac{dB}{dt}$

Electric force on charge dq is given by

$$dF = Edq = \frac{1}{2}R\left(\frac{dB}{dt}\right)dq$$

$$\Rightarrow \quad d\tau = RdF$$



$$\Rightarrow \quad d\tau = \frac{1}{2}R^2\left(\frac{dB}{dt}\right)dq$$

$$\Rightarrow \quad \tau = \frac{1}{2}R^2kq$$

Now, $\tau = \frac{1}{2}R^2kq$

$$\Rightarrow \quad I\alpha = \frac{1}{2}R^2kq \Rightarrow mR^2\alpha = \frac{1}{2}R^2kq$$

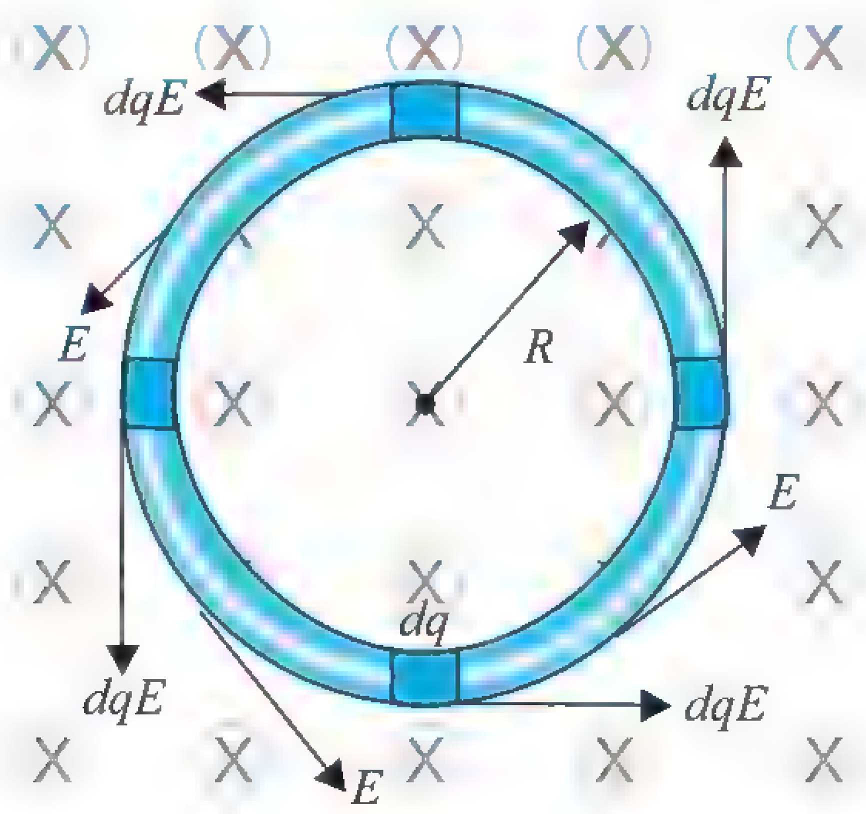
$$\Rightarrow \quad \alpha = \frac{kq}{2m} \Rightarrow \omega = \frac{kq}{2m}t$$

Illustration 4.42

A thin non-conducting ring of mass m carrying a charge q can rotate freely about its axis. At $t = 0$, the ring was at rest and no magnetic field was present. Then suddenly a magnetic field B was set perpendicular to the plane. Find the angular velocity acquired by the ring.

Sol. Due to the sudden charge in flux, an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$E = \frac{r}{2}\frac{dB}{dt}$$



Force experienced by an element of ring $dF = dqE$

Torque experienced by this element $d\tau = dFr = dq\,Er$

Total torque experienced by ring: $\tau = qEr$

$$\Rightarrow \quad \tau = q\frac{r}{2}\frac{dB}{dt}r = \frac{q}{r}r^2\frac{dB}{dt}$$

Angular impulse experienced

$$L = \int \tau\,dt = \frac{qr^2}{2}\int\frac{dB}{dt}dt = \frac{qr^2}{2}B$$

$$L = I\omega \;\Rightarrow\; mr^2\omega = \frac{qr^2}{2}B \;\Rightarrow\; \omega = \frac{qB}{2m}$$

ILLUSTRATION 4.43

A non-conducting ring of mass m and radius R has charge Q uniformaly distributed over its circumference. The ring is placed on a rough horizontal surface such that the plane of the ring is parallel to the surface. A vertical magnetic field $B = B_0t^2$ tesla is switched on. After 2 s from switching on the magnetic field, the ring is just about to rotate about vertical axis through its center.

(a) Find friction coefficient μ between the ring and the surface.

(b) If magnetic field is switched off after 4 s, then find the angular velocity of the ring just after switching off the magnetic field.

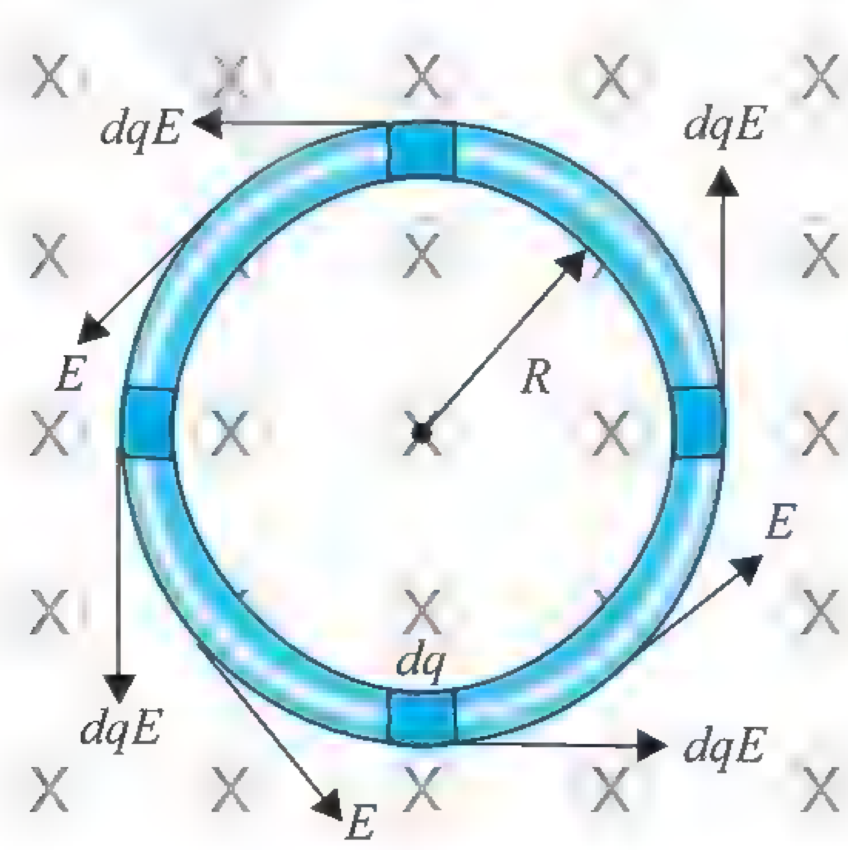
(c) Find the angle rotated by the ring before coming to rest after switching off the magnetic field.

Sol.

(a) $E = \dfrac{R}{2}\dfrac{dB}{dt} \;\Rightarrow\; E = B_0Rt$

Force on the ring $F = QE = B_0QRt$. This force is tangential to the ring. The ring starts rotating when torque of this force is greater than the torque due to maximum friction ($f_{\max} = \mu mg$).

$$\tau_F \ge \tau_{\text{fric, max}},\; FR \ge \mu mgR \;\Rightarrow\; F > \mu mg$$



$$B_0QRt = \mu mg$$

Hence, $\mu = \dfrac{B_0QRt}{mg}$

Given, $t = 2$ s $\;\Rightarrow\; \mu = \dfrac{2B_0QR}{mg}$

(b) After 2 s

Net torque $\tau = \tau_F - \tau_{f\max} = B_0QR^2t - \mu mgR$

$$= B_0QR^2t - \left(\frac{2B_0QR}{mg}\right)mgR$$

$$\Rightarrow \quad \tau = B_0QR^2(t-2) \;\Rightarrow\; I\alpha = B_0QR^2(t-2)$$

$$mR^2\left(\frac{d\omega}{dt}\right) = B_0QR^2(t-2)$$

$$\Rightarrow \quad d\omega = \frac{B_0QR^2}{mR^2}(t-2)\,dt$$

$$\Rightarrow \quad \int_0^{\omega} d\omega = \frac{B_0Q}{m}\int_2^4 (t-2)\,dt$$

$$\Rightarrow \quad \omega = \frac{2B_0Q}{m}$$

(c) If magnetic field is switched off after 4 s, only force present is frictional force which will retard the motion.

Retarding torque $\tau = \tau_{\text{friction}}$

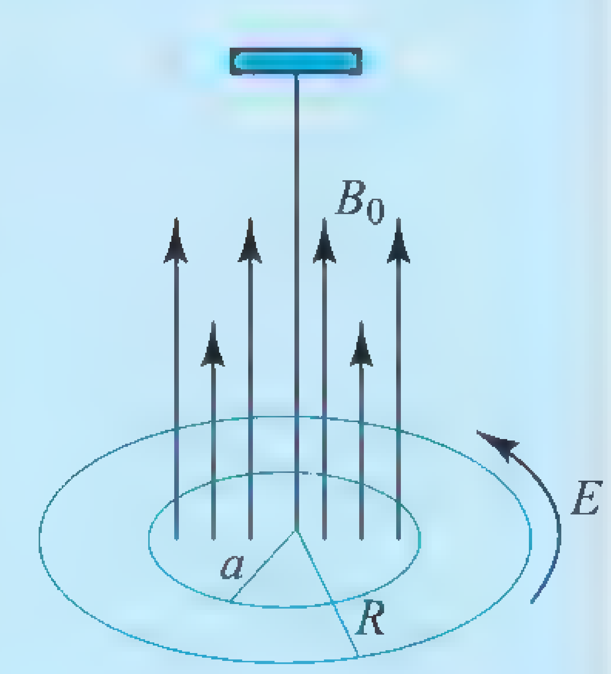
Angular retardation $\alpha = \dfrac{\tau_{\text{friction}}}{I} \;\Rightarrow\; \alpha = \dfrac{\mu mgR}{mR^2} = \dfrac{\mu g}{R}$

Using $\omega^2 = \omega_0^2 + 2\alpha\theta \;\Rightarrow\; 0 = \left(2\dfrac{B_0Q}{m}\right)^2 - 2\left(\dfrac{\mu g}{R}\right)\theta$

$$\Rightarrow \quad \theta = 2\left(\frac{B_0Q}{m}\right)^2\frac{R}{\mu g}$$

ILLUSTRATION 4.44

A line charge with linear charge density λ is wound around an insulating disc of mass M and radius R, which is then suspended horizontally as shown in figure, so that it is free to rotate. In the central region, of radius a, there is a uniform magnetic field B_0, pointing up. Now the magnetic field is switched off, which causes the disc to rotate.



Find the angular speed with which the disc starts rotating.

Sol. The induced electric field E due to the changing magnetic field is given by (from Faraday's law)

$$\Rightarrow \quad E\cdot 2\pi R = -\pi a^2\frac{dB}{dt} \;\Rightarrow\; E = \frac{-a^2}{2R}\frac{dB}{dt}$$

Hence, induced electric field is tangential to the disc as shown in figure and its magnitude is $E = \dfrac{a^2}{2R}\dfrac{dB}{dt}$.

This electric field causes the disc to rotate. Now torque on the disc is $\tau = (\lambda 2\pi R)\, ER = \pi\lambda a^2 R\,\dfrac{dB}{dt}$.

Angular impulse:

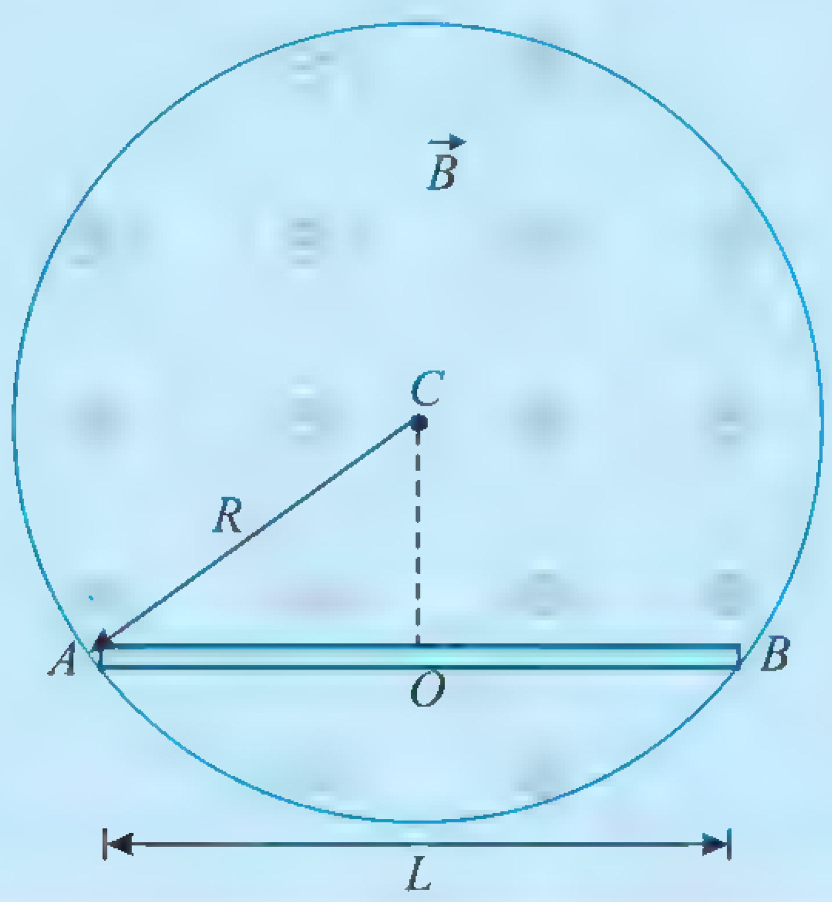
$$L = \int \tau dt = \int \pi\lambda a^2 R \frac{dB}{dt} dt = \pi\lambda a^2 R \int dB = \pi\lambda a^2 R B_0$$

Now $\quad L = I\omega$

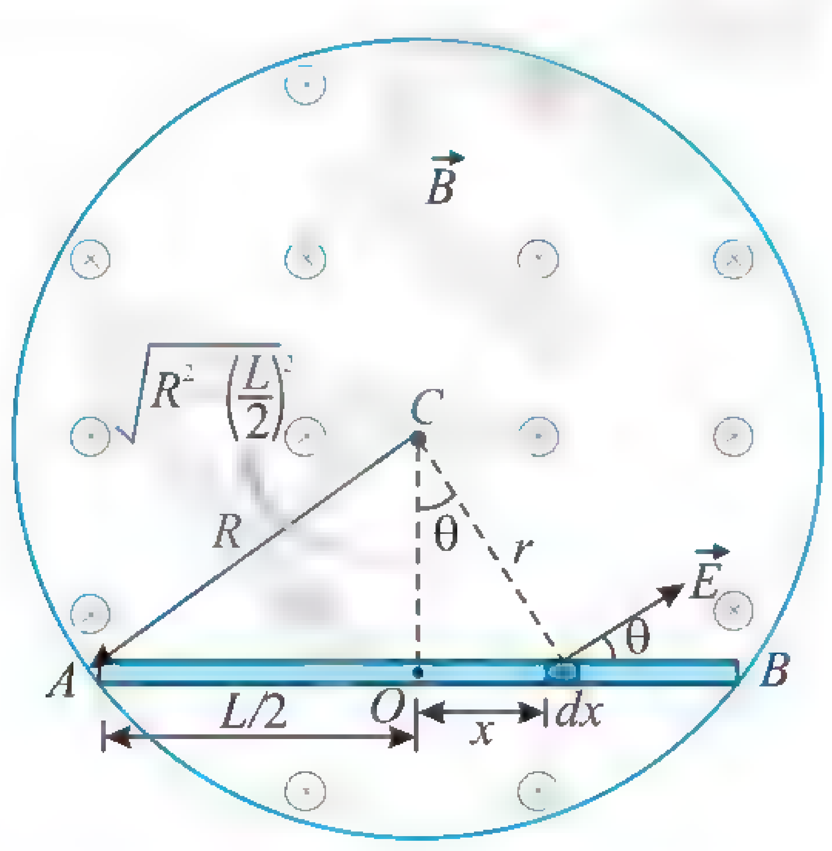
$$\Rightarrow \quad \pi\lambda a^2 R B_0 = \frac{MR^2}{2}\omega \quad \Rightarrow \quad \omega = \frac{2\pi\lambda a^2 B_0}{MR}$$

ILLUSTRATION 4.45

A cylindrical region of radius R is filled with a uniform magnetic field B as shown in the figure. A metal rod AB of length L is placed inside the field such that its ends are symmetrically located with respect to the center of the circular cross section of the region. If the magnetic field is changed at a rate $\dfrac{dB}{dt} = \alpha$, find the emf induced across the metal rod.



Sol. When the rod is placed in time varying magnetic field then due to induced electric field free electrons of the metal are displaced which causes an opposing static electric field to be developed inside the metal rod due to static charges and at every point inside the wire its magnitude is equal to that of non-conservative induced electric field of the region. To understand this we will discuss on an illustration shown in figure.



Let us consider an element of length dx on rod at a distance x from point O as shown in figure above. The induced electric field at the location of element dx can be given by

$$E_i = \frac{1}{2} r \frac{dB}{dt} = \frac{1}{2} r\alpha \quad \text{...(i)}$$

Due to the component of this electric field along the length of the rod its free electrons will drift toward left and left end A of rod becomes negative and due to deficiency of electrons end B becomes positive. These charges establishes another electric field inside the rod from B to A. In steady condition this electric field balances the component of E_i along the rod so that no further drift of free electrons take place. Thus the electric field inside the rod due to static charges which is conservative in nature is given as

$$E = E_i \cos\theta \quad \text{...(ii)}$$

$$\Rightarrow \quad E = \frac{1}{2} r\alpha \times \frac{\sqrt{R^2 - \left(\frac{L}{2}\right)^2}}{r}$$

$$\Rightarrow \quad E = \frac{1}{2}\alpha \sqrt{R^2 - \left(\frac{L}{2}\right)^2} \quad \text{...(iii)}$$

The expression in above Eq. (iii) shows that the static electric field inside the rod is uniform and depends only upon the distance of axis of rod from the center of the cylindrical region in which magnetic field is confined.

If this electric field is uniform, the potential difference across the rod is given as

$$V_B - V_A = EL$$

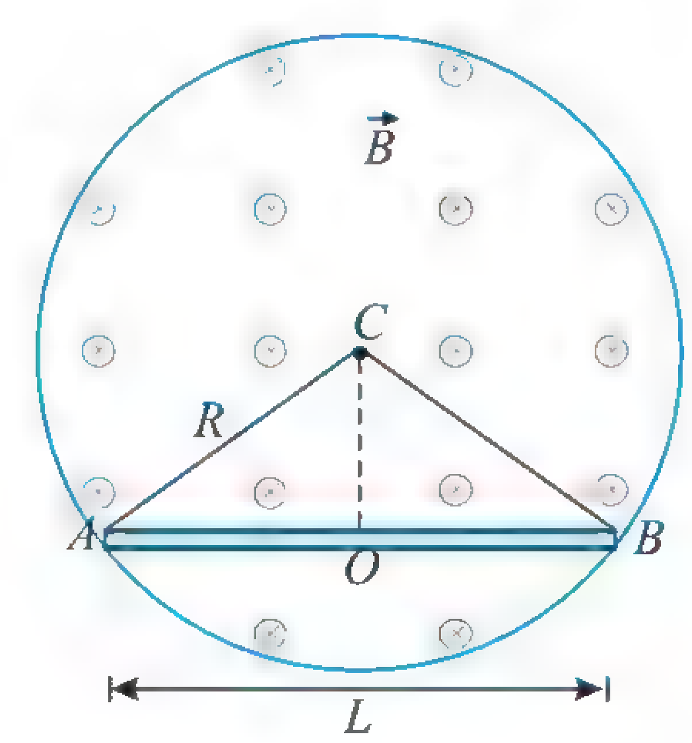
$$\Rightarrow \quad V_B - V_A = \left(\frac{1}{2}\alpha \sqrt{R^2 - \left(\frac{L}{2}\right)^2}\right) L$$

$$\Rightarrow \quad V_B - V_A = \frac{1}{2}\alpha L \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

2nd Approach

Let us join end A and B of the rod with center of the circle and consider a triangular closed loop ABC

Area of the loop $\quad A = \dfrac{1}{2} \times L \times \sqrt{R^2 - \left(\dfrac{L}{2}\right)^2}$

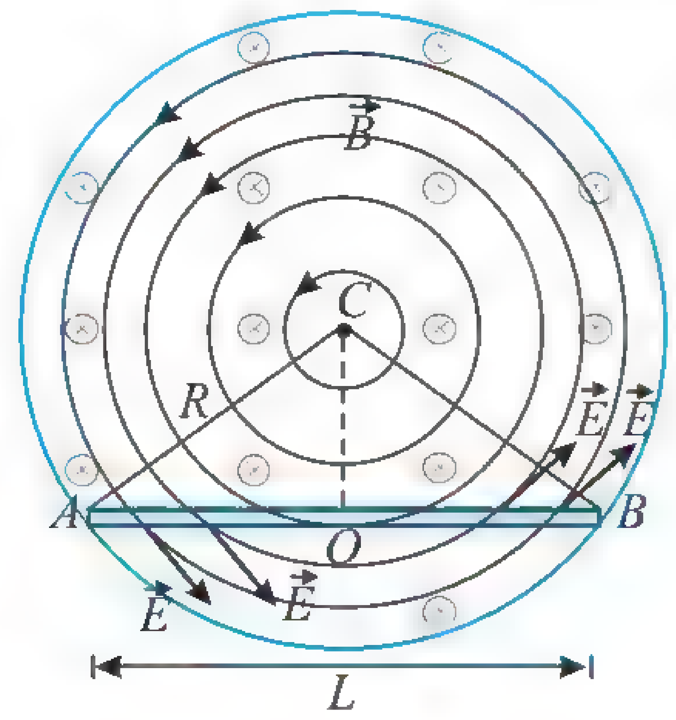


Flux through loop ABC is $\phi = B.\left(\dfrac{1}{2} \times L \times \sqrt{R^2 - \left(\dfrac{L}{2}\right)^2}\right)$

The magnitude of the emf induced in the loop is

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt} = \left(\frac{1}{2} \times L \times \sqrt{R^2 - \left(\frac{L}{2}\right)^2}\right).\alpha$$

Because of symmetry the induced field lines will be circular.



It means field lines of E are perpendicular to CA and CB. The emf in the loop ABC can also be written as

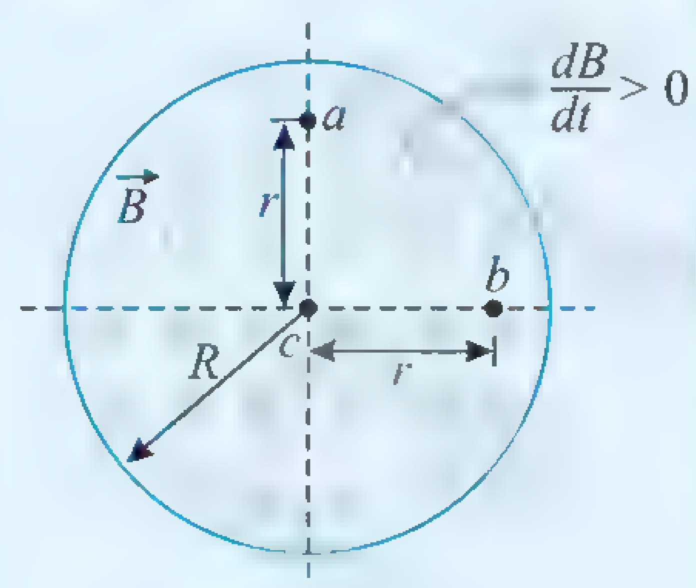
$$\int_C^A \vec{E}\cdot d\vec{l} + \int_A^B \vec{E}\cdot d\vec{l} + \int_B^C \vec{E}\cdot d\vec{l} = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$0 + \int_A^B \vec{E}\cdot d\vec{l} + 0 = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

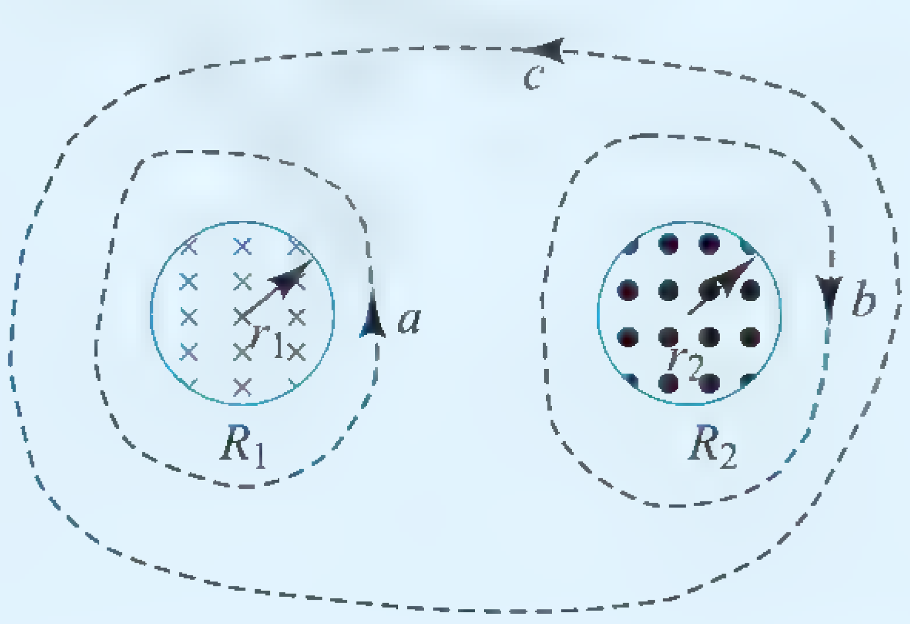
$$\int_A^B \vec{E}\cdot d\vec{l} = V_B - V_A = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

Concept Application Exercise 4.4

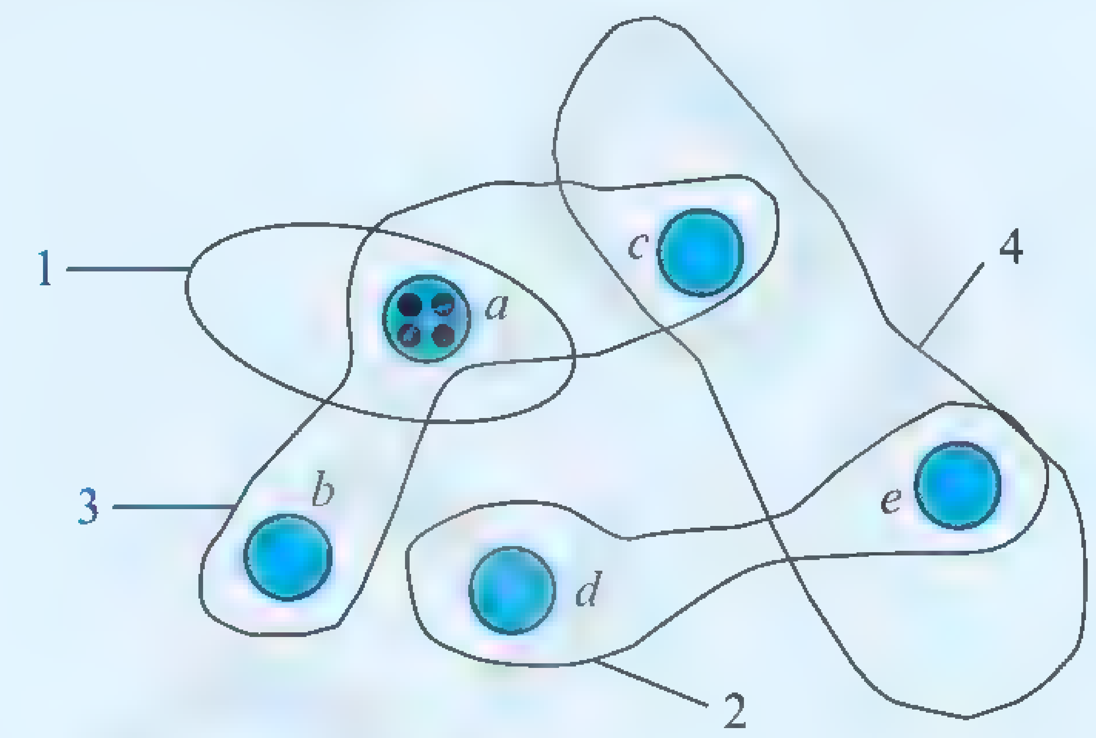
1. The magnetic field at all points within a circular region of radius R is uniform in space and directed into the plane of the page in figure (the region could be a cross section inside the windings of a long, straight solenoid). If the magnetic field is increasing at a rate dB/dt, what are the magnitude and direction of the force on a stationary positive point charge q located at points a, b, and c? (Point a is a distance r above the center of the region, point b is at a distance r to the right of the center, and point c is at the center of the region.)



2. Figure shows two circular regions R_1 and R_2 with radii $r_1 = 20.0$ cm and $r_2 = 25.0$ cm, respectively. In R_1 there is a uniform magnetic field $B_1 = 48.6$ mT into the page and in R_2 there is a uniform magnetic field $B_2 = 77.2$ mT out of the page (ignore any fringing of these fields). Both fields are decreasing at the rate 8.0 mT s^{-1}. Calculate the integral $\oint \vec{E}\cdot d\vec{l}$ for each of the three identical paths.

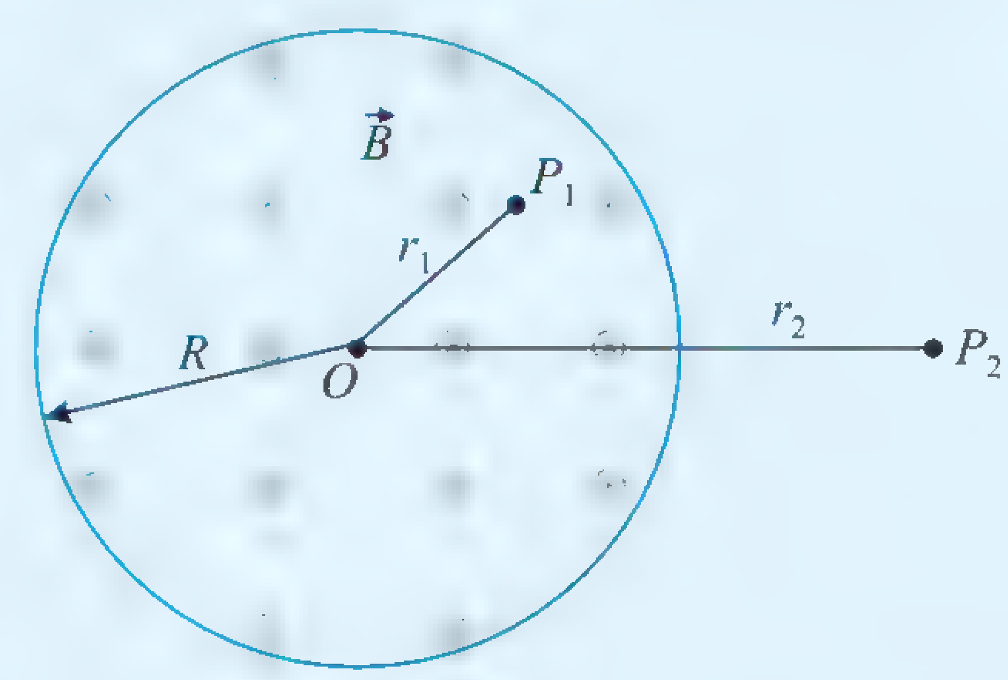


3. Figure shows five lettered regions in which a uniform magnetic field extends directly either out of the page (as in region a) or into the page. The field is increasing in magnitude at the same steady rate in all the five regions; the regions are identical in area. Also shown are four numbered paths along which $\oint \vec{E}\cdot d\vec{l}$ has the magnitudes given below in terms of a quantity *mag*. Determine whether the magnetic fields in regions b and e are directed into or out of the page.

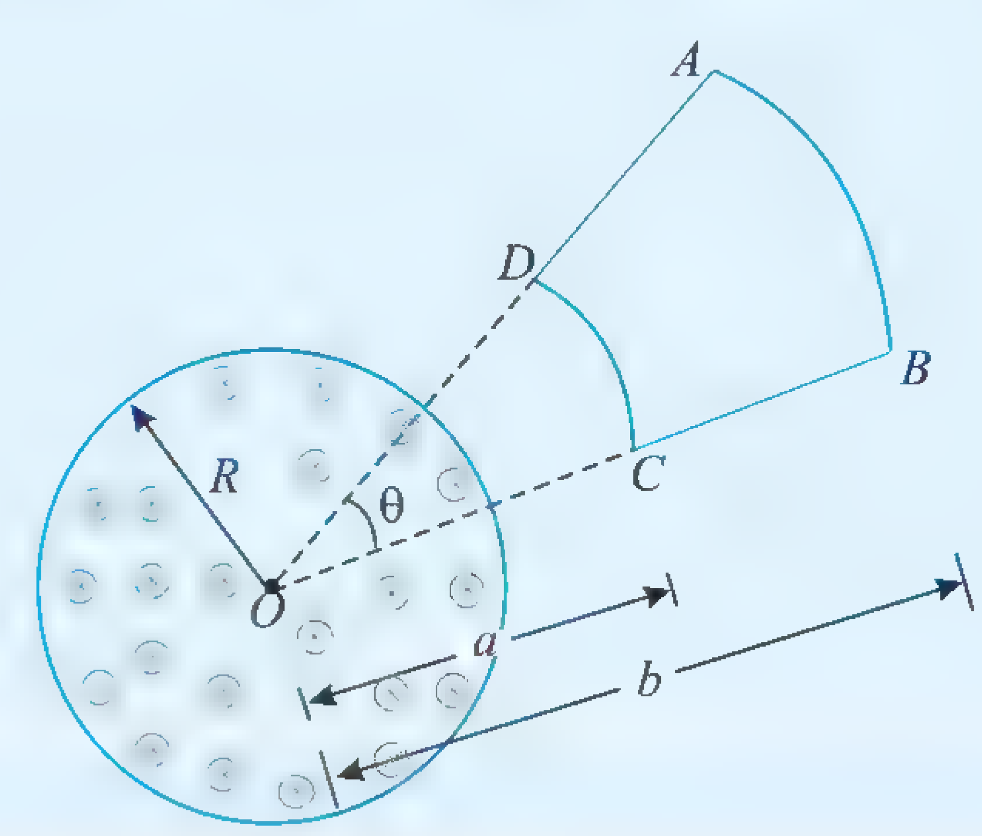


Path:	1	2	3	4
$\oint \vec{E}\cdot d\vec{l}$	*mag*	2(*mag*)	3(*mag*)	0

4. In a cylindrical region of radius $R = 2.5$ cm the magnetic field changes with time according to,
   $B = (2.00t^3 - 4.00t^2 + 0.8)$ T and
   (a) Find the force on an electron located at point P_2 at a distance $r_2 = 5.0$ cm, from the centre of magnetic field region at time $t = 2$ s.
   (b) What is the magnitude and direction of the electric field at P_1 when $t = 3$ s and $r_1 = 0.02$ m.

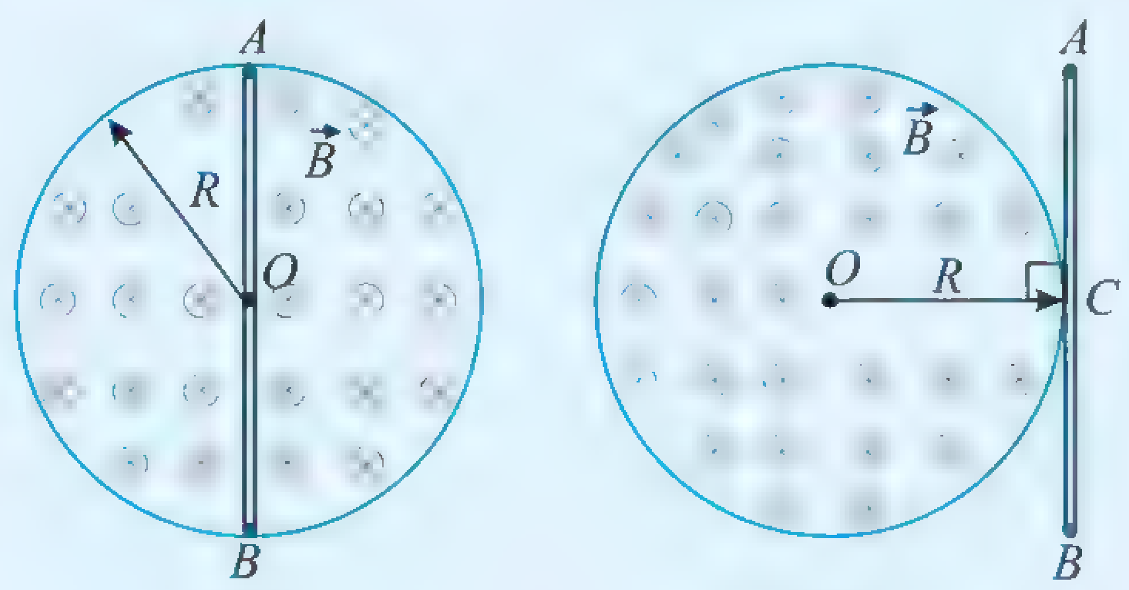


5. A uniform magnetic field present in cylindrical region of radius R centered at O as shown in figure. The magnetic field is changing at a rate $\frac{dB}{dt} = \beta$. There present a conducting loop $ABCD$ in the plane of the figure with its arms BC and DA along two radial direction from O having an angle θ between them. AB and CD are two circular arcs of radius b and a respectively centered at O. Find



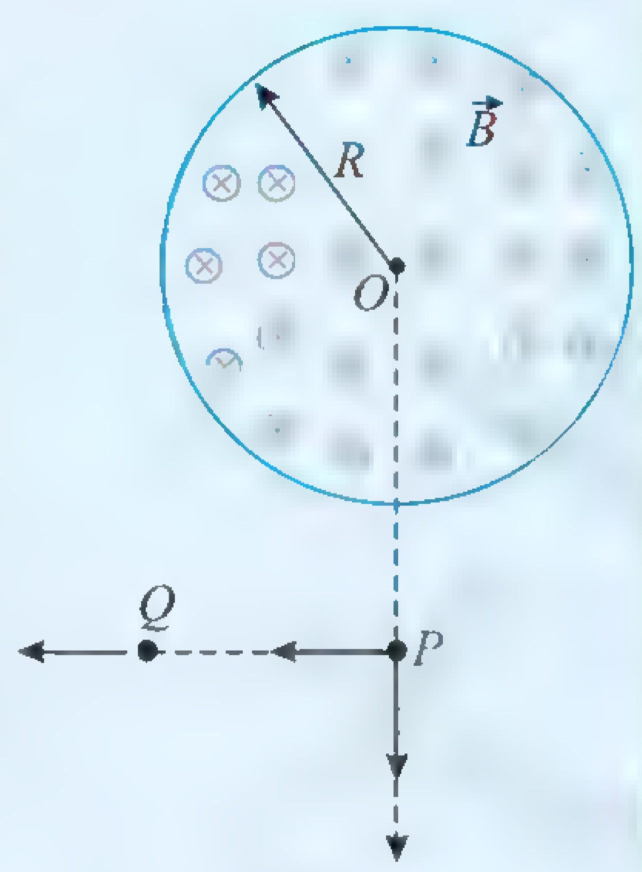
(a) the emf induced in the loop *ABCD*.
(b) the emf induced in arc *AB*
(c) the emf induced in arc *BC*

6. A uniform magnetic field B is present in a cylindrical region of radius R. The field is increasing at a constant rate of $\frac{dB}{dt} = \alpha$. A straight conducting rod AB of length $2R$ is placed in two situations as shown in Figs. (a) and (b). Find the emf induced in the rod when it is placed
(a) along the diameter shown in Fig. (a)
(b) along the tangent shown in Fig. (b) point C is midpoint of the rod.



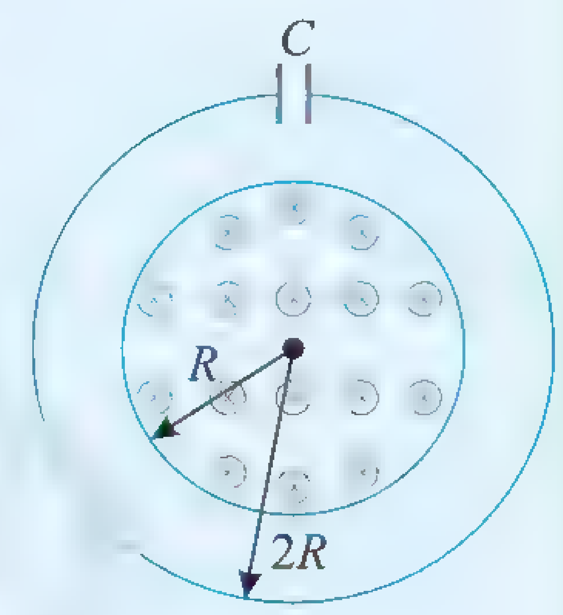
(a) (b)

7. *A* particle having charge q is placed at a point P outside the cylindrical magnetic field region, where uniform magnetic field is perpendicular to the plane of the figure in a cylindrical region of radius R. The magnetic field is increasing at a constant rate of βTs^{-1}. If the particle is slowly moved to infinity, calculate work done by the external agent on the particle in case



(a) the particle is moved in radial direction *OP*.
(b) the particle is moved in a direction perpendicular to *OP* along *PQ*.

8. A long solenoid of radius R and n be the number of turns per unit length carries a linearly increasing current $i = \frac{i_0}{T}t$. A co-axial circular conducting loop of radius $2R$ is fitted with a parallel capacitor whose plate area is A and separation is d. Find the:



(a) charge deposited in the capacitor
(b) static electric field in the capacitor
(c) induced electric field in the capacitor
(d) total electric field in the capacitor

ANSWERS

1. Force on a: $\frac{qr}{2}\left(\frac{dB}{dt}\right)(-\hat{i})$; on b: $\frac{qr}{2}\left(\frac{dB}{dt}\right)(-\hat{j})$; on c: zero

2. For loop a: $-3.2\pi \times 10^{-4}$ V, For loop b: $= -5\pi \times 10^{-4}$ V, For loop c: $-1.8\pi \times 10^{-4}$ V

3. b, $c \to$ out of the page; d, $e \to$ into the page

4. (a) 8.0×10^{-21} N (b) 0.3 V/m

5. (a) Zero (b) $\frac{R^2\theta\beta}{2}$ (c) Zero

6. (a) Zero (b) $\frac{\pi R^2}{4}\cdot\alpha$ **7.** (a) Zero (b) $\frac{\pi q R^2 \beta}{4}$

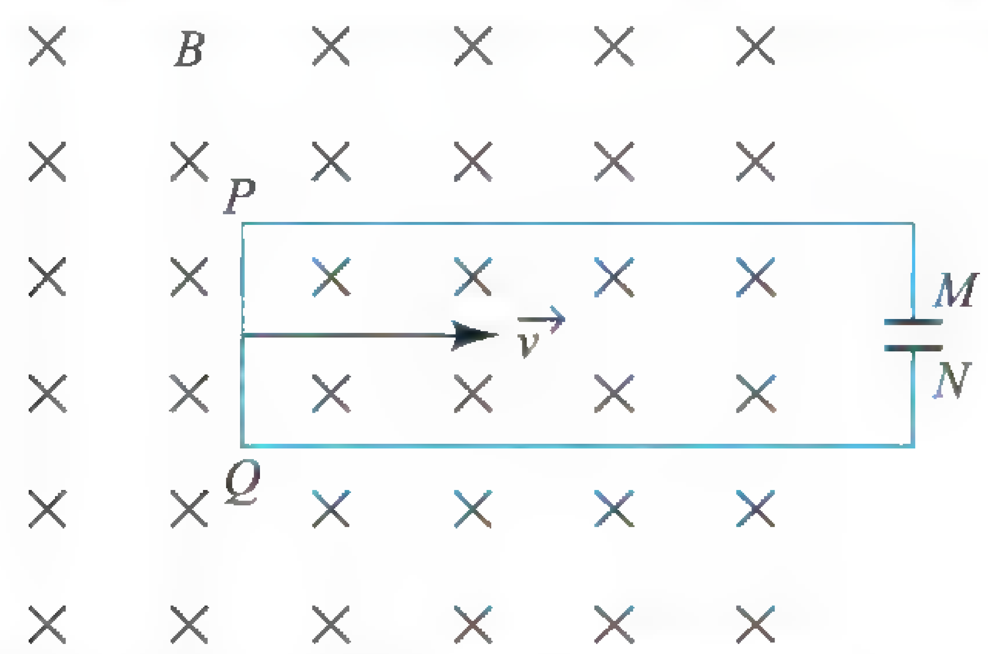
8. (a) $\frac{\pi R^2 \mu_0 n i_0 \varepsilon_0 A}{dT}$ (b) $\frac{\mu_0 \pi R^2 n i_0}{dT}$ (c) $\frac{R\mu_0 n i_0}{4T}$

(d) $\frac{\mu_0 n i R}{T}\left[\frac{\pi R}{d} - 1\right]$

Exercises

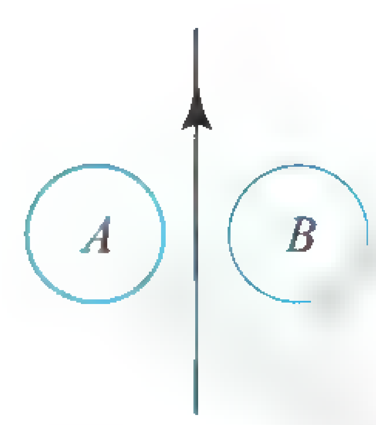
Single Correct Answer Type

1. A rod PQ is connected to the capacitor plates. The rod is placed in a magnetic field (B) directed downward perpendicular to the plane of the paper. If the rod is pulled out of magnetic field with velocity $\vec{v}$ as shown in figure,



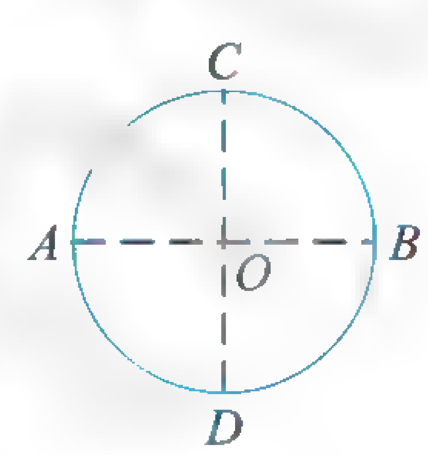
(1) Plate M will be positively charged.
(2) Plate N will be positively charged.
(3) Both plates will be similarly charged.
(4) No charge will be collected on plates.

2. A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown in figure. If current in the wire is slowly decreased, the direction of the induced current will be



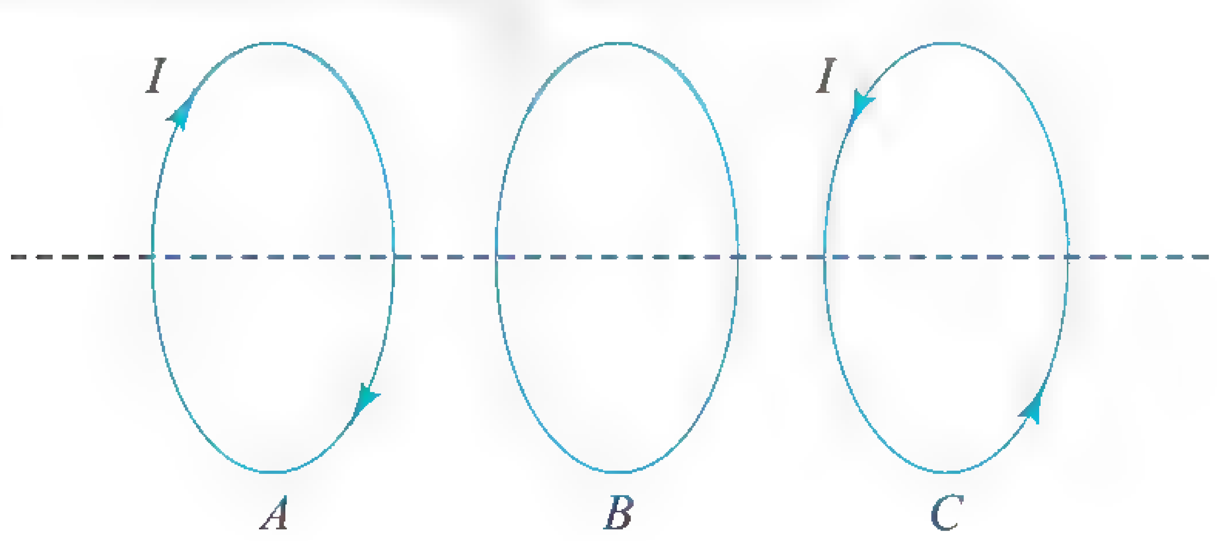
(1) clockwise in A and anticlockwise in B
(2) anticlockwise in A and clockwise in B
(3) clockwise in both A and B
(4) anticlockwise in both A and B

3. A vertical conducting ring of radius R falls vertically with a speed V in a horizontal uniform magnetic field B which is perpendicular to the plane of the ring. Which of the following statements is correct?



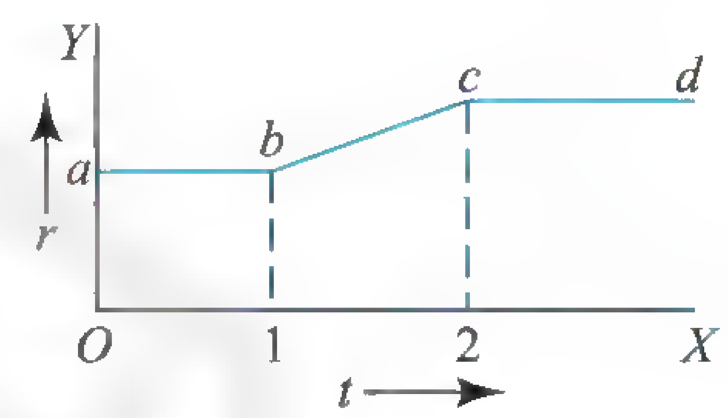
(1) A and B are at the same potential
(2) C and D are at the same potential
(3) current flows in clockwise direction
(4) current flows in anticlockwise direction

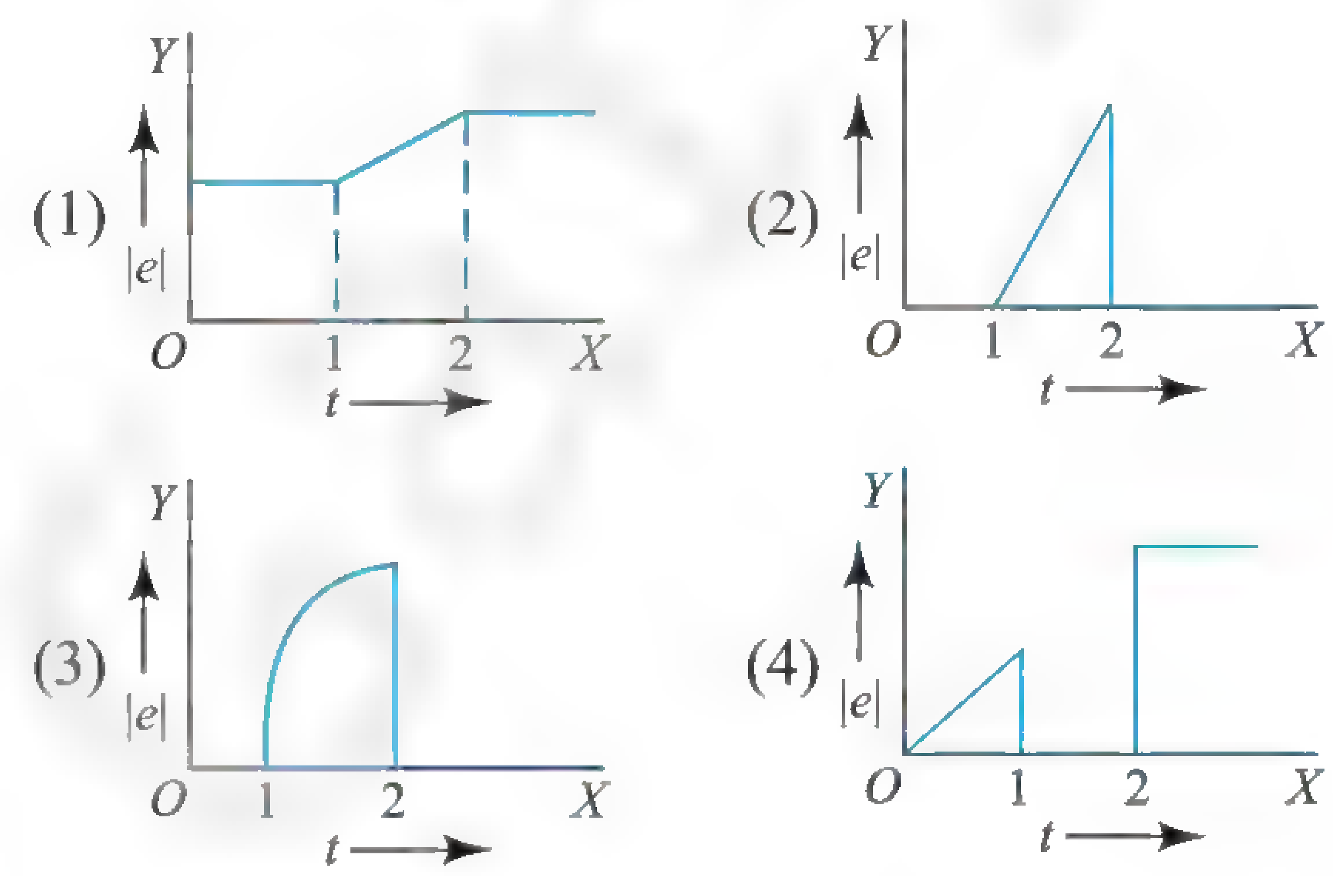
4. Three identical coils A, B, and C carrying currents are placed coaxially with their planes parallel to one another. A and C carry currents as shown in figure. B is kept fixed, while A and C both are moved toward B with the same speed. Initially, B is equally separated from A and C. The direction of the induced current in the coil B is



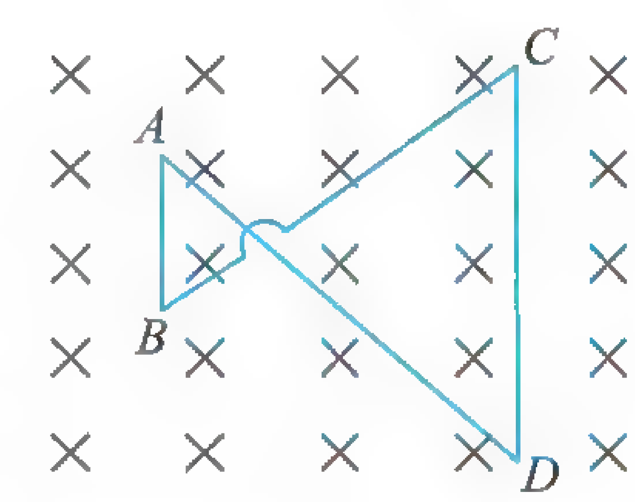
(1) same as that in coil A (1) same as that in coil B
(3) zero (4) none of these

5. A flexible wire bent in the form of a circle is placed in a uniform magnetic field perpendicularly to the plane of the coil. The radius of the coil changes as shown in figure. The graph of magnitude of induced emf in the coil is represented by



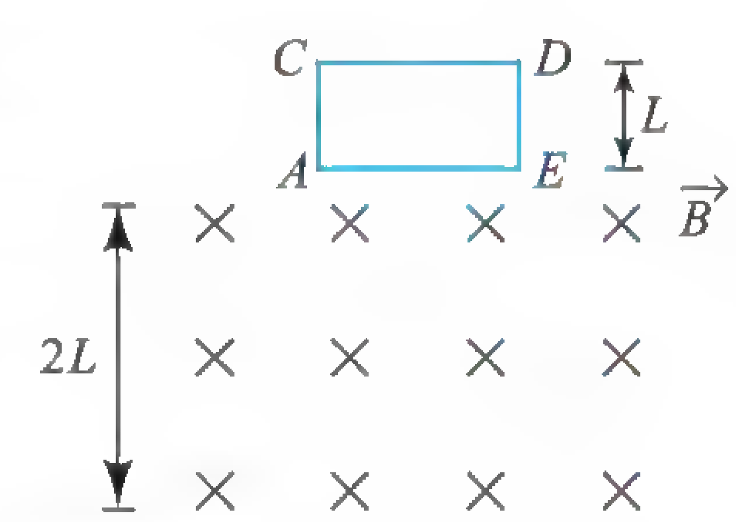


6. A conducting wire frame is placed in a magnetic field which is directed into the plane of the paper (figure). The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



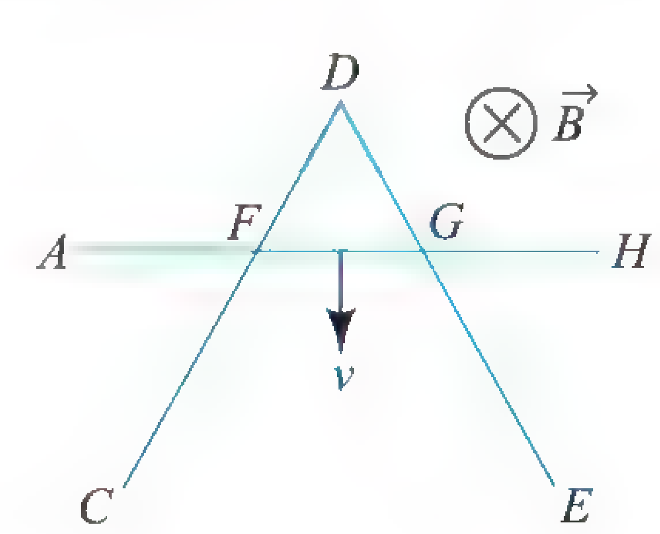
(1) B to A and D to C (2) A to B and C to D
(3) A to B and D to C (4) B to A and C to D

7. A square coil $ACDE$ with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length $2L$ (figure). The acceleration of the coil is



(1) less than g for all the time till the loop crosses the magnetic field completely
(2) less than g when it enters the field and greater than g when it comes out of the field
(3) g all the time
(4) less than g when it enters and comes out of the field but equal to g when it is within the field

8. A long conducting wire AH is moved over a conducting triangular wire CDE with a constant velocity v in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. Resistance per unit length of each wire is ρ. Then

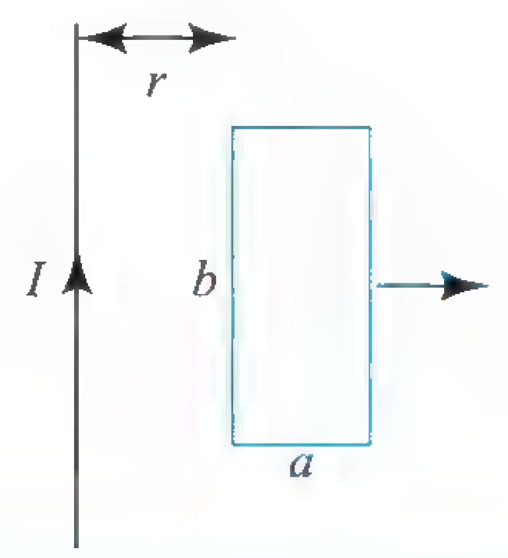


(1) a constant clockwise induced current will flow in the closed loop
(2) an increasing anticlockwise induced current will flow in the closed loop

(3) a decreasing anticlockwise induced current will flow in the closed loop

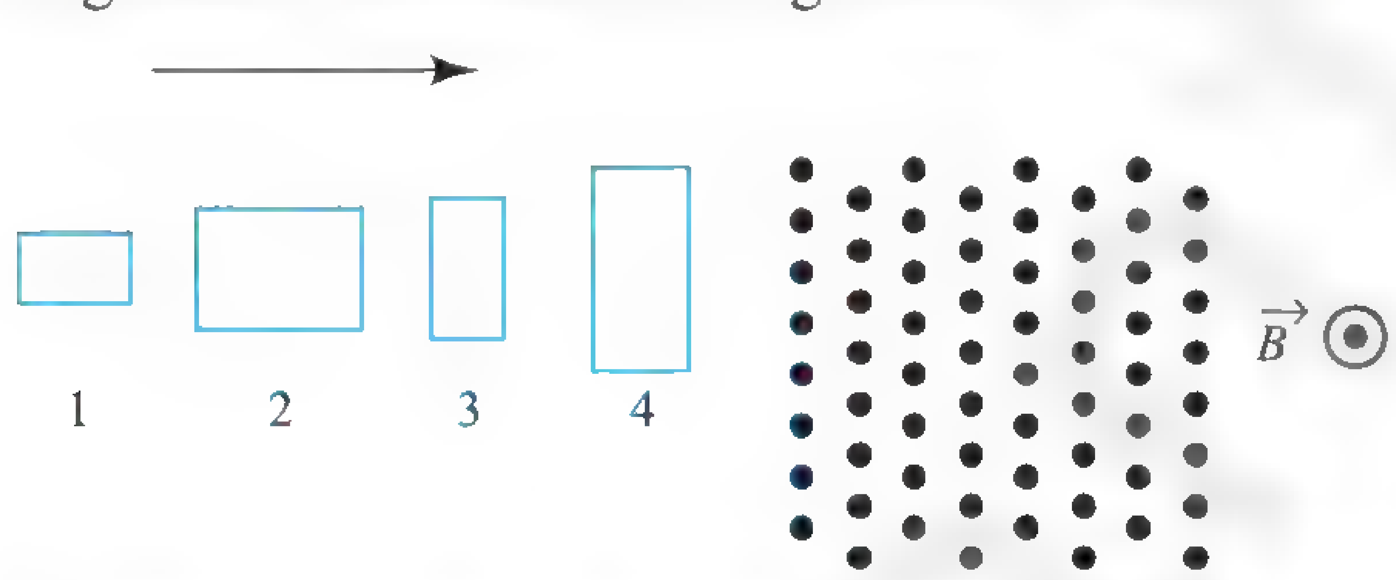
(4) a constant anticlockwise induced current will flow in the closed loop

9. A rectangular loop of wire with dimensions shown in figure is coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop when the loop is pulled?



Induced current	Force on left side	Force on right side
(1) Counterclockwise	To the left	To the left
(2) Counterclockwise	To the right	To the left
(3) Clockwise	To the right	To the left
(4) Clockwise	To the left	To the right

10. The four wire loops shown in figure have vertical edge lengths of either L, $2L$, or $3L$. They will move with the same speed into a region of uniform magnetic field $\vec{B}$ directed out of the page. Rank them according to the maximum magnitude of the induced emf greatest to least.



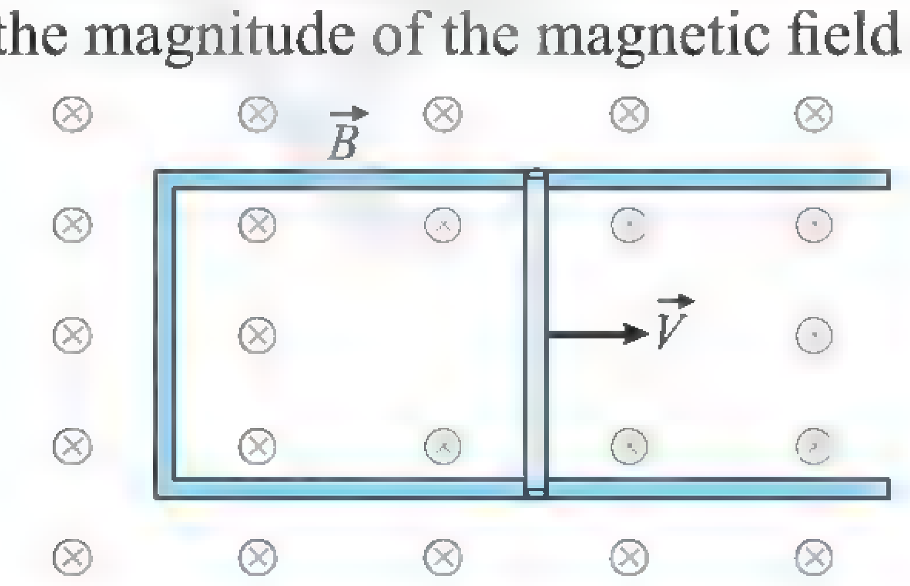
(1) 1 and 2 tie, then 3 and 4 tie

(2) 3 and 4 tie, then 1 and 2 tie

(3) 4, 2, 3, 1

(4) 4 then 2 and 3 tie, and then 1

11. A rod lies across frictionless rails in a uniform magnetic field $\vec{B}$ as shown in figure. The rod moves to the right with speed V. In order to make the induced emf in the circuit to be zero, the magnitude of the magnetic field should



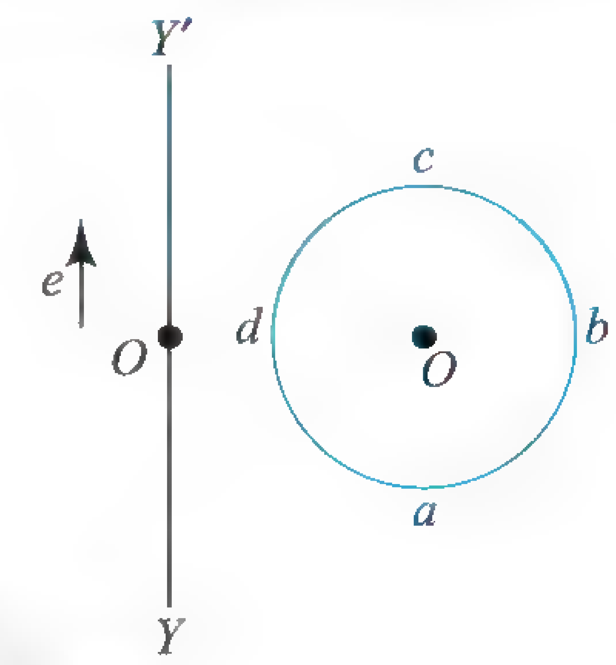
(1) not change

(2) increase linearly with time

(3) decrease linearly with time

(4) decrease nonlinearly with time

12. An electron moves on a straight line path YY' as shown in figure. A coil is kept on the right such that YY' is in the plane of the coil. At the instant when the electron gets closest to the coil (neglect self-induction of the coil),



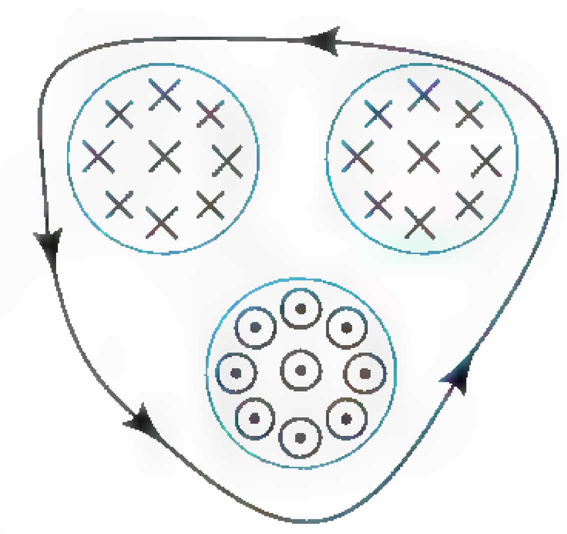
(1) the current in the coil flows clockwise

(2) the current in the coil flows anticlockwise

(3) the current in the coil is zero

(4) the current in the coil does not change the direction as the electron crosses point O

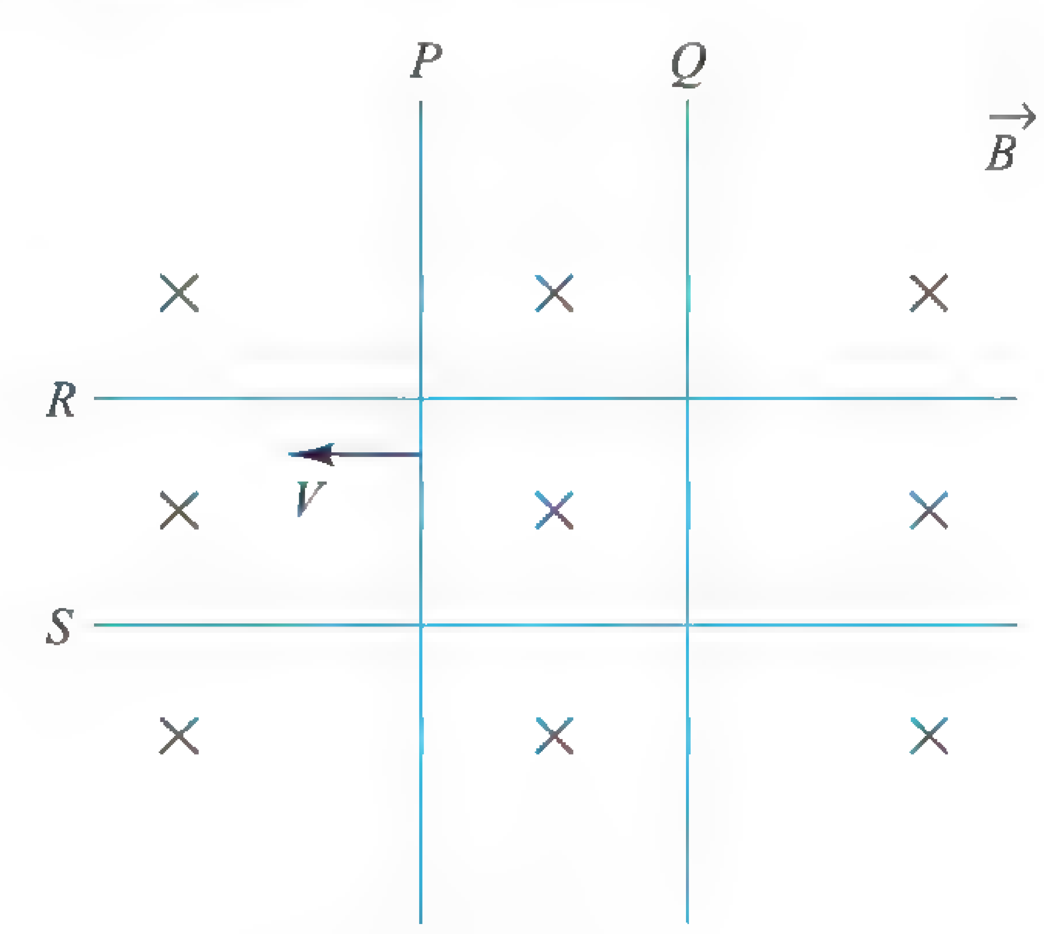
13. Figure shows three regions of magnetic field each of area A, and in each region, magnitude of magnetic field decreases at a constant rate α. If $\vec{E}$ is the induced electric field, then the value of the line integral $\oint \vec{E}\cdot d\vec{r}$ along the given loop is equal to



(1) αA (2) $-\alpha A$

(3) $3\alpha A$ (4) $-3\alpha A$

14. Two identical conductors P and Q are placed on two frictionless rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed, then rod Q



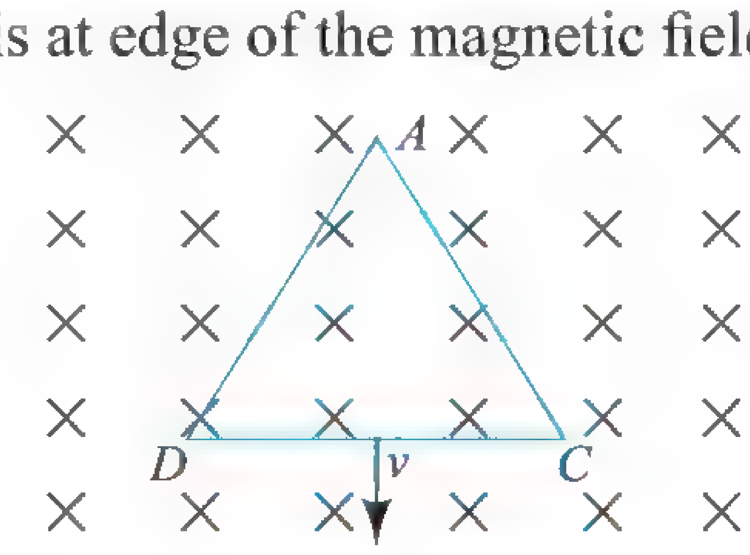
(1) will be attracted toward P

(2) will be repelled away from P

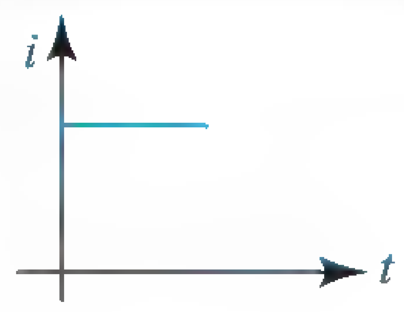
(3) will remain stationary

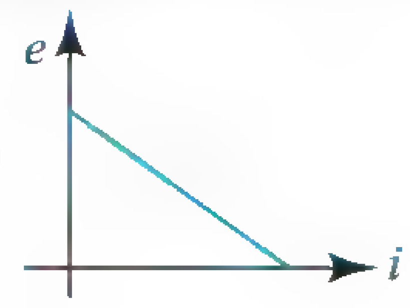
(4) may be repelled away or attracted toward P

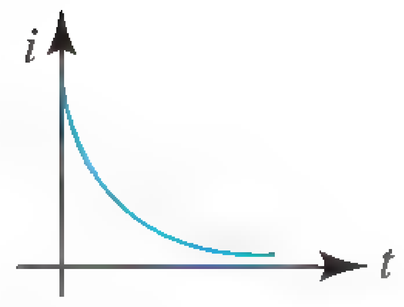
15. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper (figure). At time $t = 0$, side DC of the loop is at edge of the magnetic field.

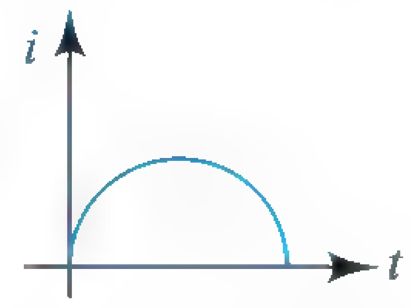


The induced current (i) versus time (t) graph will be as

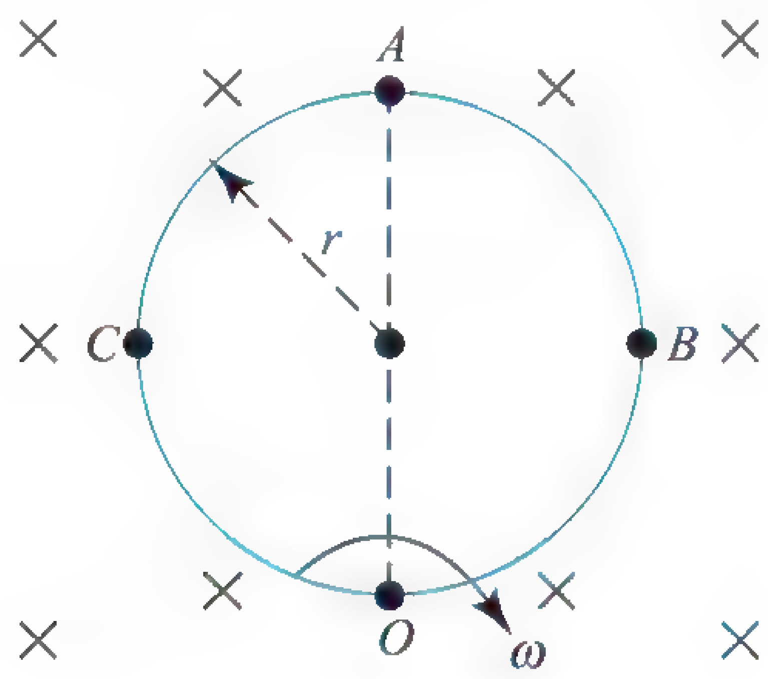
(1) 

(2) 

(3) 

(4) 

16. In figure, there is a conducting ring having resistance R placed in the plane of paper in a uniform magnetic field B_0. If the ring is rotating in the plane of paper about an axis passing through point O and perpendicular to the plane of paper with constant angular speed ω in clockwise direction, then

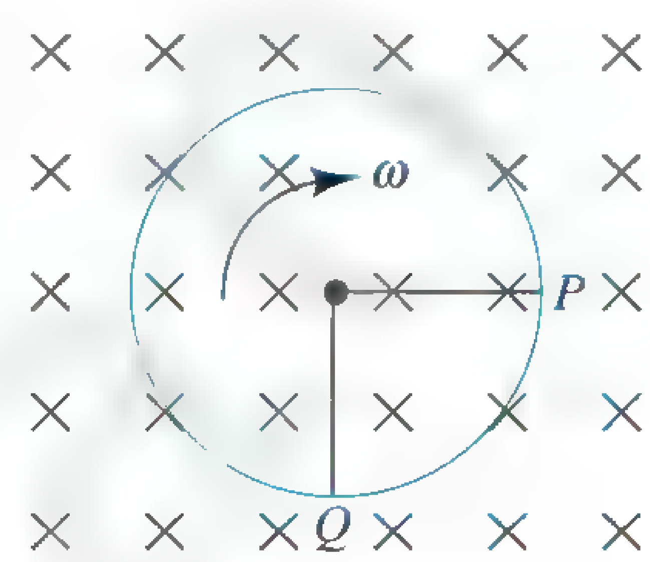


(1) point O will be at higher potential than A
(2) the potential of point B and C will be different
(3) the current in the ring will be zero
(4) the current in the ring will be $\dfrac{2B_0\omega r^2}{R}$

17. A 0.1 m long conductor carrying a current of 50 A is perpendicular to a magnetic field of 1.25 mT. The mechanical power to move the conductor with a speed of 1 m s^{-1} is

(1) 0.25 mW (2) 6.25 mW
(3) 0.625 W (4) 1 W

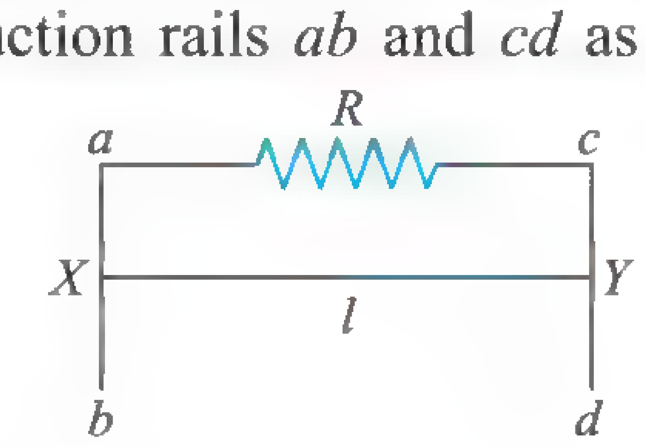
18. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω (figure). If the magnetic field strength is B and is directed into the page then the emf induced across PQ is



(1) $B\omega r^2$ (2) $\dfrac{B\omega r^2}{2}$

(3) $4B\omega r^2$ (4) $\dfrac{\pi^2 r^2 B\omega}{8}$

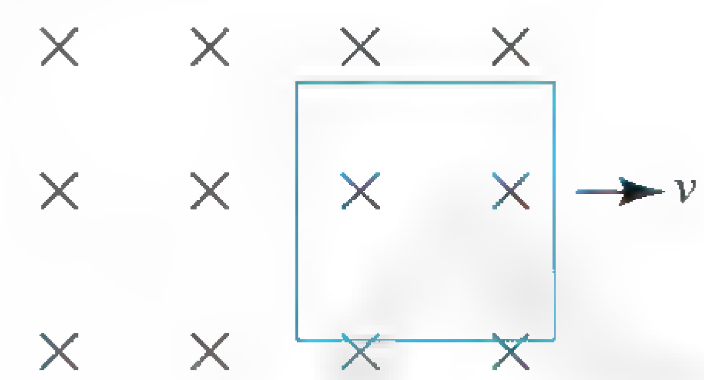
19. A conducting wire xy of length l and mass m is sliding without friction on vertical conduction rails ab and cd as shown in figure. A uniform magnetic field B exists perpendicular to the plane of the rails, x moves with a constant velocity of



(1) $\dfrac{mgR}{Bl}$ (2) $\dfrac{mgR}{Bl^2}$

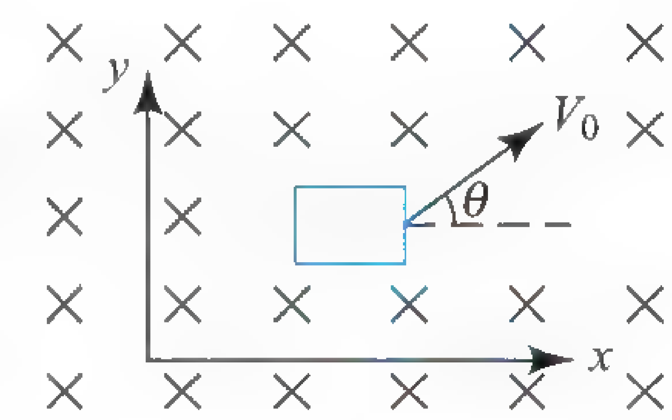
(3) $\dfrac{mgR}{B^2l^2}$ (4) $\dfrac{mgR}{B^2l}$

20. Figure shows a square loop of side 0.5 m and resistance 10 Ω. The magnetic field has a magnitude B = 1.0 T. The work done in pulling the loop out of the field slowly and uniformly in 2.0 s is



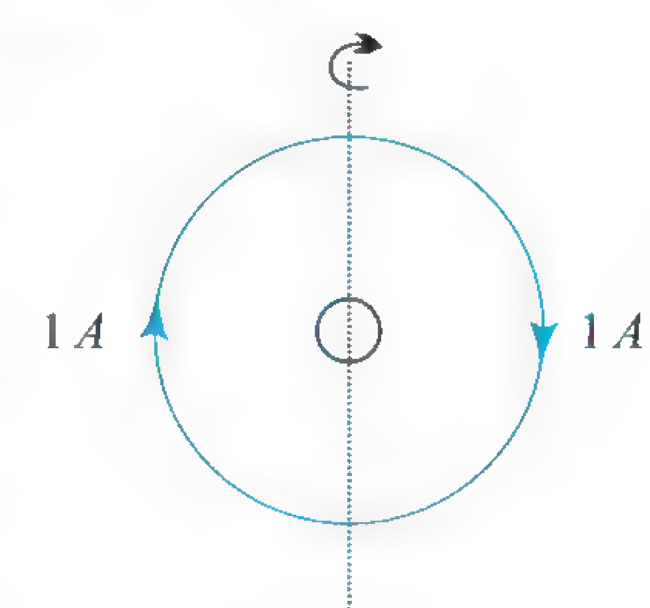
(1) 3.125×10^{-3} J (2) 6.25×10^{-4} J
(3) 1.25×10^{-2} J (4) 5.0×10^{-4} J

21. In the space shown a non-uniform magnetic field $\vec{B} = B_0(1+x)(-\hat{k})$ tesla is present. A closed loop of small resistance, placed in the x–y plane is given velocity V_0. The force due to magnetic field on the loop is



(1) zero (2) along $+x$ direction
(3) along $-x$ direction (4) along $+y$ direction

22. The inner loop has an area of 5×10^{-4} m² and a resistance of 2 Ω (figure). The larger circular loop is fixed and has a radius of 0.1 m. Both the loops are concentric and coplanar. The smaller loop is rotated with an angular velocity ω rad s^{-1} about its diameter. The magnetic flux with the smaller loop is



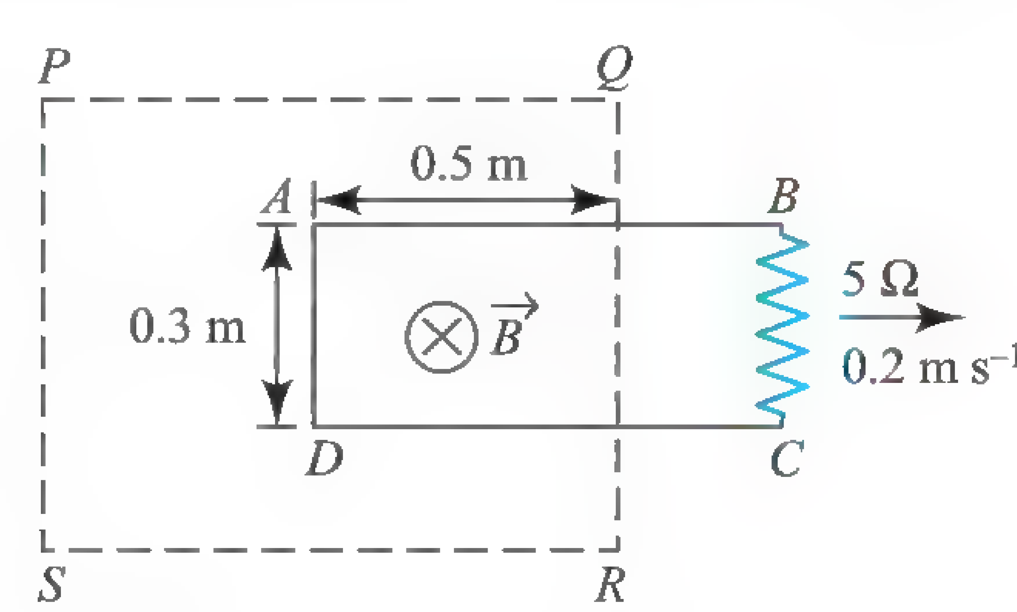
(1) $2\pi \times 10^{-6}$ Wb (2) $\pi \times 10^{-9}$ Wb
(3) $\pi \times 10^{-9} \cos \omega t$ Wb (4) zero

23. A gold rod of length l is accelerated in the horizontal direction with an acceleration a_0. The rod is held between two perfectly insulating clamps. Calculate the electric field set up in the rod. Take the mass of electron as m.

(1) $E = \dfrac{ma_0}{e}$ (2) $E = ma_0 l$

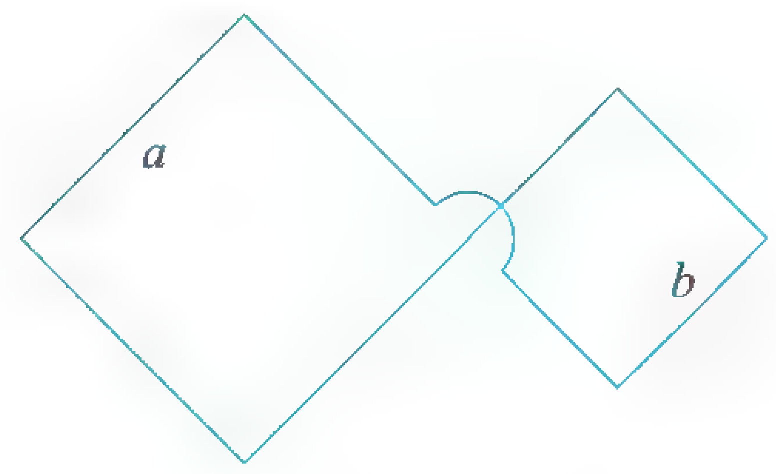
(3) zero (4) none of these

24. A circuit $ABCD$ is held perpendicular to the uniform magnetic field of $B = 5 \times 10^{-2}$ T extending over the region $PQRS$ and directed into the plane of the paper. The circuit is moving out of the field at a uniform speed of 0.2 m s^{-1} for 1.5 s. During this time, the current in the 5 Ω resistor is



(1) 0.6 mA from B to C (2) 0.9 mA from B to C
(3) 0.9 mA from C to B (4) 0.6 mA from C to B

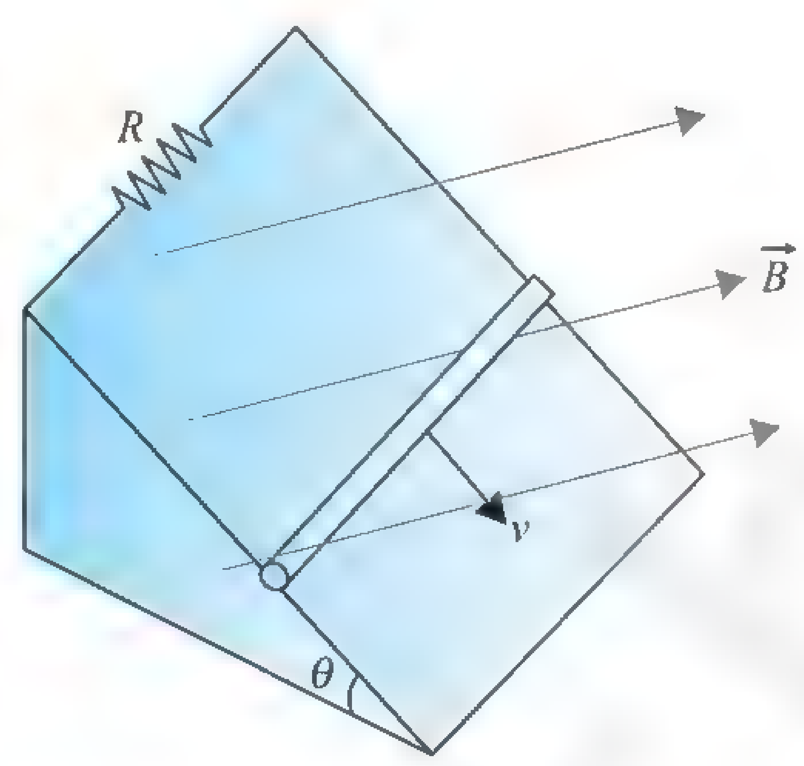
25. A plane loop, shaped as two squares of sides $a = 1$ m and $b = 0.4$ m is introduced into a uniform magnetic field $\perp$ to the plane of loop (figure). The magnetic field varies as $B = 10^{-3}\sin(100t)$ T. The amplitude of the current induced in the loop if its resistance per unit length is $r = 5$ mΩ m^{-1} is



(1) 2 A (2) 3 A
(3) 4 A (4) 5 A

26. A conductor AB of length l moves in x–y plane with velocity $\vec{v} = v_0(\hat{i} - \hat{j})$. A magnetic field $\vec{B} = B_0(\hat{i} + \hat{j})$ exists in the region. The induced emf is
(1) zero (2) $2B_0lv_0$
(3) B_0lv_0 (4) $\sqrt{2}B_0lv_0$

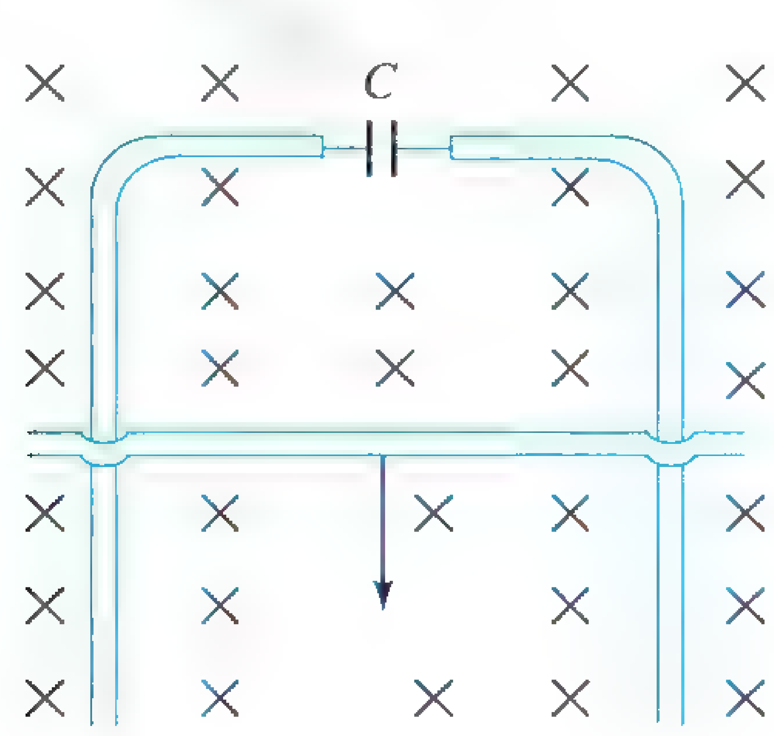
27. A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in figure. The separation between the bars is l. The system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The constant velocity of the wire is



(1) $\dfrac{mgR\sin\theta}{B^2l^2}$ (2) $\dfrac{mgR\sin\theta}{Bl^3}$
(3) $\dfrac{mgR\theta}{B^2l^5}$ (4) $\dfrac{mgR\sin\theta}{Bl^4}$

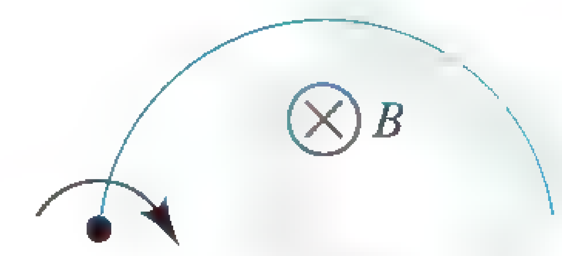
28. A uniform magnetic field exists in a region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m along y-axis moves with a constant speed of 1 ms^{-1} along x axis. Then the induced emf in the rod will be
(1) 0 (2) 25 V
(3) 20 V (4) 15 V

29. A conductor of length l and mass m can slide without any friction along the two vertical conductors connected at the top through a capacitor (figure). A uniform magnetic field B is set up $\perp$ to the plane of paper. The acceleration of the conductor



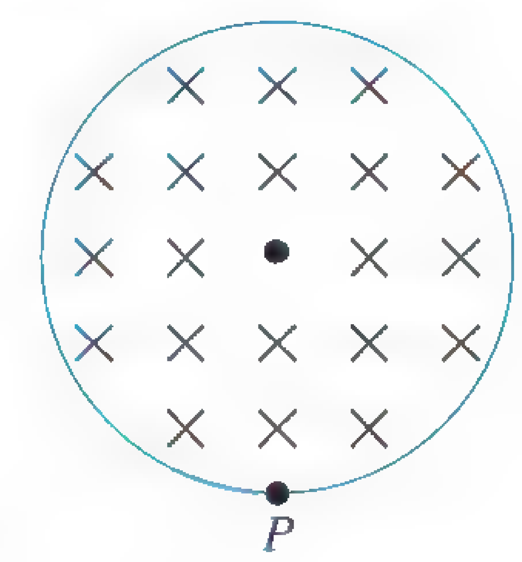
(1) is constant (2) increases
(3) decreases (4) cannot say

30. A semicircular wire of radius R is rotated with constant angular velocity about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced emf between the ends is



(1) $B\omega R^2/2$ (2) $2B\omega R^2$
(3) is variable (4) none of these

31. A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of dB/dt T s^{-1}. An electron placed at the point P on the periphery of the field, experiences an acceleration



(1) $\dfrac{1}{2}\dfrac{eR}{m}\dfrac{dB}{dt}$ toward left (2) $\dfrac{1}{2}\dfrac{eR}{m}\dfrac{dB}{dt}$ toward right
(3) $\dfrac{eR}{m}\dfrac{dB}{dt}$ toward left (4) zero

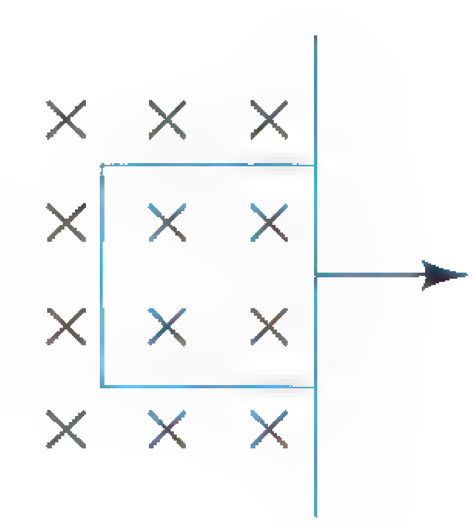
32. Magnetic flux linked with a stationary loop of resistance R varies with respect to time during the time period T as follows:

$$\phi = at(T - t)$$

The amount of heat generated in the loop during that time (inductance of the coil is negligible) is

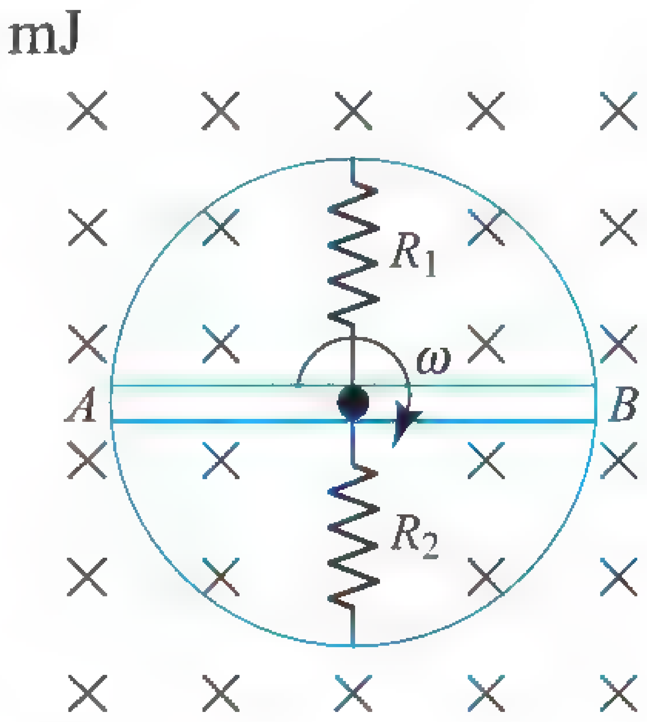
(1) $\dfrac{aT}{3R}$ (2) $\dfrac{a^2T^2}{3R}$
(3) $\dfrac{a^2T^2}{R}$ (4) $\dfrac{a^2T^3}{3R}$

33. A square loop of area 2.5×10^{-3} m^2 and having 100 turns with a total resistance of 100 Ω is moved out of a uniform magnetic field of 0.40 T in 1 s with a constant speed. Then the work done in pulling the loop is



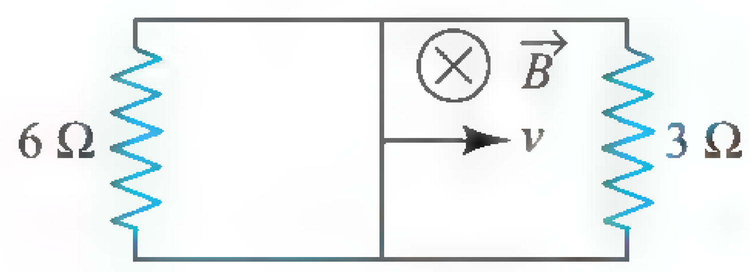
(1) 0 (2) 1 mJ
(3) 1 μJ (4) 0.1 mJ

34. AB is a resistanceless conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown in figure. The resistors R_1 and R_2 do not rotate. Then the current through the resistor R_1 is



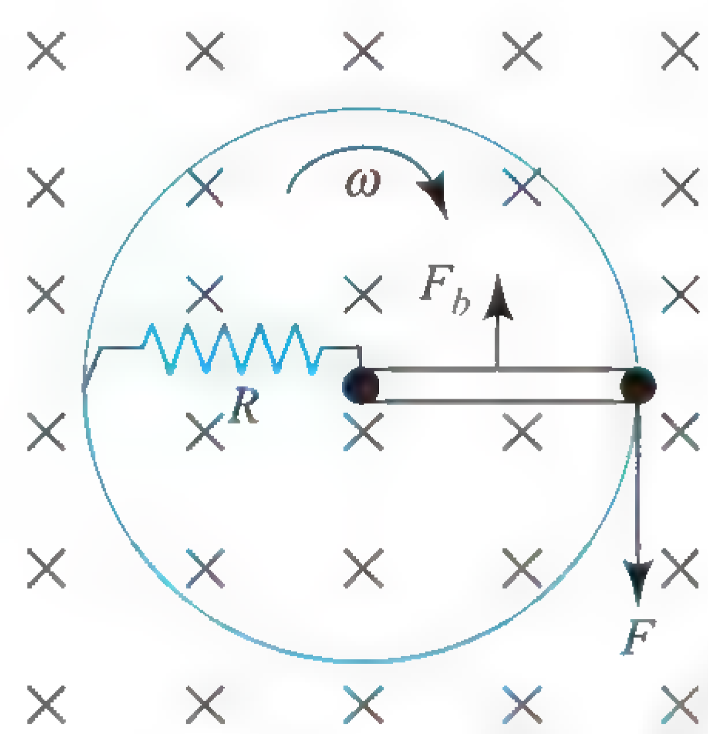
(1) $\dfrac{B\omega r^2}{2R_1}$ (2) $\dfrac{B\omega r^2}{2R_2}$
(3) $\dfrac{B\omega r^2}{2R_1R_2}(R_1 + R_2)$ (4) $\dfrac{B\omega r^2}{2(R_2 + R_2)}$

35. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$ T perpendicular to the plane of loop. Resistance of connector is $r = 2\ \Omega$. Two resistances of $6\ \Omega$ and $3\ \Omega$ are connected as shown in figure. The external force required to keep the connector moving with a constant velocity $v = 2\ \text{m s}^{-1}$ is



(1) 6 N (2) 4 N
(3) 2 N (4) 1 N

36. A metallic ring of radius r with a uniform metallic spoke of negligible mass and length r is rotated about its axis with angular velocity ω in a perpendicular uniform magnetic field B as shown in figure. The central end of the spoke is connected to the rim of the wheel through a resistor R as shown. The resistor does not rotate, its one end is always at the center of the ring and the other end is always in contact with the ring. A force F as shown is needed to maintain constant angular velocity of the wheel. F is equal to (the ring and the spoke has zero resistance)



(1) $\dfrac{B^2\omega r^2}{8R}$ (2) $\dfrac{B^2\omega r^2}{2R}$

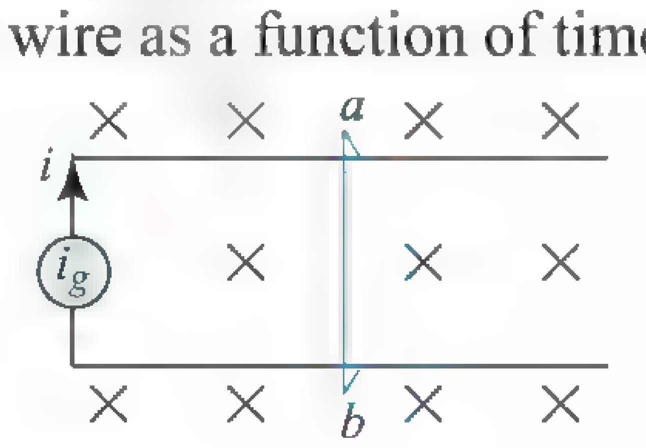
(3) $\dfrac{B^2\omega r^3}{2R}$ (4) $\dfrac{B^2\omega r^3}{4R}$

37. A metal rod of resistance $20\ \Omega$ is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x–y plane. There is a magnetic field $\vec{B} = (50\ \text{T})\hat{k}$. The ring rotates with an angular velocity $\omega = 20\ \text{rad s}^{-1}$ about its axis. An external resistance of $10\ \Omega$ is connected across the center of the ring and rim. The current through external resistance is

(1) $\dfrac{1}{4}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{1}{3}$ (4) 0

38. The current generator i_g, shown in figure, sends a constant current i through the circuit. The wire ab has a length l and mass m slide on the smooth, horizontal rails connected to i_g. The entire system lies in a vertical magnetic field B. The velocity of the wire as a function of time is



(1) $\dfrac{ilBt}{m}$ (2) $\dfrac{ilBt}{2m}$

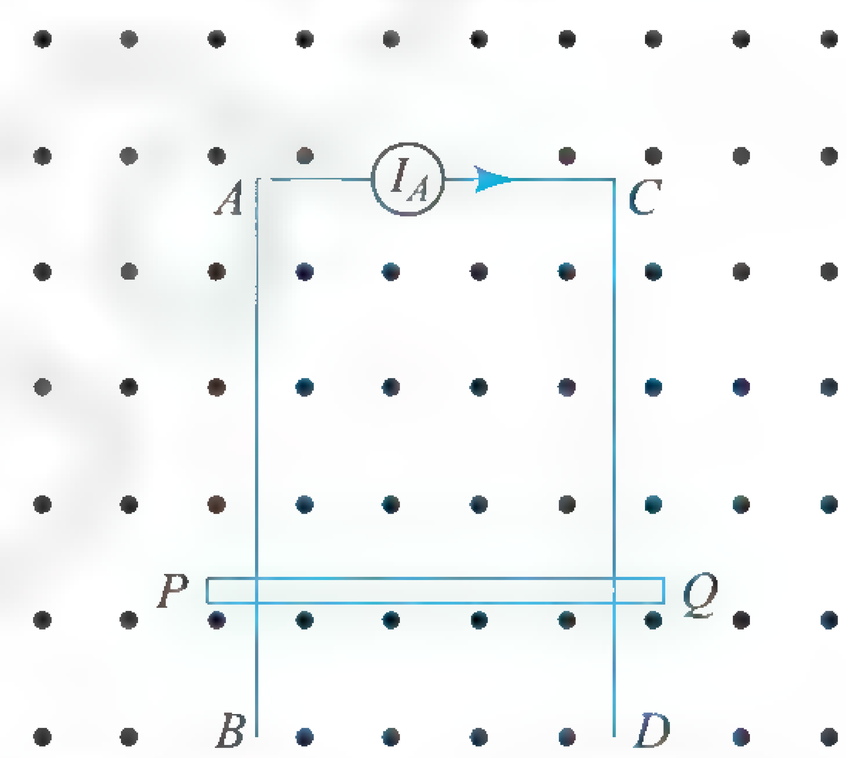
(3) $\dfrac{2ilBt}{m}$ (4) $\dfrac{ilBt}{3m}$

39. A metal disc of radius a rotates with a constant angular velocity ω about its axis. The potential difference between the center and the rim of the disc is (m = mass of electron, e = charge on electron)

(1) $\dfrac{m\omega^2a^2}{e}$ (2) $\dfrac{1}{2}\dfrac{m\omega^2a^2}{e}$

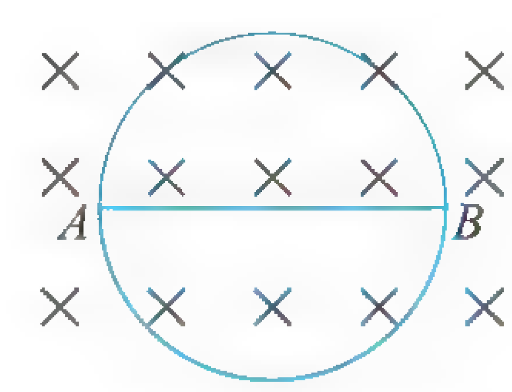
(3) $\dfrac{e\omega^2a^2}{2m}$ (4) $\dfrac{e\omega^2a^2}{m}$

40. AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then,



(1) the rod PQ will move downward with constant acceleration
(2) the rod PQ will move upward with constant acceleration
(3) the rod will remain at rest
(4) any of the above

41. The radius of the circular conducting loop shown in figure is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ.



Then, the current in wire AB is (AB is one of the diameters)

(1) $\dfrac{R\alpha}{2\rho}$ from A to B (2) $\dfrac{R\alpha}{2\rho}$ from B to A

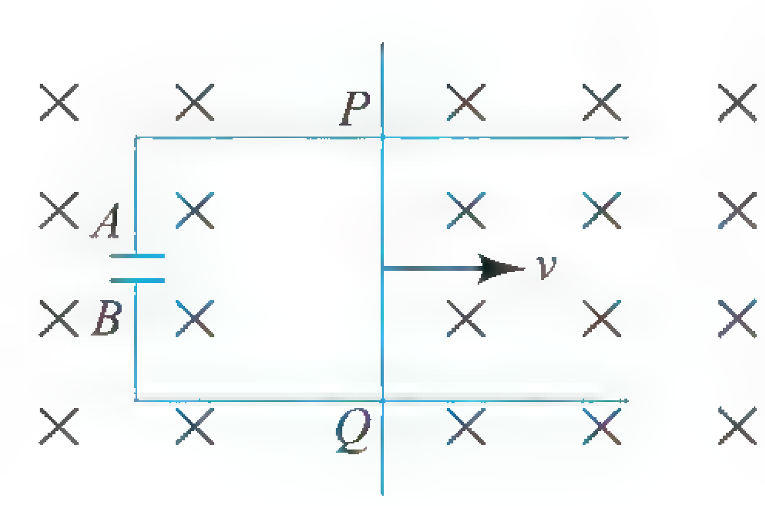
(3) $\dfrac{R\alpha}{\rho}$ from A to B (4) 0

42. The magnetic field in a region is given by $\vec{B} = B_0\left(1 + \dfrac{x}{a}\right)\hat{k}$. A square loop of edge length d is placed with its edge along the x- and y-axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is

(1) $\dfrac{v_0B_0d^2}{a}$ (2) $\dfrac{v_0B_0d^3}{a^2}$

(3) v_0B_0d (4) zero

43. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2.0\ \text{m s}^{-1}$ in a uniform magnetic field $B = 4.0$ T directed into the plane of the paper.



A capacitor of capacity $C = 10\ \mu F$ is connected as shown in figure, then

(1) $q_A = +80\ \mu C$ and $q_B = -80\ \mu C$

(2) $q_A = -80\ \mu C$ and $q_B = +80\ \mu C$

(3) $q_A = 0 = q_B$

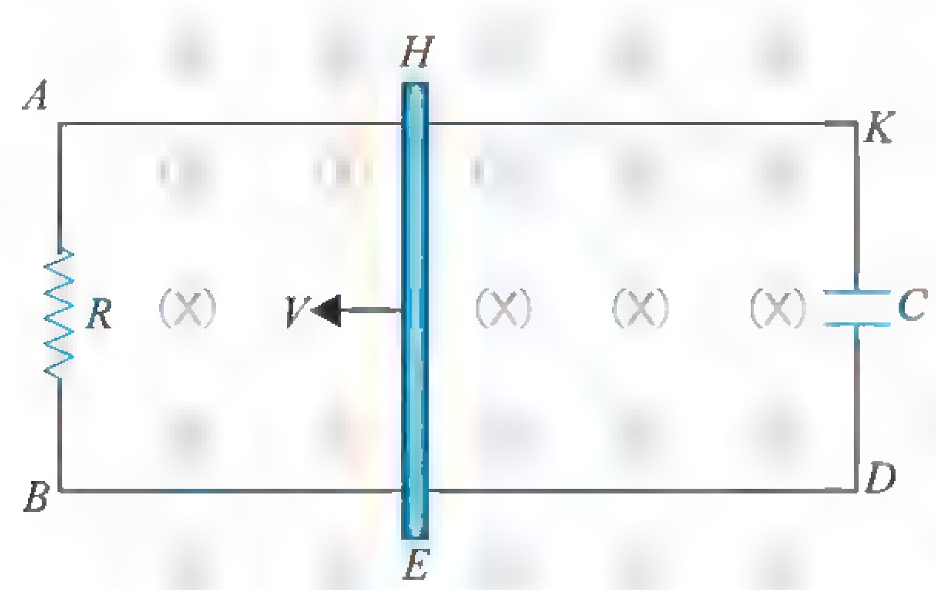
(4) charge stored in the capacitor increases exponentially with time

44. A metallic wire is folded to form a square loop of side a. It carries current i and is kept perpendicular to the region of uniform magnetic field B. If the shape of the loop is changed from square to an equilateral triangle without changing the length of the wire and current, the amount of work done in doing so is

(1) $Bia^2\left(1-\frac{4\sqrt{3}}{9}\right)$ (2) $Bia^2\left(1-\frac{\sqrt{3}}{9}\right)$

(3) $\frac{2}{3}Bia^2$ (4) zero

45. In the circuit shown in figure, a conducting wire HE is moved with a constant speed v toward left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the circuit inward. The current in $HKDE$ is



(1) clockwise (2) anticlockwise

(3) alternating (4) zero

46. A flexible wire loop in the shape of a circle has a radius that grows linearly with time. There is a magnetic field perpendicular to the plane of the loop that has a magnitude inversely proportional to the distance from the center of the loop, $B(r) \propto \frac{1}{r}$. How does the emf E vary with time?

(1) $E \propto t^2$ (2) $E \propto t$

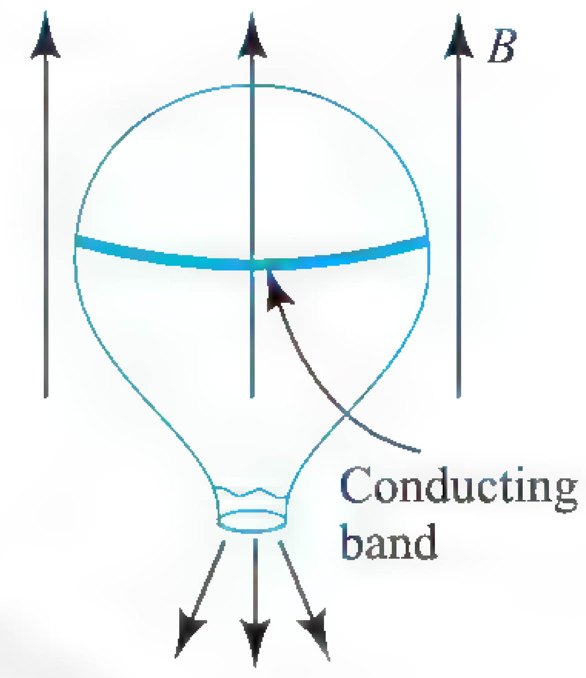
(3) $E \propto \sqrt{t}$ (4) E is constant

47. A vertical ring of radius r and resistance R falls vertically. It is in contact with two vertical rails which are joined at the top. The rails are without friction and resistance. There is a horizontal uniform magnetic field of magnitude B perpendicular to the plane of the ring and the rails. When the speed of the ring is v, the current in the top horizontal of the rail section is

(1) 0 (2) $\frac{2Brv}{R}$

(3) $\frac{4Brv}{R}$ (4) $\frac{8Brv}{R}$

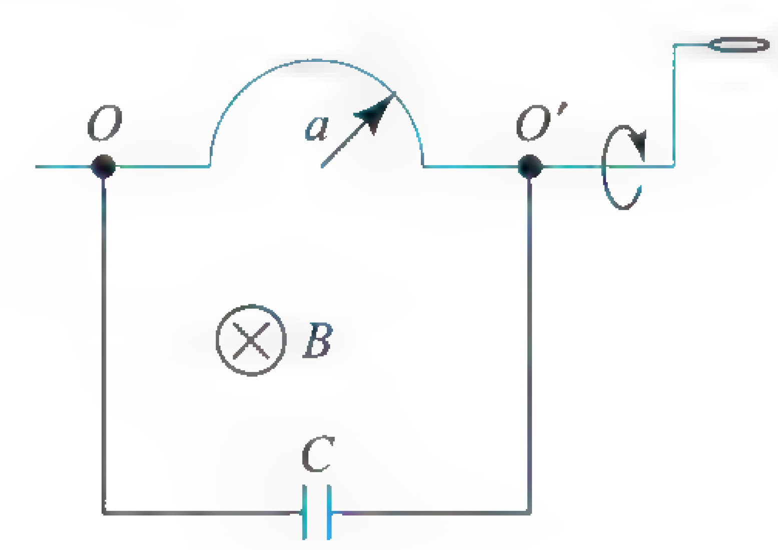
48. An elasticized conducting band is around a spherical balloon (figure). Its plane passes through the center of the balloon. A uniform magnetic field of magnitude 0.04 T is directed perpendicular to the plane of the band. Air is let out of the balloon at 100 cm^3s^{-1} at an instant when the radius of the balloon is 10 cm. The induced emf in the band is



(1) 15 μV (2) 25 μV

(3) 10 μV (4) 20 μV

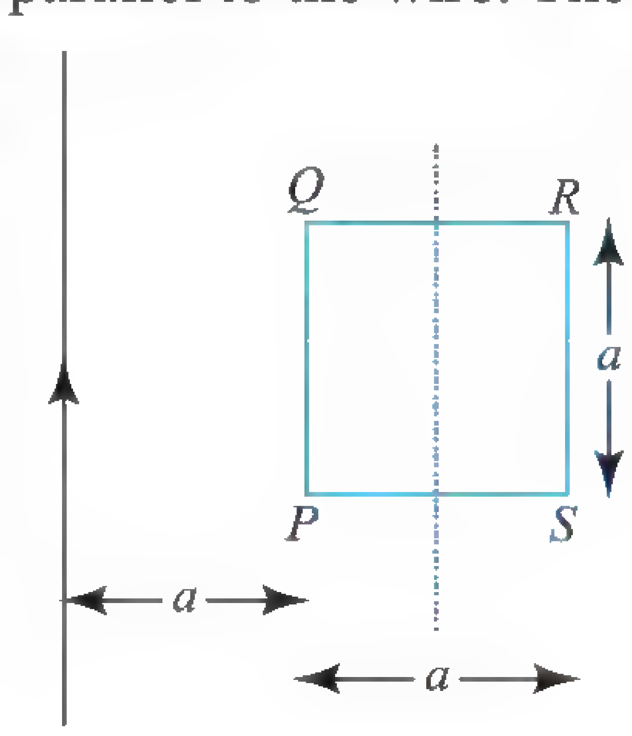
49. A copper rod is bent into a semi-circle of radius a and at ends straight parts are bent along diameter of the semi-circle and are passed through fixed, smooth, and conducting ring O and O' as shown in figure. A capacitor having capacitance C is connected to the rings. The system is located in a uniform magnetic field of induction B such that axis of rotation OO' is perpendicular to the field direction. At initial moment of time (t = 0), plane of semi-circle was normal to the field direction and the semicircle is set in rotation with constant angular velocity ω. Neglect the resistance and inductance of the circuit. The current flowing through the circuit as function of time is



(1) $\frac{1}{4}\pi\omega^2a^2CB\cos\omega t$ (2) $\frac{1}{2}\pi\omega^2a^2CB\cos\omega t$

(3) $\frac{1}{4}\pi\omega^2a^2CB\sin\omega t$ (4) $\frac{1}{2}\pi\omega^2a^2CB\sin\omega t$

50. In figure, a square loop $PQRS$ of side a and resistance r is placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid-points of the sides QR and PS. The total amount of charge which passes through any point of the loop during rotation is



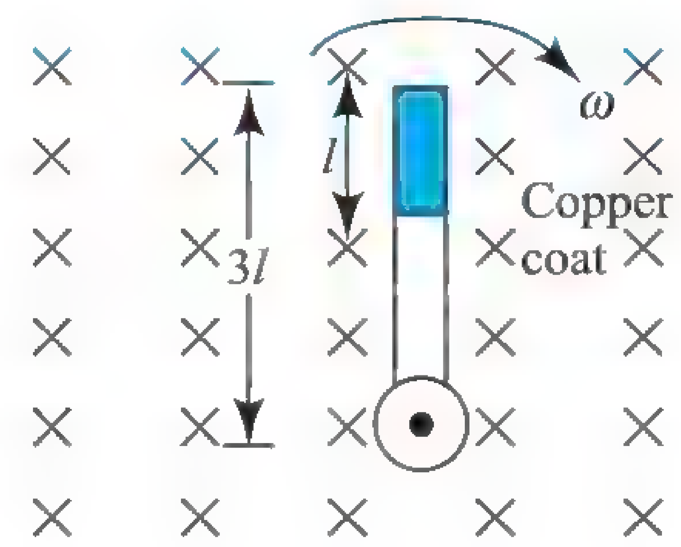
(1) $\frac{\mu_0 Ia}{2\pi r}\ln 2$

(2) $\frac{\mu_0 Ia}{\pi r}\ln 2$

(3) $\frac{\mu_0 Ia^2}{2\pi r}$

(4) Cannot be found because time of rotation is not given.

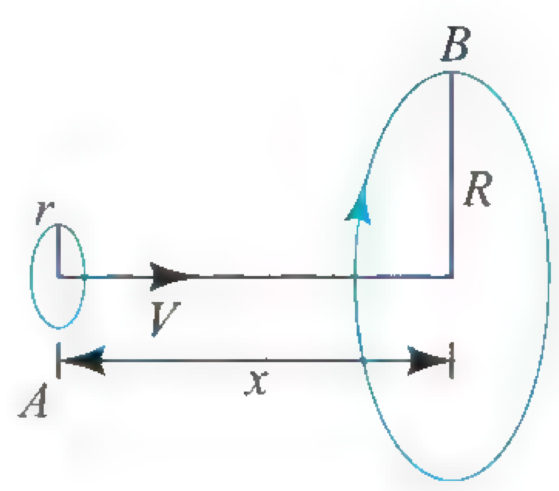
51. A wooden stick of length $3l$ is rotated about an end with constant angular velocity ω in a uniform magnetic field B perpendicular to the plane of motion. If the upper one-third of its length is coated with copper, the potential difference across the whole length of the stick is



(1) $\frac{9B\omega l^2}{2}$ (2) $\frac{4B\omega l^2}{2}$

(3) $\frac{5B\omega l^2}{2}$ (4) $\frac{B\omega l^2}{2}$

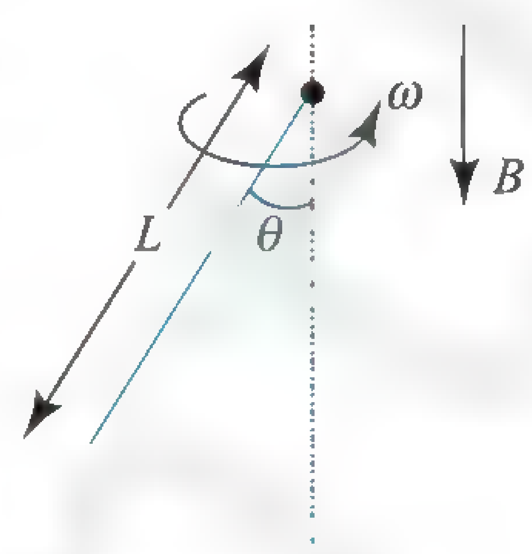
52. Loop A of radius $r << R$ moves toward loop B with a constant velocity V in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced emf in loop A is maximum?



(1) R (2) $\frac{R}{\sqrt{2}}$

(3) $\frac{R}{2}$ (4) $R\left(1-\frac{1}{\sqrt{2}}\right)$

53. A rod of length L rotates in the form of a conical pendulum with an angular velocity ω about its axis as shown in figure. The rod makes an angle θ with the axis. The magnitude of the motional emf developed across the two ends of the rod is



(1) $\frac{1}{2}B\omega L^2$ (2) $\frac{1}{2}B\omega L^2\tan^2\theta$

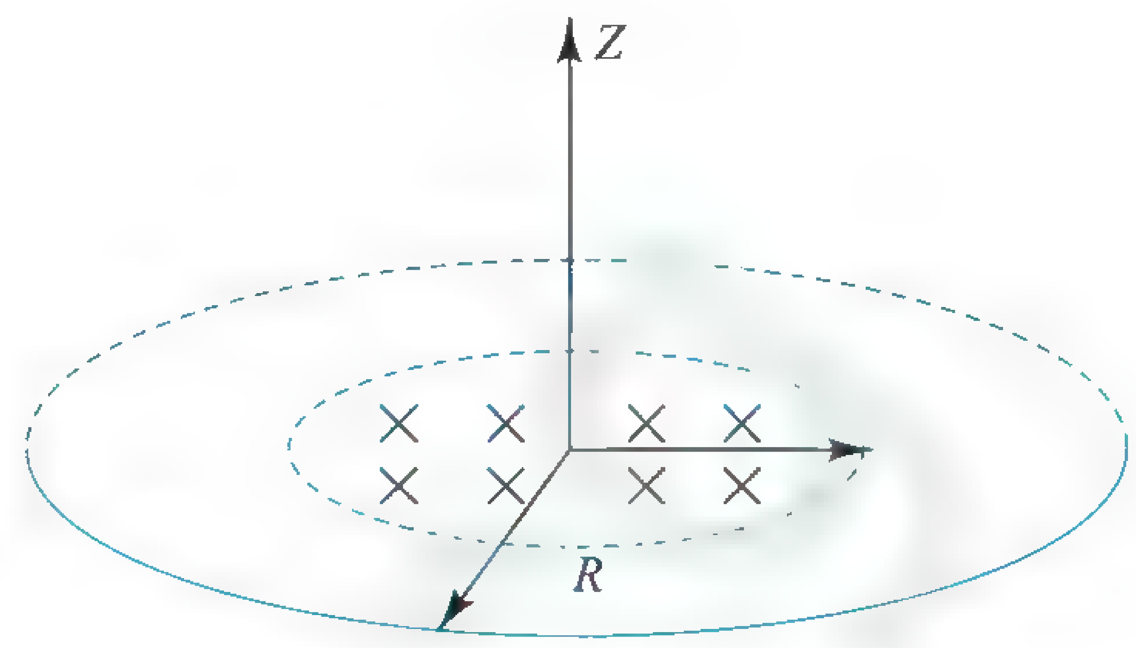
(3) $\frac{1}{2}B\omega L^2\cos^2\theta$ (4) $\frac{1}{2}B\omega L^2\sin^2\theta$

54. The magnetic flux density B is changing in magnitude at a constant rate dB/dt. A given mass m of copper, drawn into a wire of radius a and formed into a circular loop of radius r is placed perpendicular to the field B. The induced current in the loop is i. The resistivity of copper is ρ and density is d. The value of the induced current i is

(1) $\frac{m}{2\pi\rho d}\frac{dB}{dt}$ (2) $\frac{m}{4\pi a^2 r}\frac{dB}{dt}$

(3) $\frac{m}{4\pi a d}\frac{dB}{dt}$ (4) $\frac{m}{4\pi\rho d}\frac{dB}{dt}$

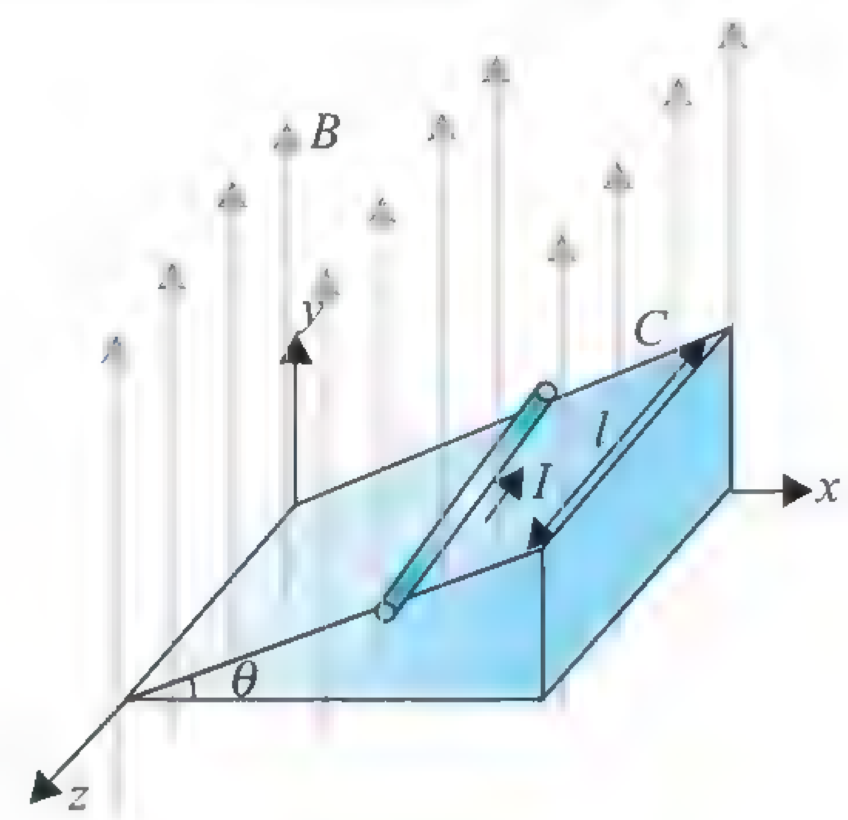
55. A line charge λ per unit length is pasted uniformly on to the rim of a wheel of mass m and radius R. The wheel has light non-conducting spokes and is free to rotate about a vertical axis as shown in figure. A uniform magnetic field extends over a radial region of radius r given by $B = -B_0\,\hat{k}\,(r \le a;\, a < R) = 0$ (otherwise). What is the angular velocity of the wheel when this field is suddenly switched off?



(1) $\frac{-2B_0\pi a^2 r}{mR}\hat{k}$ (2) $\frac{-B_0\pi a^2 r}{3mR}\hat{k}$

(3) $\frac{B_0\pi a^2\lambda}{mR}\hat{k}$ (4) $\frac{-B_0\pi a^2\lambda}{mR}\hat{k}$

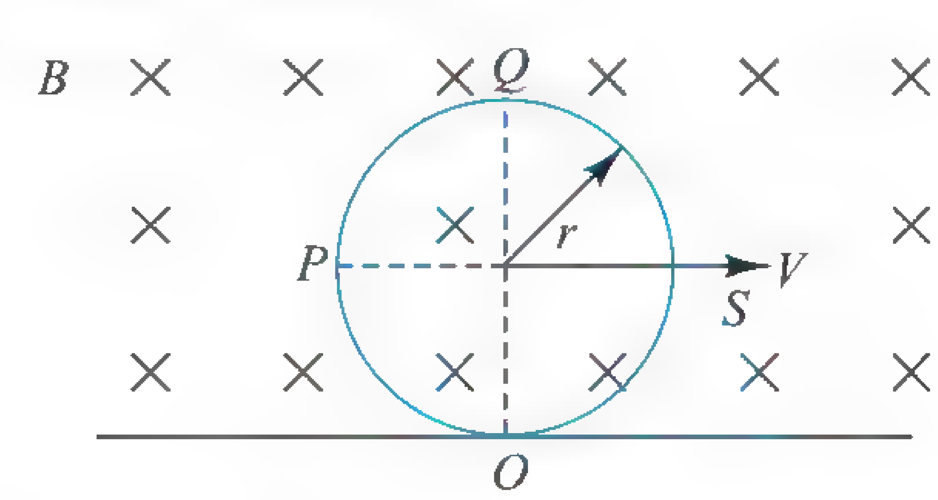
56. A conducting wire of length l and mass m is placed on two inclined rails as shown in figure. A current I is flowing in the wire in the direction shown. When no magnetic field is present in the region, the wire is just on the verge of sliding. When a vertically upward magnetic field is switched on, the wire starts moving up the incline. The distance travelled by the wire as a function of time t will be



(1) $\frac{1}{2}\left[\frac{IBl}{m}-2g\right]t^2$ (2) $\frac{1}{2}\left[\frac{IBl}{m}\times\frac{1}{\cos\theta}-2g\sin\theta\right]t^2$

(3) $\frac{1}{2}\left[\frac{IBl}{m}-2g\sin\theta\right]t^2$ (4) $\frac{1}{2}\left[\frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta}-2g\sin\theta\right]t^2$

57. A conducting ring of radius r and resistance R rolls on a horizontal surface with constant velocity v. The magnetic field B is uniform and is normal to the plane of the loop. Choose the correct option.



(1) The induced emf between O and Q is Brv.

(2) An induced current $I = \frac{2Bvr}{R}$ flows in the clockwise direction.

(3) An induced current $I = \dfrac{2Bvr}{R}$ flows in the anticlockwise direction.
(4) No current flows.

58. In a region at a distance r from z-axis, magnetic field $\vec{B} = B_0 rt\hat{k}$ is present where B_0 is constant and t is time. Then the magnitude of induced electric field at a distance r from z-axis is given by

(1) $\dfrac{r}{2}B_0$ (2) $\dfrac{r^2}{2}B_0$

(3) $\dfrac{r^2}{3}B_0$ (4) none

59. A small square loop of edge length a and resistance R is moved with velocity v_0 away from an infinitely long current carrying conductor carrying current i so that the conductor and side of square are always in same plane. Find induced current in loop at a separation of r.

(1) $\dfrac{\mu_0 i a^2 v}{\pi r^2 R}$ (2) $\dfrac{\mu_0 i a^2 v}{4\pi r^2 R}$

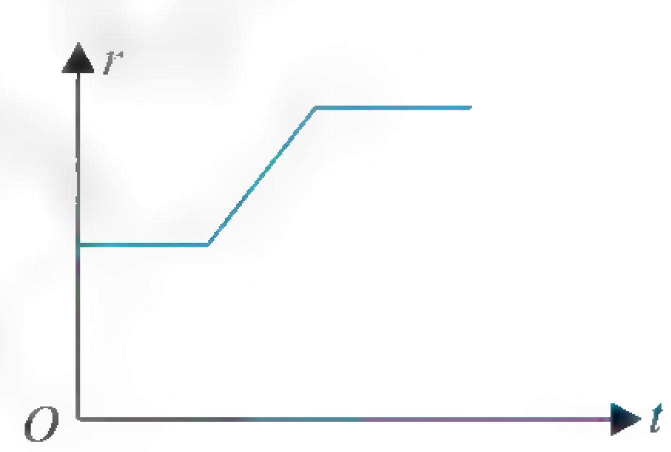
(3) $\dfrac{\mu_0 i a^2 v}{2\pi r^2 R}$ (4) $\dfrac{\mu_0 i a v}{2\pi r R}$

60. In Figs. (a) and (b), two air-cored solenoids P and Q have been shown. They are placed near each other. In Fig. (a), when I_p the current in P, changes at the rate of 5 A/s, an emf of 2 mV is induced in Q. The current in P is then switched off, and a current changing at 2 A/s is fed through Q as shown in diagram. What emf will be induced in P?

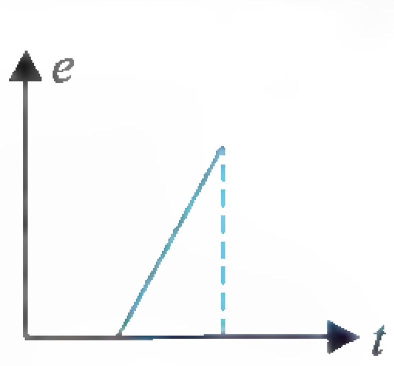
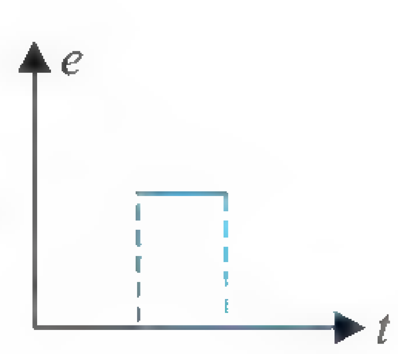
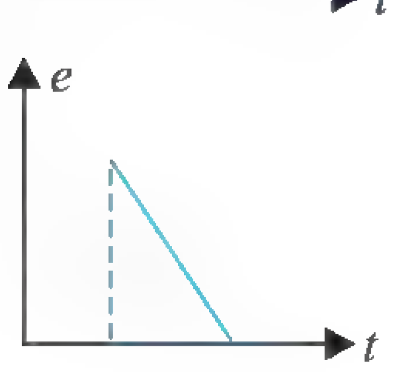
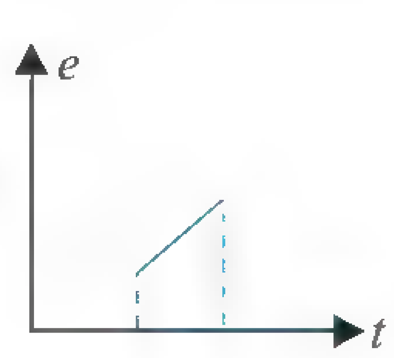
(a) (b)

(1) 8×10^{-4} V (2) 2×10^{-3} V
(3) 5×10^{-3} V (4) 8×10^{-2} V

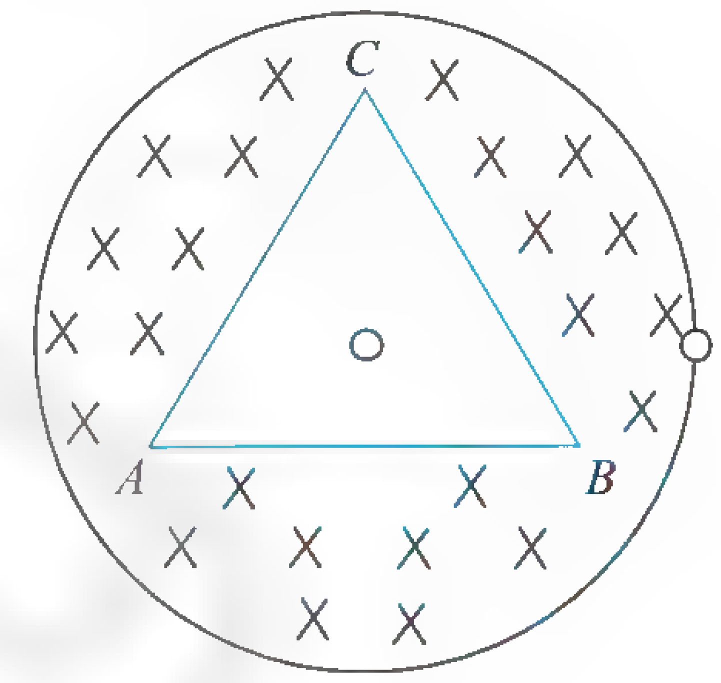
61. Radius of a circular ring is changing with time and the coil is placed in uniform magnetic field perpendicular to its plane. The variation of 'r' with time 't' is shown in figure.



Then induced emf e with time t will be best represented by

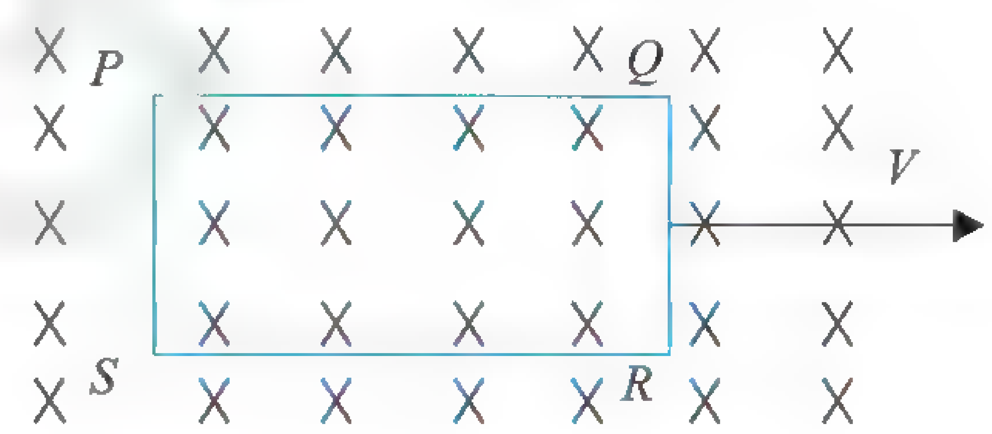
(1) 
(2) 
(3) 
(4) 

62. A triangular wire frame (each side = 2 m) is placed in a region of time variant magnetic field $dB/dt = \sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle and its centre coincides with the centre of triangle. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2 Ω each. The magnitude of potential difference between the points A and B will be



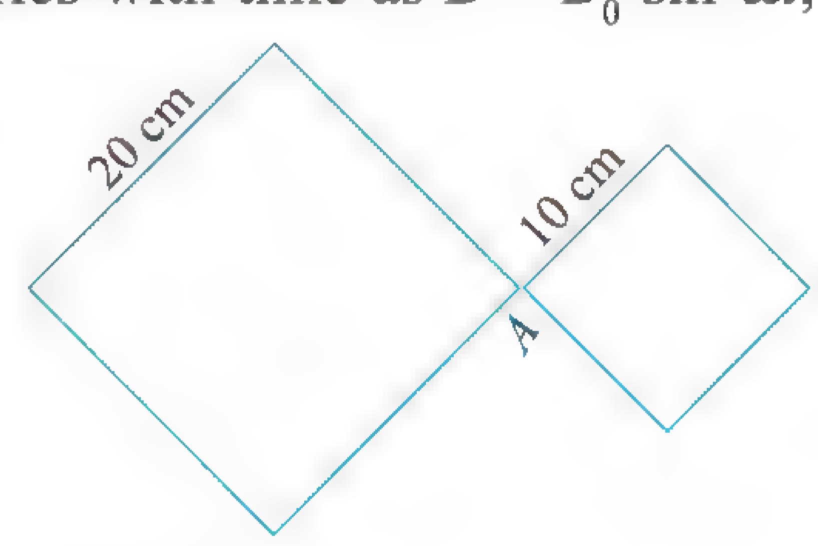
(1) 0.4 V (2) 0.6 V
(3) 1.2 V (4) None

63. A metallic square loop $PQRS$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in figure. If V_P, V_Q, V_R and V_S are the potentials of points P, Q, R and S, then which of the following is an incorrect statement?



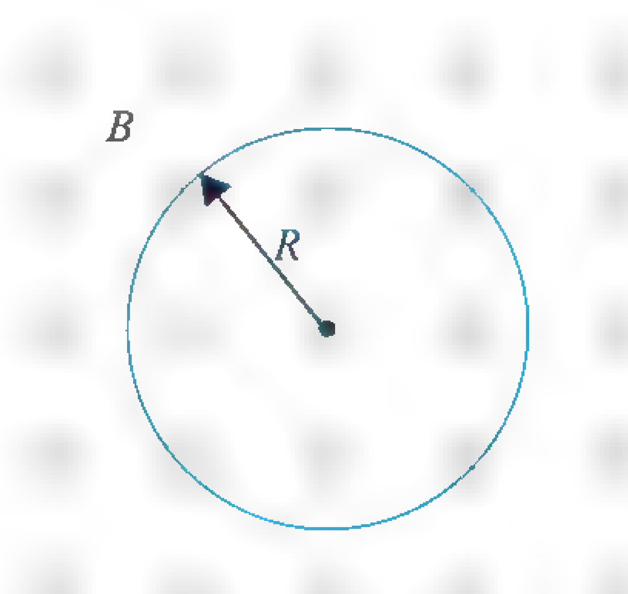
(1) $V_P = V_Q$ (2) $V_P > V_S$
(3) $V_P > V_R$ (4) $V_S > V_R$

64. A plane loop is shaped as two squares (figure) and placed in a uniform magnetic field at right angle to the loop's plane. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where $B_0 = 10$ mT and $\omega = 100$ rads^{-1}. The wires do not touch at point A. If resistance per unit length of the loop is 50 mΩ/m, then amplitude of current induced in the loop is



(1) 1.5 A (2) 1.0 A
(3) 0.5 A (4) 2.0 A

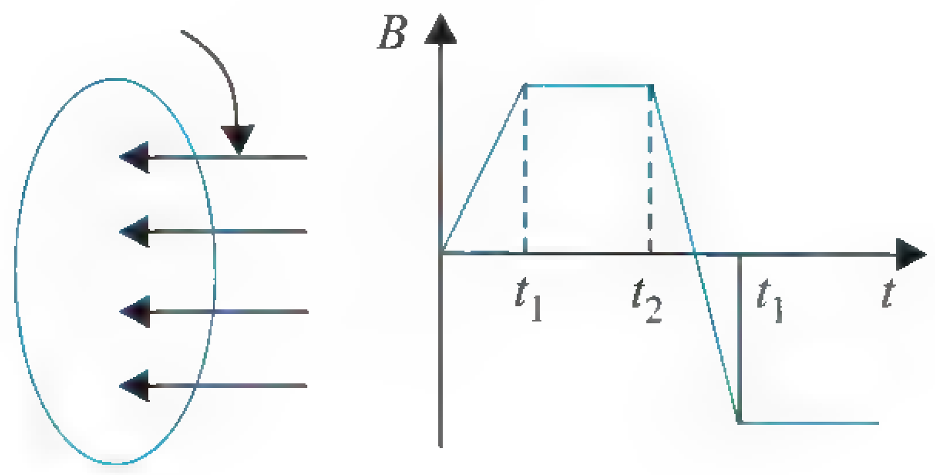
65. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time t, as $R = R_0 + t$. The emf induced in the loop is

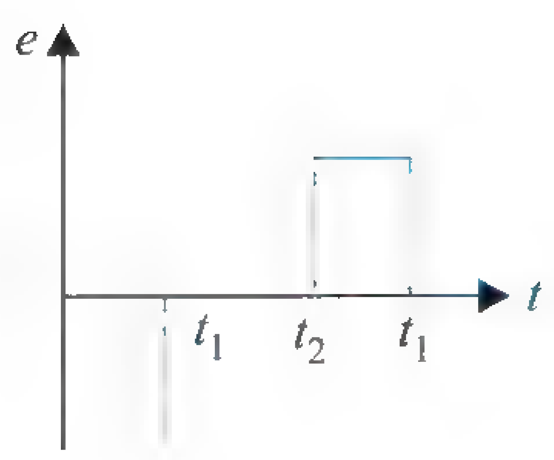


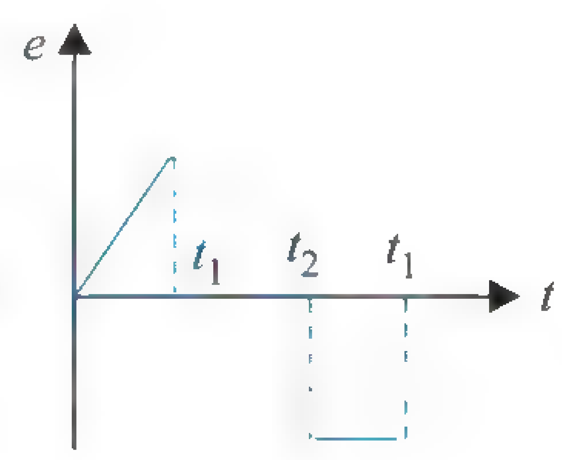
(1) $2\pi(R_0 + t)$B clockwise
(2) $\pi(R_0 + t)$B clockwise
(3) $2\pi(R_0 + t)$B anticlockwise
(4) zero.

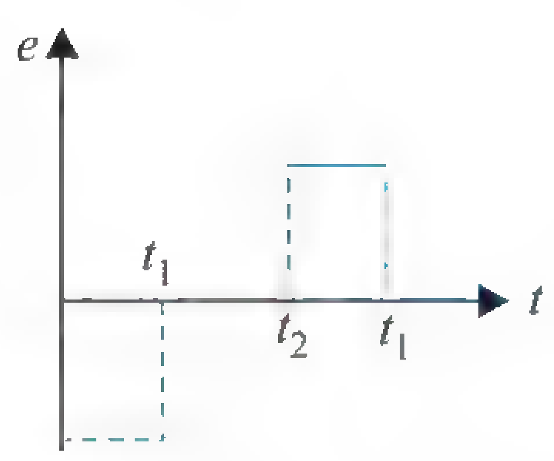
66. A wire loop is placed in a region of time varying magnetic field which is oriented orthogonally to the plane of the loop as shown in figure. The graph shows the magnetic field variation as the function of time. Assume the positive emf is the one which drives a current in the clockwise direction and seen by the observer in the direction of B. Which of

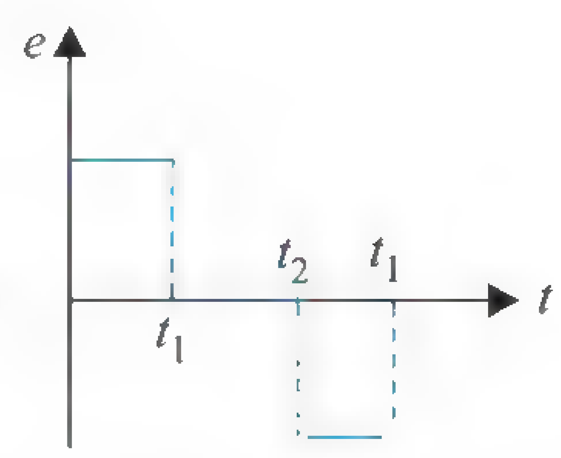
the following graphs best represents the induced emf as a function of time ?



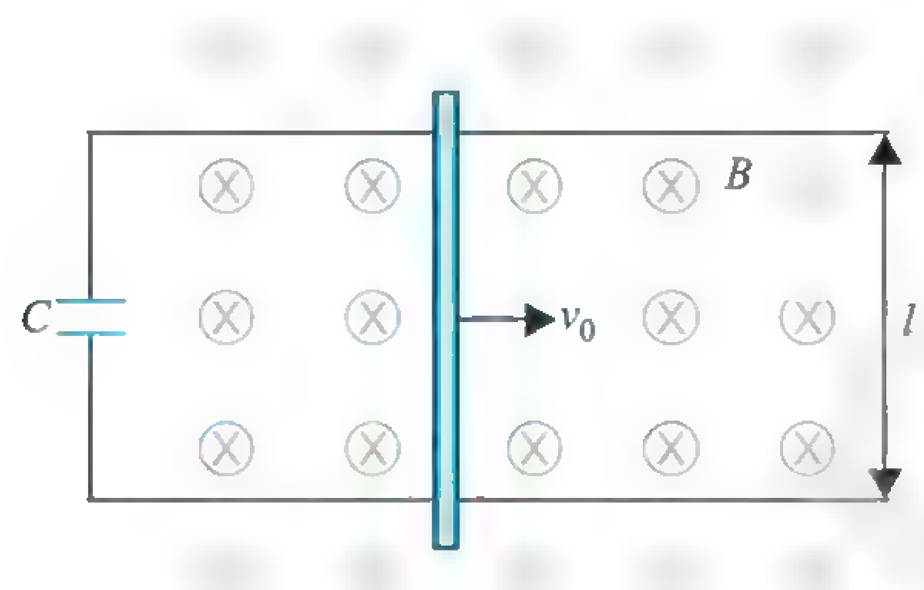
(1) 

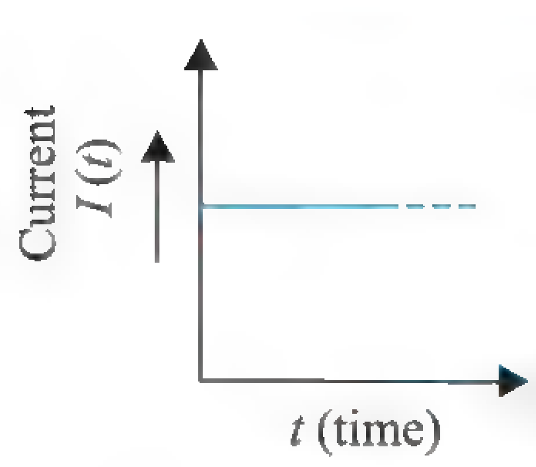
(2) 

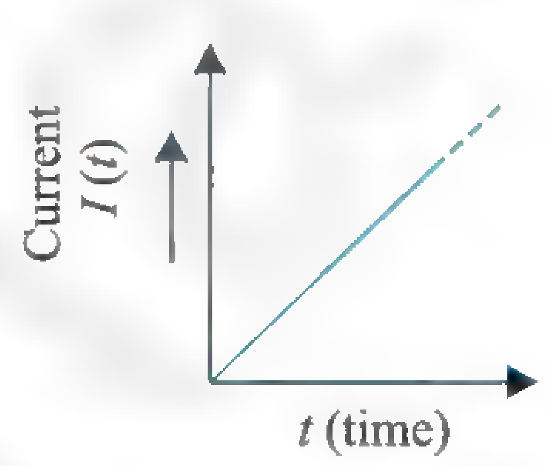
(3) 

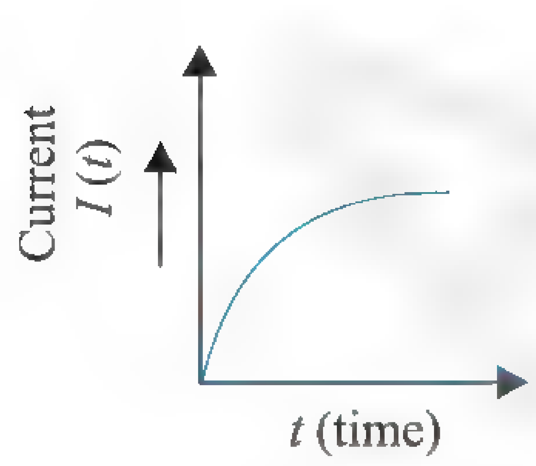
(4) 

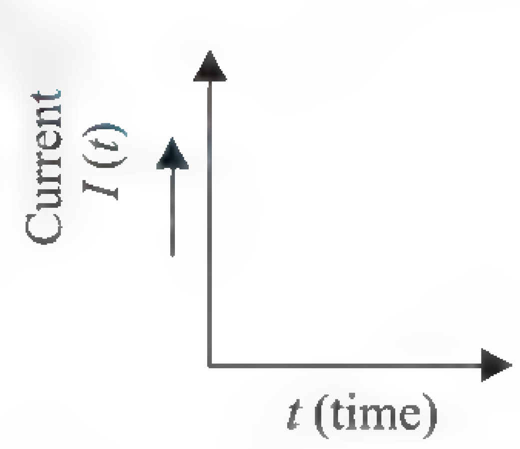
67. Two infinitely long conducting parallel rails are connected through a capacitor C as shown in figure. A conductor of length l is moved with constant speed v_0. Which of the following graph truly depicts the variation of current through the conductor with time ?



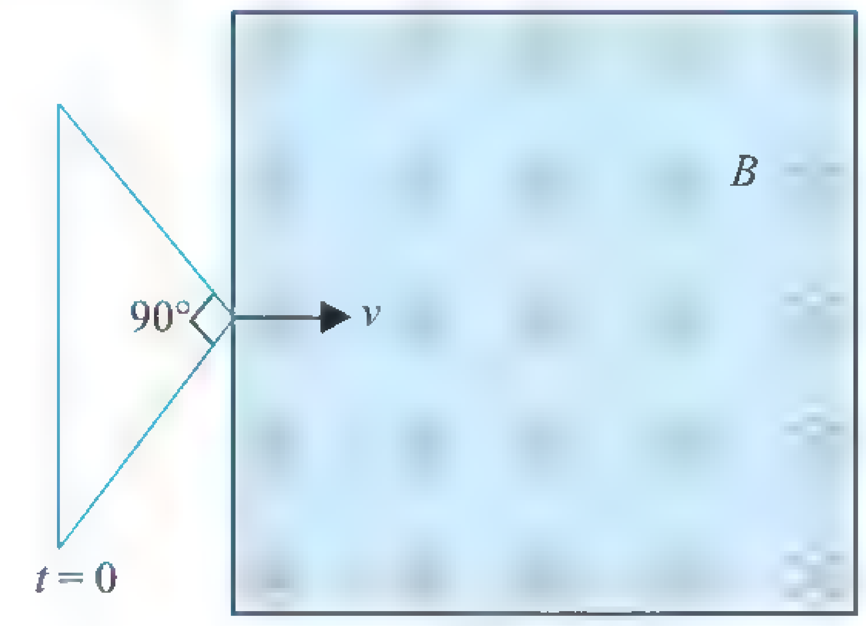
(1) 

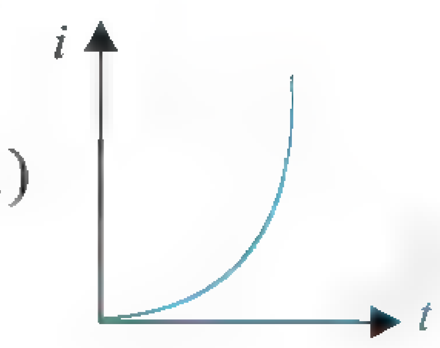
(2) 

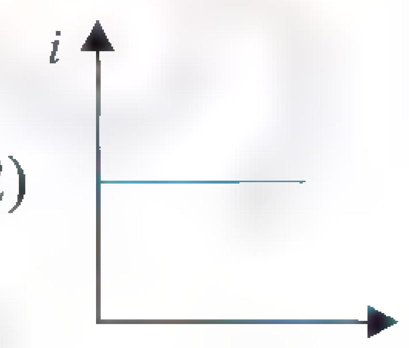
(3) 

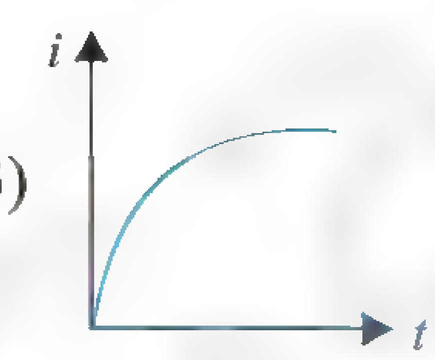
(4) 

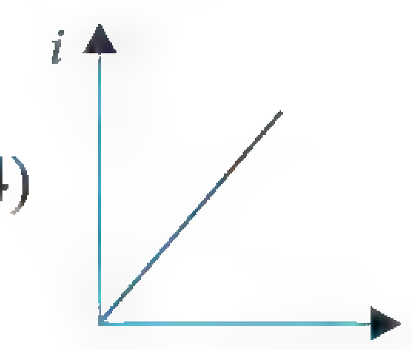
68. Figure shows an isosceles triangle wire frame with apex angle equal to $\pi/2$. The frame starts entering into the region of uniform magnetic field B with constant velocity v at $t = 0$. The longest side of the frame is perpendicular to the direction of velocity. If i is the instantaneous current through the frame then choose the alternative showing the correct variation of i with time.



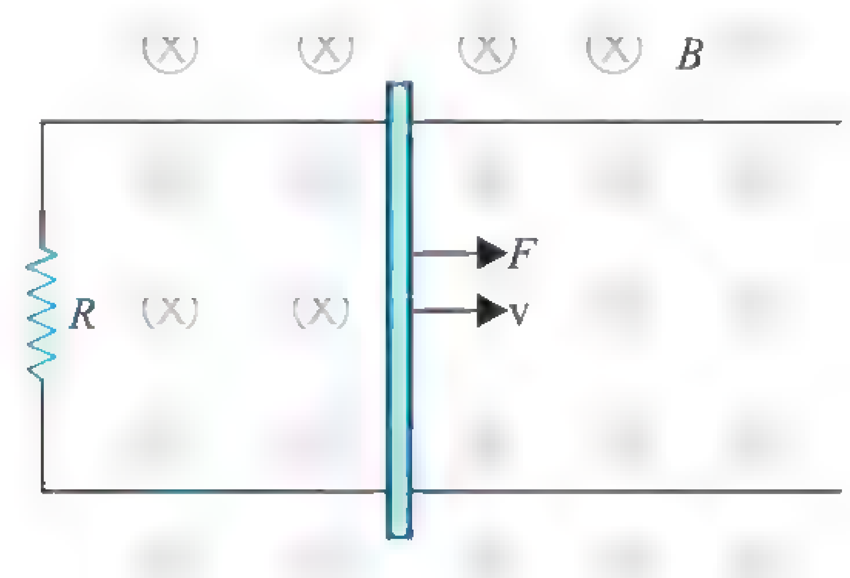
(1) 

(2) 

(3) 

(4) 

69. A rod closing the circuit shown in figure moves along a U shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicular to the plane. Calculate F if the rate of heat generation in the circuit is Q.



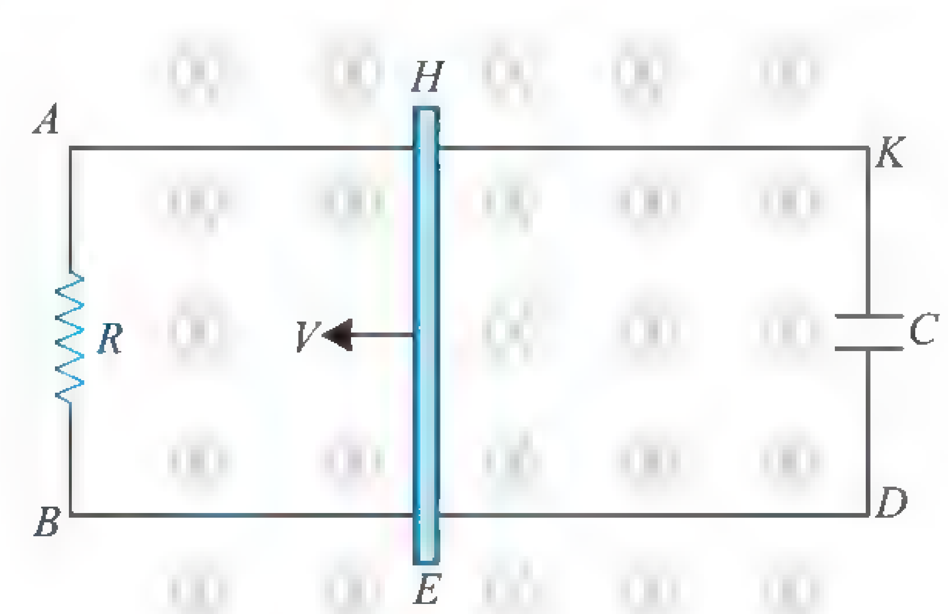
(1) $F = Qv$

(2) $F = \dfrac{Q}{v}$

(3) $F = \dfrac{v}{Q}$

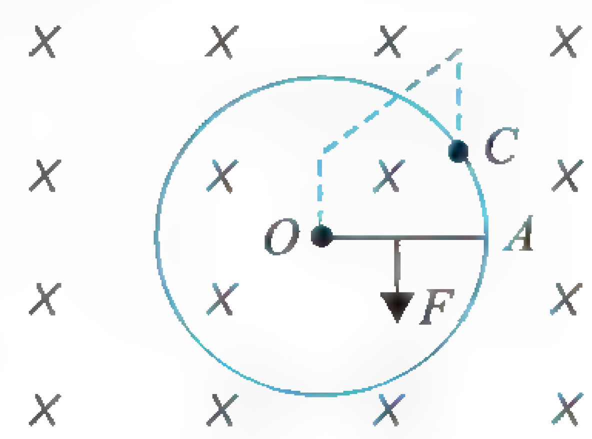
(4) $F = \sqrt{Qv}$

70. In the circuit shown in figure, a conducting wire HE is moved with a constant speed v towards left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of circuit inwards. The current in $HKDE$ is



(1) clockwise
(2) anticlockwise
(3) direction will change with time
(4) zero

71. Figure shows a conducting circular loop of radius a placed in a uniform, perpendicular magnetic field B. A thick metal rod OA is pivoted at the centre O. The other end of the rod touches the loop at A. The centre O and a fixed point C on the loop are connected by a wire OC of resistance R. A force is applied at the middle point of the rod OA perpendicularly, so that the rod rotates clockwise at a uniform angular velocity ω. Find the force.



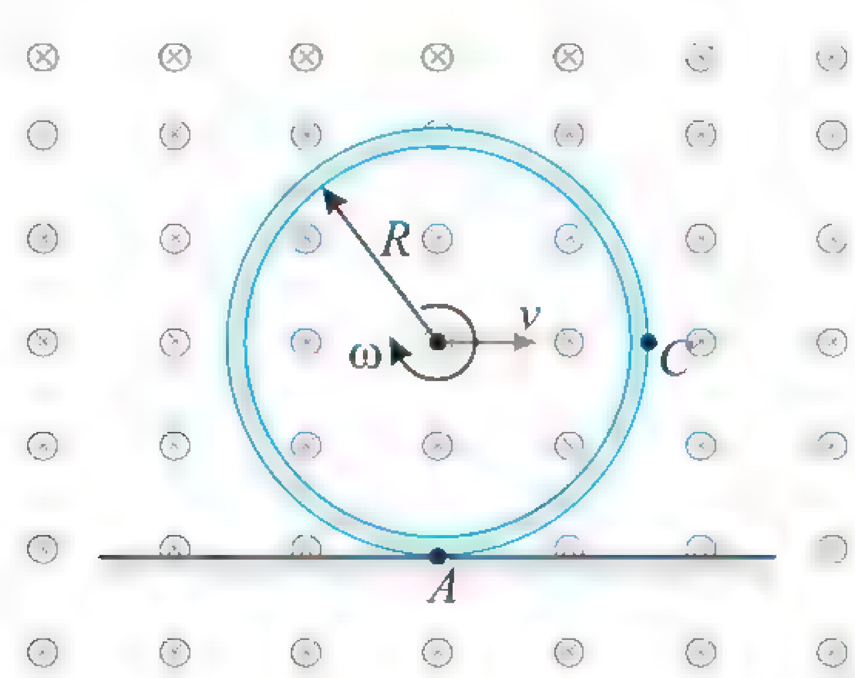
(1) $\dfrac{\omega a^3 B^2}{R}$ to the right of OA in the figure

(2) $\dfrac{\omega a^3 B^2}{2R}$ to the right of OA in the figure

(3) $\dfrac{\omega a^3 B^2}{2R}$ to the left of OA in the figure

(4) None of these

72. A ring of radius R is rolling on a horizontal plane with constant velocity v. There is a constant and uniform magnetic field B which is perpendicular to the plane of the ring. Emf across the lowest point A and right-most point C as shown in the figure will



(1) increase with time
(2) decrease with time
(3) remains constant and equal to $\sqrt{2}BvR$
(4) remains constant and equal to BvR

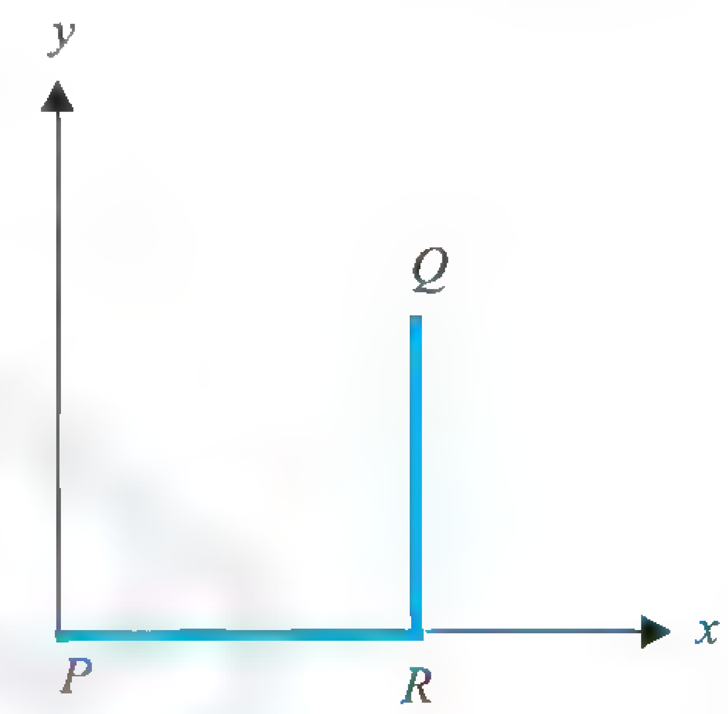
73. A current $I = 3.36\ (1 + 2t) \times 10^{-2}$ A increases at a steady state in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and is placed at a distance of 1 m from the wire. The resistance of loop is $8.4 \times 10^{-4}\ \Omega$. Find the approximate value of induced current in the loop.

(1) 5.024×10^{-11} A
(2) 3.8×10^{-11} A
(3) 2.75×10^{-11} A
(4) 1.23×10^{-11} A

74. The magnetic field in a region is given by $\vec{B} = \hat{k}\dfrac{B_0}{L}y$ where L is a fixed length. A conducting rod of length L lies along the Y-axis between the origin and the point $(0, L, 0)$. If the rod moves with a velocity $\vec{v} = v_0\hat{i}$, find the emf induced between the ends of the rod.

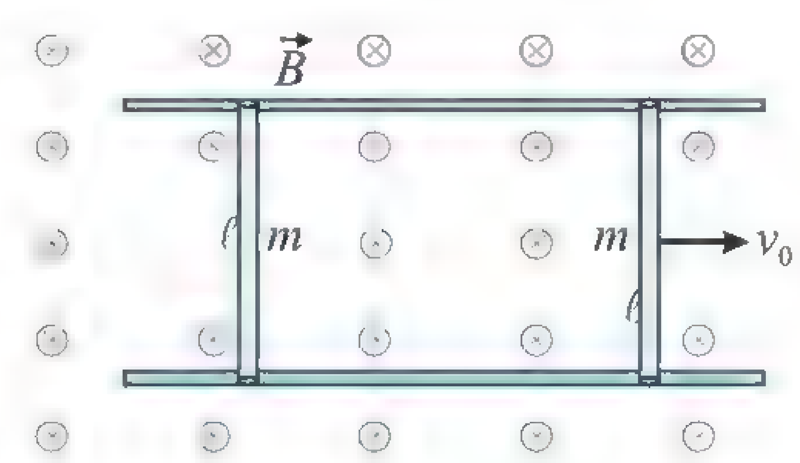
(1) $2B_0v_0l$
(2) B_0v_0l
(3) $\dfrac{B_0v_0l}{2}$
(4) None of these

75. An inverted L shaped conductor PRQ is made by joining two perpendicular conducting rods, each of length $1.5\ L$, at end R. This structure is moving in x–y plane containing variable magnetic field, $\vec{B} = -3x\hat{k}$ with a velocity $v\hat{i} + v\hat{j}$. If potential of P is V_P and that of Q is V_Q, then value of $V_P - V_Q$ at the instant when P is at origin as shown in figure, will be



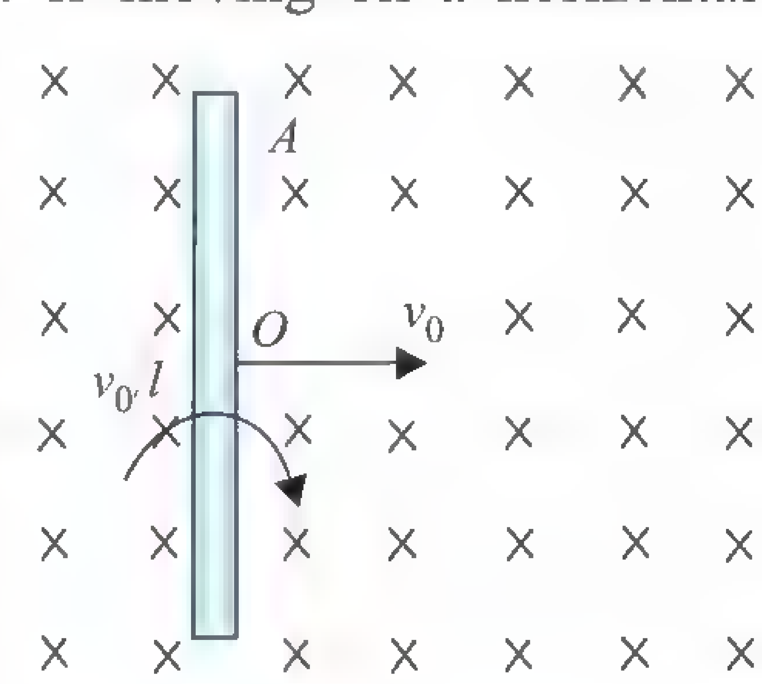
(1) $\dfrac{9vL^2}{8}$
(2) $\dfrac{27vL^2}{8}$
(3) $-\dfrac{9vL^2}{8}$
(4) $-\dfrac{27vL^2}{8}$

76. Consider parallel conducting rails separated by a distance l. There exists a uniform magnetic field B perpendicular to the plane of the rails as shown in figure. Two conducting wires each of length l are placed so as to slide on parallel conducting rails. One of the wires is given a velocity v_0 parallel to the rails. Till steady state is achieved, loss in kinetic energy of the system is



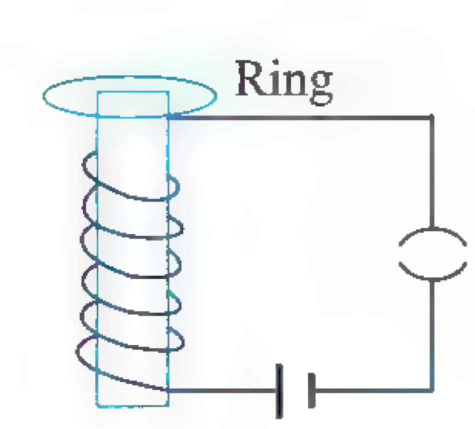
(1) zero
(2) $\dfrac{3}{4}mv_0^2$
(3) $\dfrac{1}{4}mv_0^2$
(4) $\dfrac{3}{8}mv_0^2$

77. A conducting rod of length l is moving on a horizontal smooth surface. Magnetic field in the region is vertically downward and of magnitude B_0. If centre of mass (COM) of the rod is translating with velocity v_0 and rod rotates about COM with angular velocity v_0/l, then potential difference between points O and A will be



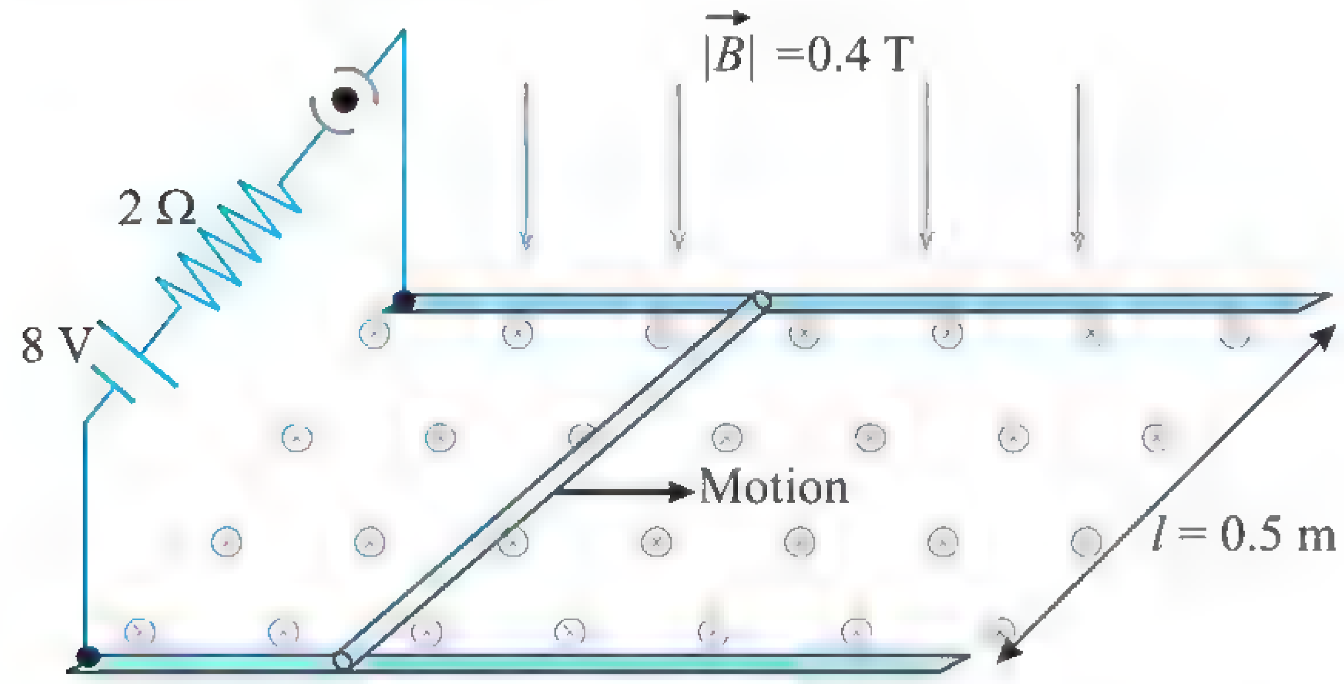
(1) $\dfrac{5}{8}B_0v_0l$
(2) $\dfrac{3}{8}B_0v_0l$
(3) $\dfrac{1}{8}B_0v_0l$
(4) $\dfrac{1}{2}B_0v_0l$

78. Figure shows a powerful electromagnet arrangement. A copper ring, which is free to move, is placed on the projecting part of the core as shown. When the key is inserted, the ring



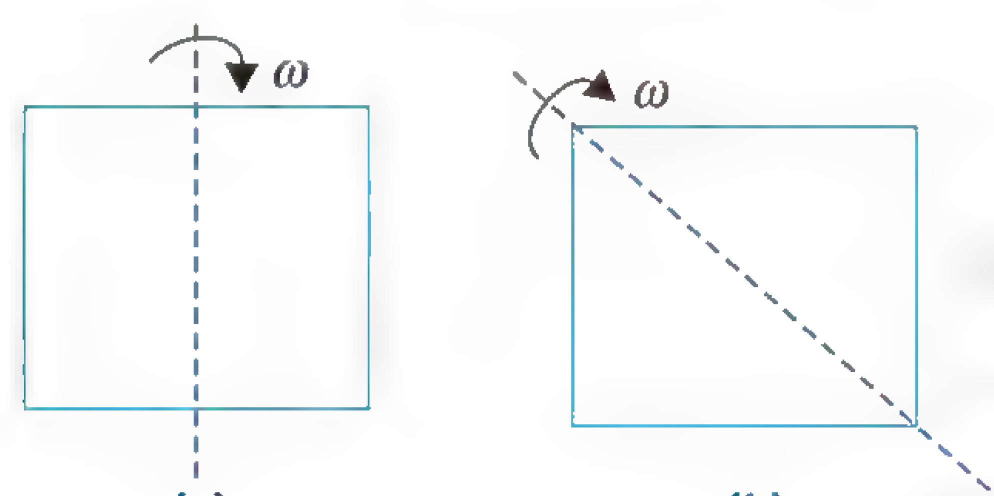
(1) is thrown up (2) remains stationary
(3) sticks to the core (4) slips down along the core

79. Figure shows a conducting frame having battery and a resistance on which a movable conductor of length 0.5 m can slide. The whole arrangement is placed in a uniform magnetic field of $B = 0.4$ T directed perpendicular and into the plane of frame. Initially the circuit is open. When the key is inserted, the conductor begins to move. It is found that a force 0.5 N has to be applied on the conductor to the left to keep it moving at constant speed to the right. Current flowing in the conductor is:



(1) 5 A (2) 2.5 A
(3) 1.25 A (4) 1 A

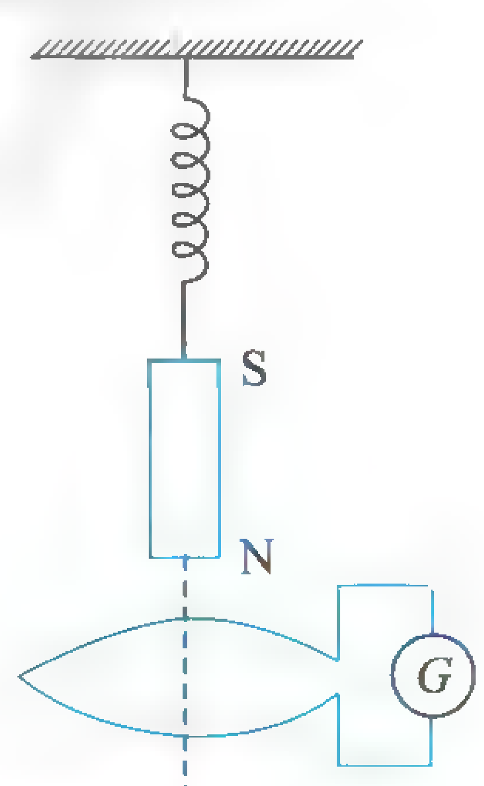
80. Figures (a) and (b) show a square loop of side a rotating about the given axes in a uniform magnetic field such that the field is perpendicular to the axis in both cases. Angular speed of rotation is both cases being the same,



(a) (b)

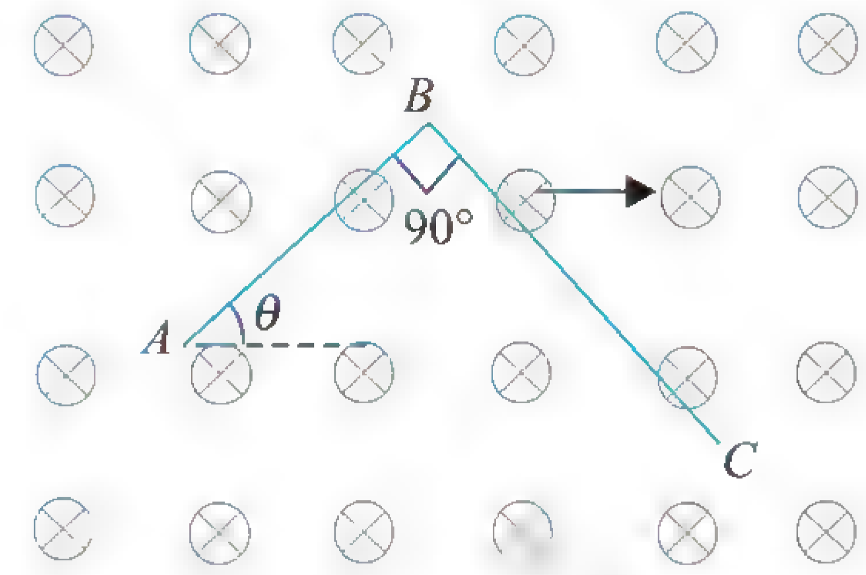
(1) induced emf is more in (a) than in (b)
(2) induced emf is more in (b) than in (a)
(3) induced emf in the two cases are equal and non-zero
(4) induced emf in the two cases are equal and zero.

81. Figure shows a magnet suspended at the lower end of a spring while its length lies along the axis of a fixed circular conducting coil. The magnet is made to oscillate. Deflection in the galvanometer is



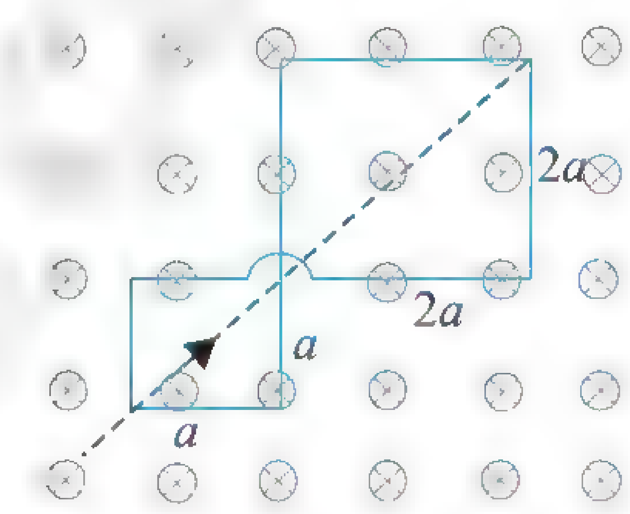
(1) minimum when the magnet is at mean position
(2) maximum when the magnet is at mean position
(3) zero when the magnet is at mean position
(4) maximum when the magnet is at extreme position

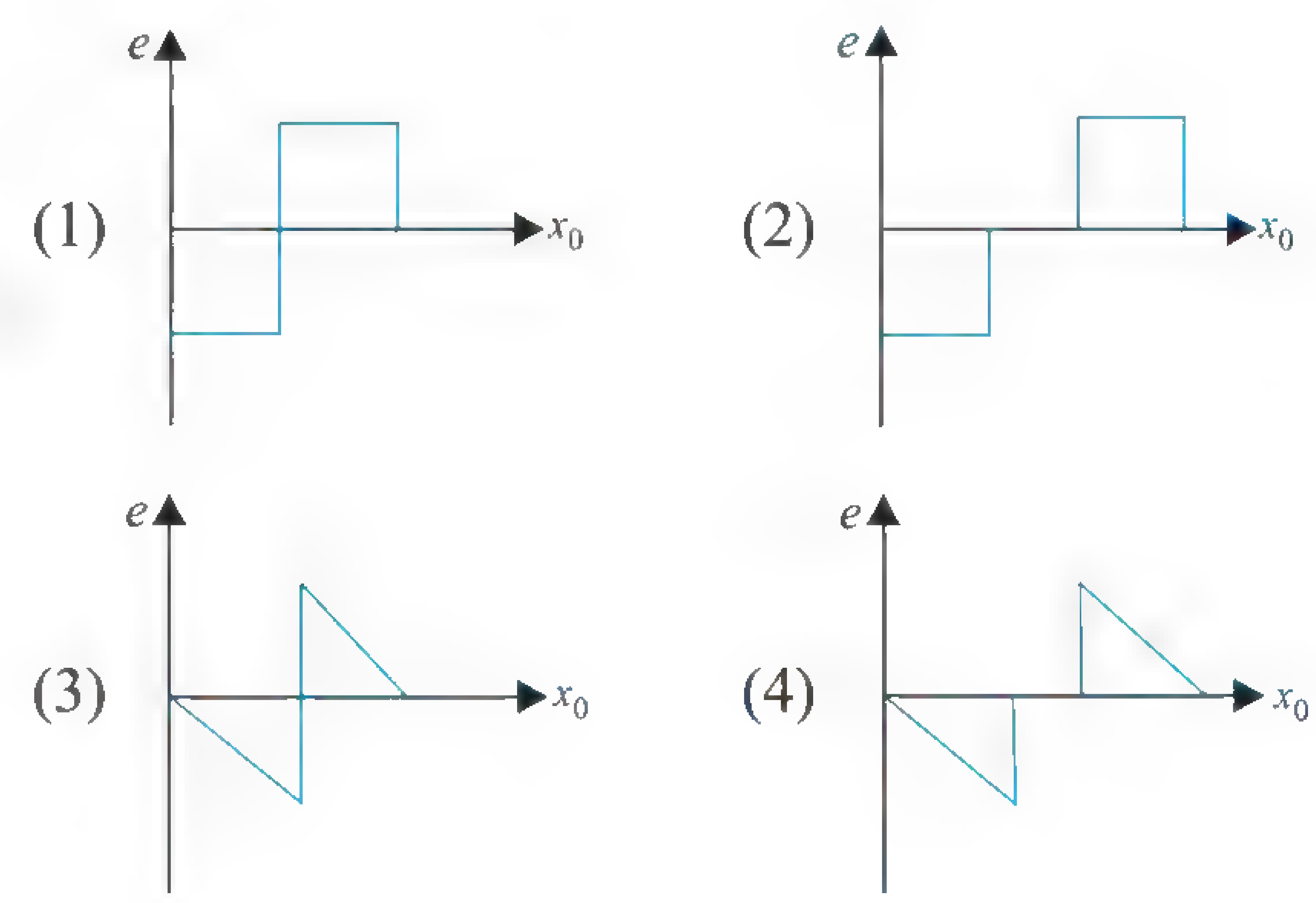
82. A conducting wire ABC (as shown) is moving with a constant velocity along horizontal direction. The magnetic field is perpendicular to the wire and directed into the page. $AB = BC$. Find the range of angle θ made by rod AB with horizontal at A so that value of induced emf at A is greater than at C.



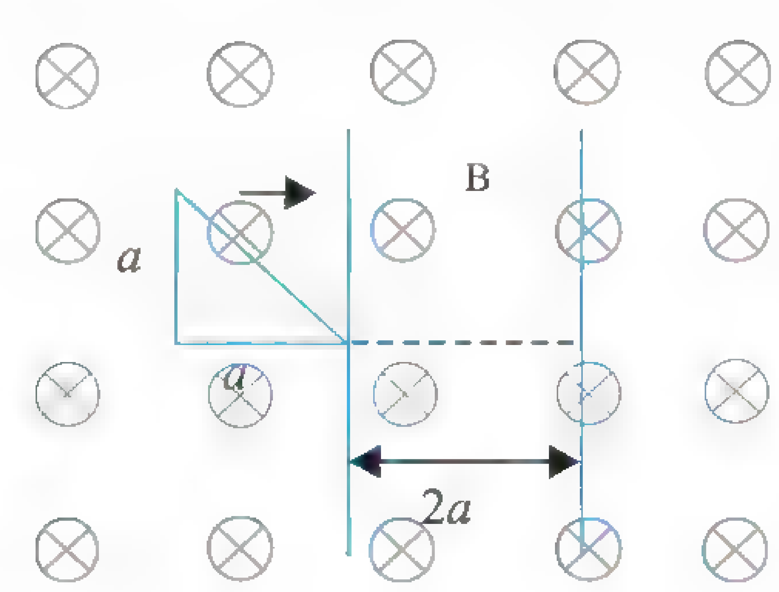
(1) $\theta > 45°$ (2) $\theta < 45°$
(3) $\theta > 60°$ (4) $\theta < 75°$

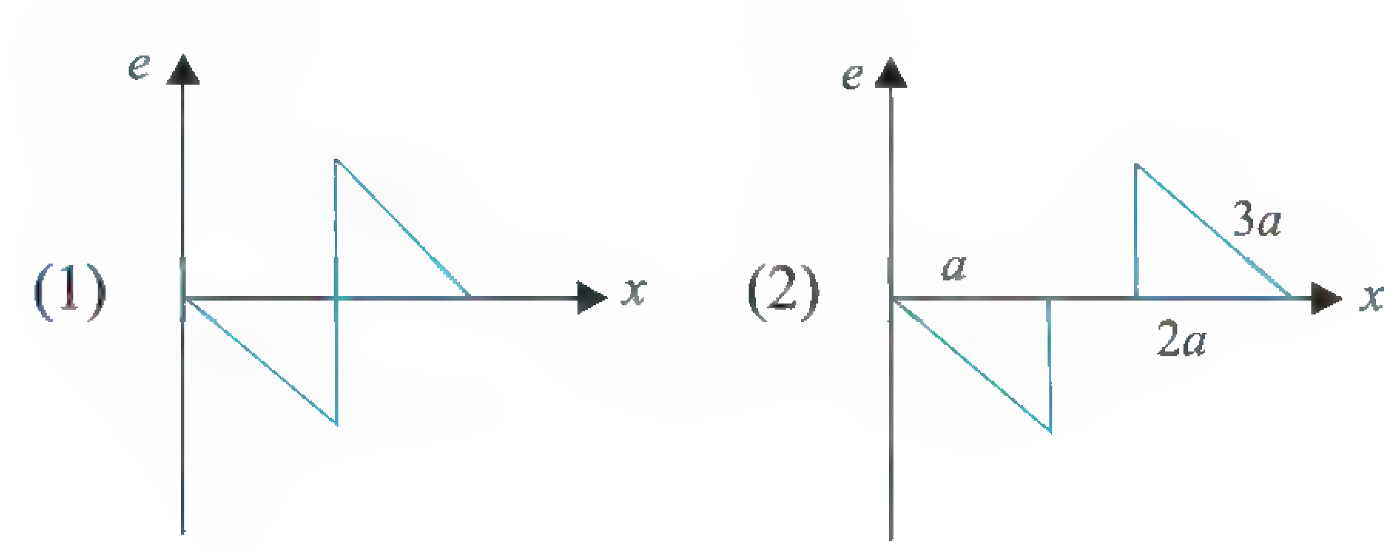
83. A uniform magnetic field exists in a square of side $2a$ (as shown in figure). A square loop of side a enters the field along a diagonal and leave it at a constant speed. Draw the curve between induced emf e and distance along the diagonal, say x_0.

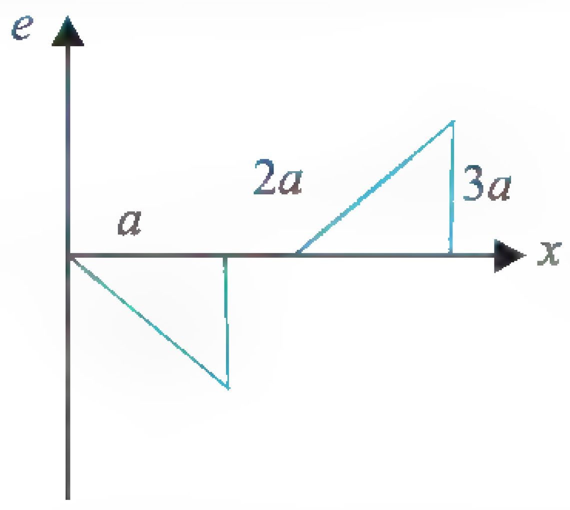


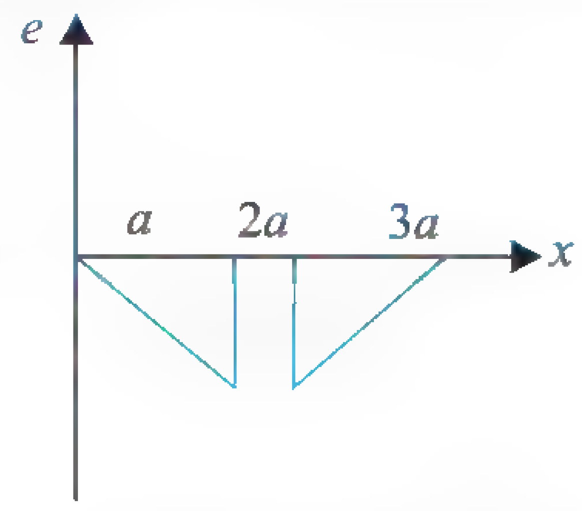


84. A right angled triangular loop as shown below enters uniform magnetic field (at right angle to the boundary of the field) directed into the paper. Draw the graph between induced emf e and the distance along the perpendicular to the boundary of the field, (say x) along which loop moves.

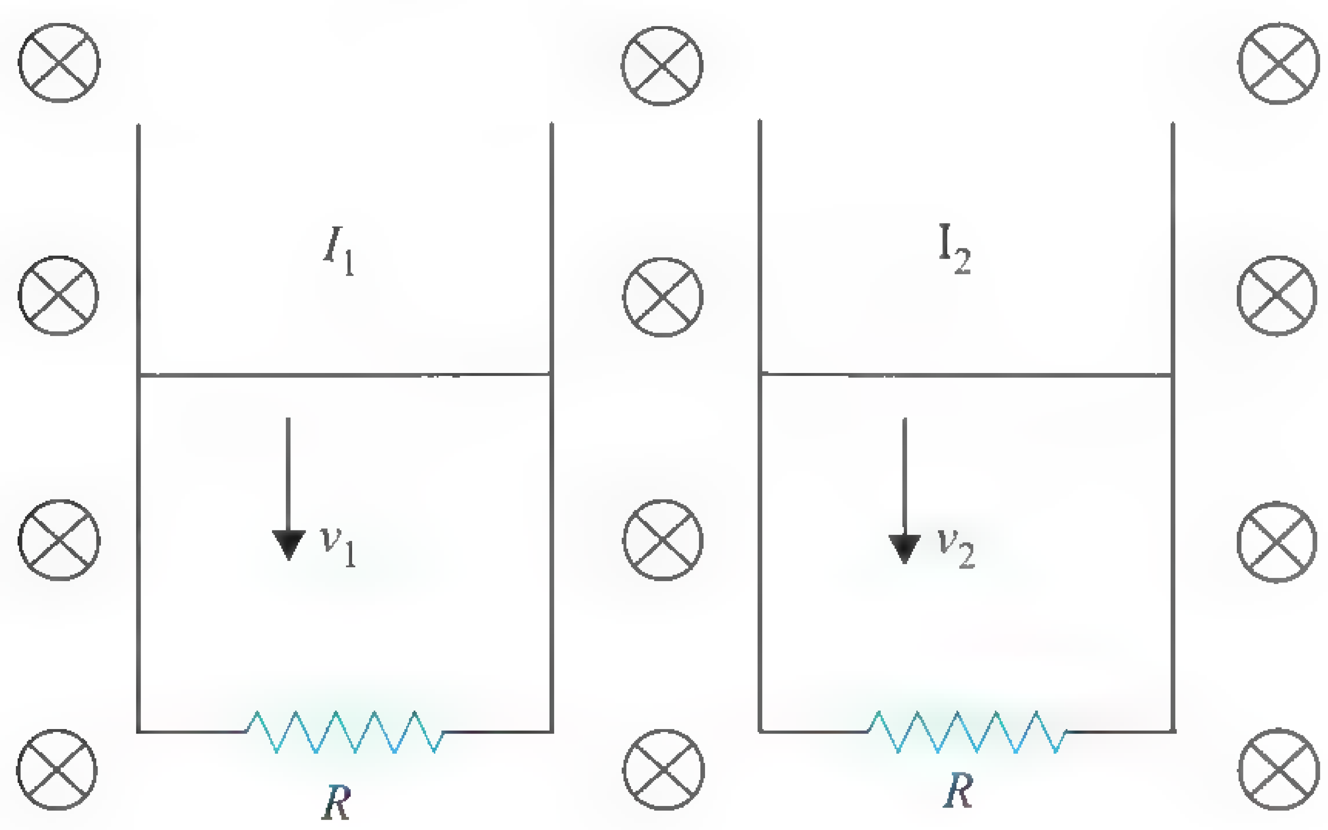




(3) 

(4) 

85. In a magnetic field as shown, in figure two horizontal wires of same mass and lengths l_1 and l_2 are free to slide on different vertical rails with velocities v_1 and v_2 respectively. If the resistances of two circuits are same and a_1 and a_2 are accelerations of two horizontal wires respectively, the condition for $a_1 > a_2$ is



(1) $\dfrac{l_1}{l_2} = \dfrac{v_2}{v_1}$ (2) $\dfrac{l_1}{l_2} > \left(\dfrac{v_2}{v_1}\right)^{1/2}$

(3) $\dfrac{l_1}{l_2} < \left(\dfrac{v_1}{v_2}\right)^{1/2}$ (4) $\dfrac{l_1}{l_2} < \dfrac{v_1}{v_2}$

86. In the previous problem, if two falling conductors attain velocities v_1 and v_2 respectively after falling through same height h, ratio of energy dissipated as heat to the energy dissipated in resistor per unit time for two conductors is same. Find which of the following conditions will be satisfied

(1) $gh + \dfrac{1}{2}v_1v_2 = 0$ (2) $gh + v_1v_2 = 0$

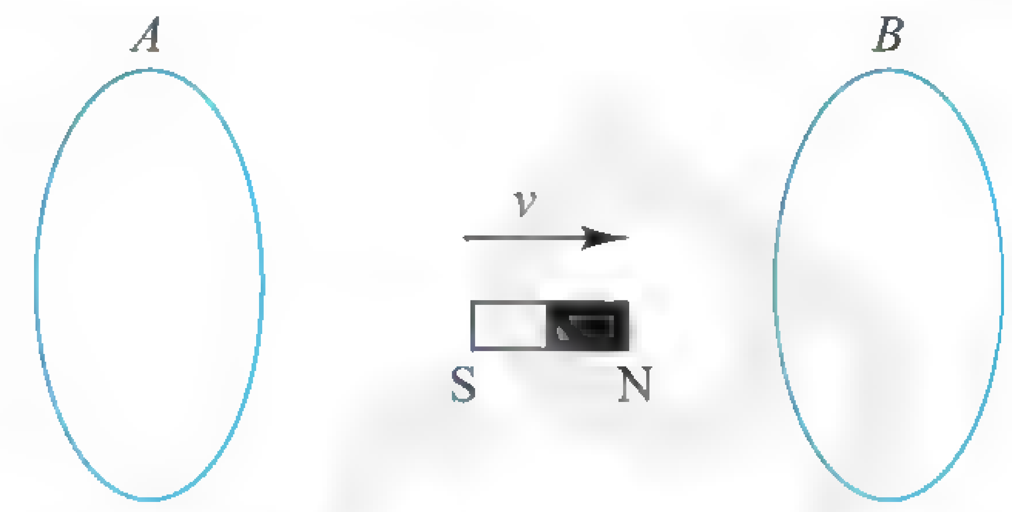
(3) $\dfrac{1}{2}gh + v_1v_2 = 0$ (4) $gh + \dfrac{v_1^2 v_2}{v_1 + v_2} = 0$

Multiple Correct Answers Type

1. An infinite current-carrying conductor is placed along the z-axis and a wire loop is kept in the x–y plane. The current in the conductor is increasing with time. Then the
(1) emf induced in the wire loop is zero
(2) magnetic flux passing through the wire loop is zero
(3) emf induced is zero but magnetic flux is not zero
(4) emf induced is not zero but magnetic flux is zero

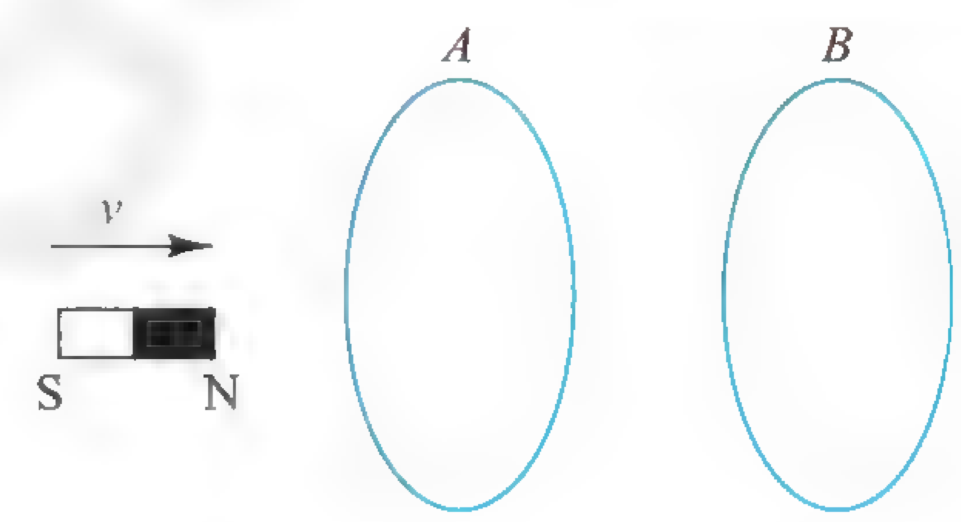
2. A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field, whose direction is perpendicular to that fixed diameter.
(1) The emf will be maximum at the moment when flux is zero
(2) The emf will be 0 at the moment when flux is maximum
(3) The emf will be maximum at the moment when plane of the loop is parallel to the magnetic field
(4) The phase difference between the flux and the emf is $\pi/2$

3. A bar magnet is moved between two parallel circular loops A and B with a constant velocity v as shown in figure.



(1) The current in each loop flows in the same direction
(2) The current in each loop flows in opposite directions
(3) The loops will repel each other
(4) The loops will attract each other

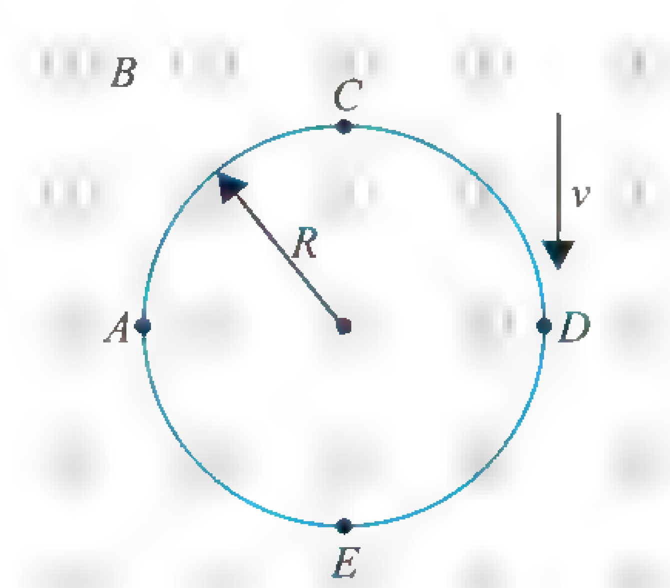
4. A bar magnet moves toward two identical parallel circular loops with a constant velocity v as shown in figure.



(1) Both the loops will attract each other
(2) Both the loops will repel each other
(3) The induced current in A is more than that in B
(4) The induced current is same in both the loops

5. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M will be
(1) $<g$ when it is above R and moving toward R
(2) $>g$ when it is above R and moving toward R
(3) $<g$ when it is below R and moving away from R
(4) $>g$ when it is below R and moving away from R

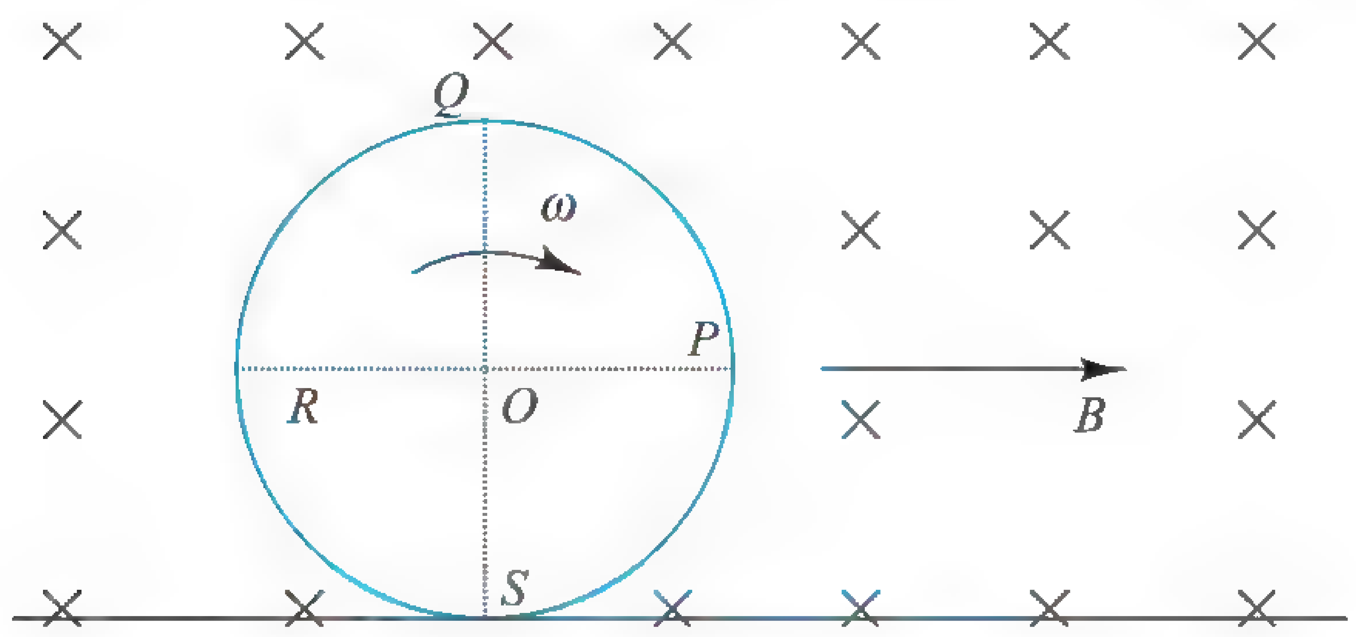
6. A vertical conducting ring of radius R falls vertically in a horizontal magnetic field of magnitude B. The direction of B is perpendicular to the plane of the ring. When the speed of the ring is v,



(1) no current flows in the ring
(2) A and D are at the same potential
(3) C and E are at the same potential
(4) the potential difference between A and D is $2BRv$, with D at a higher potential

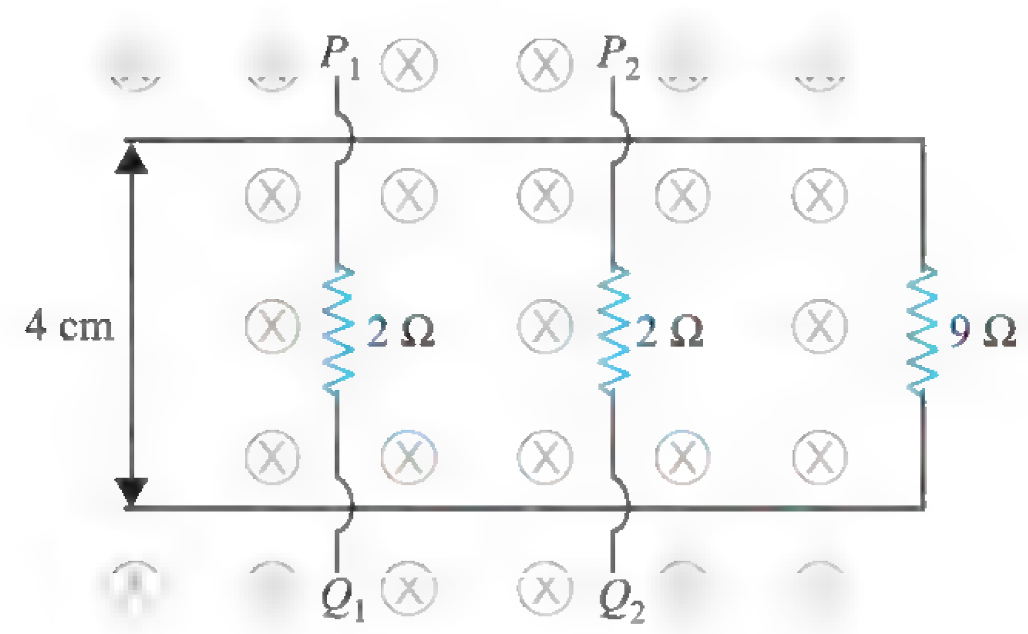
7. A disc of radius R is rolling without sliding on a horizontal surface with a velocity of center of mass v and angular

velocity ω in a uniform magnetic field B which is perpendicular to the plane of the disc as shown in figure. O is the center of the disc and P, Q, R, and S are the four points on the disc. Which of the following statements is true?



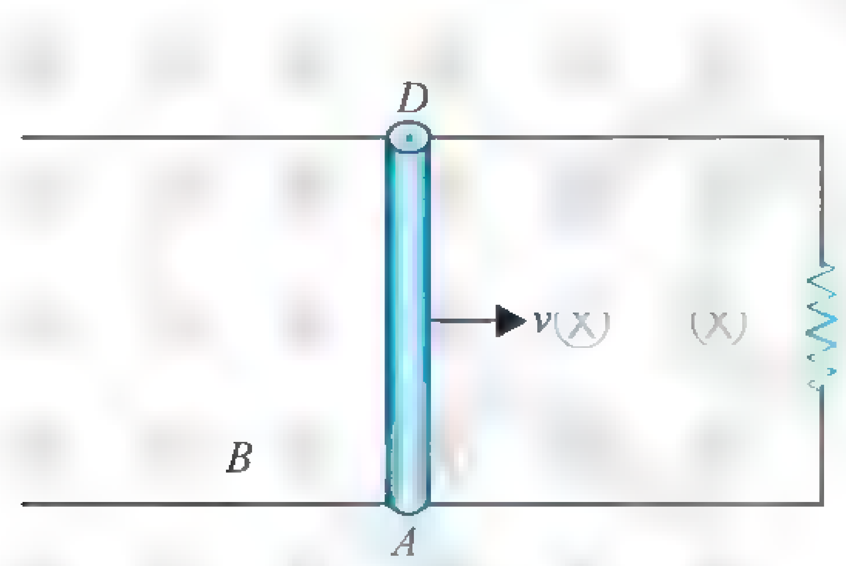
(1) Due to translation, induced emf across $PS = Bvr$
(2) Due to rotation, induced emf across $QS = 0$
(3) Due to translation, induced emf across $RO = 0$
(4) Due to rotation, induced emf across $OQ = Bvr$

8. In figure, the wires P_1Q_1 and P_2Q_2 are made to slide on the rails with same speed of 5 cm s^{-1}. In this region, a magnetic field of 1 T exists. The electric current in the 9 Ω resistance is



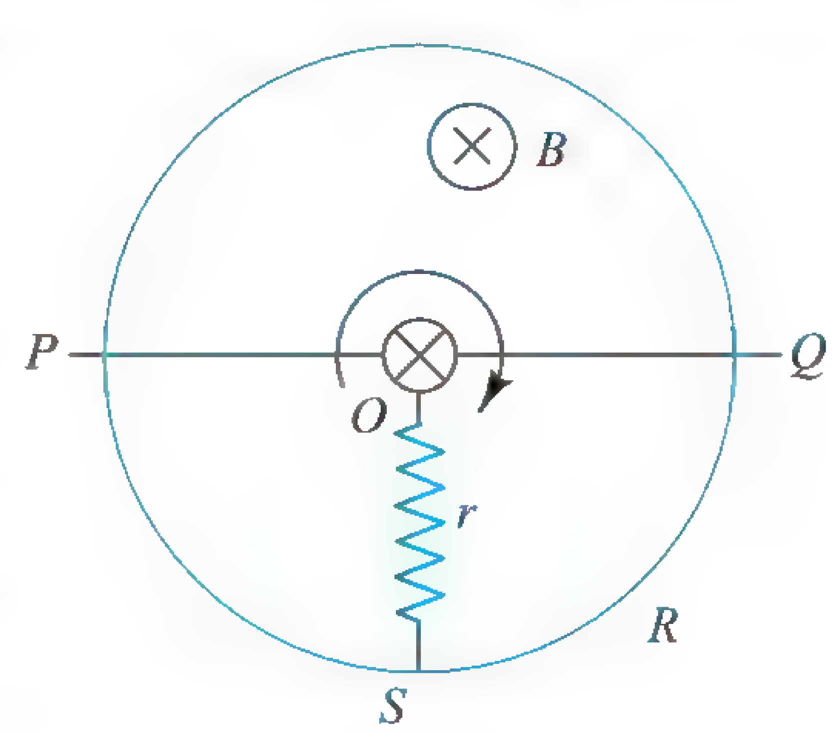
(1) zero if both wires slide toward left
(2) zero if both wires slide in opposite directions
(3) 0.2 mA if both wires move toward left
(4) 0.2 mA if both wires move in opposite directions

9. The conductor AD moves to the right in a uniform magnetic field directed into the plane of the paper.



(1) The free electron in AD will move toward A
(2) D will acquire a positive potential with respect to A
(3) A current will flow from A to D in AD in closed loop
(4) The current in AD flows from lower to higher potential

10. In figure, R is a fixed conducting ring of negligible resistance and radius a. PQ is a uniform rod of resistance r. It is hinged at the center of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There is a uniform magnetic field of strength B pointing inward and r is a stationary resistance. Then

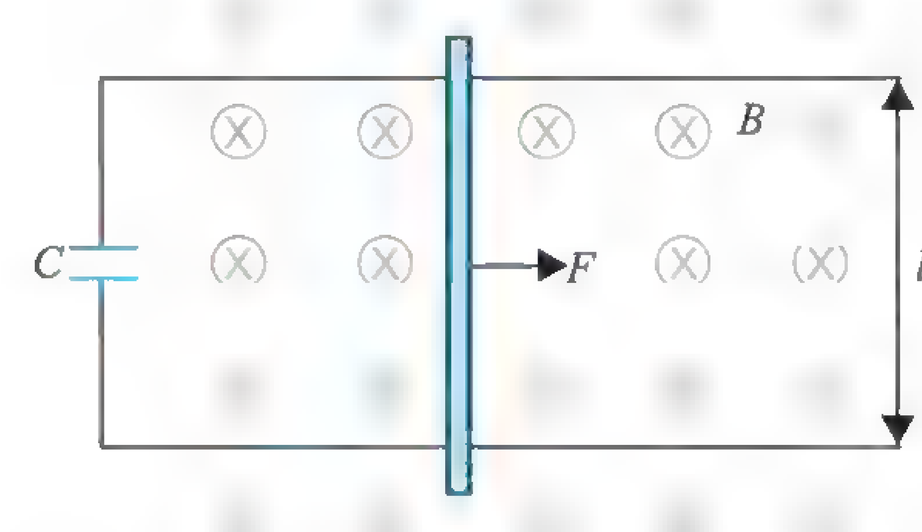


(1) current through r is zero
(2) current through r is $(2B\omega a^2)/5r$
(3) direction of current in external resistance r is from center to circumference
(4) direction of current in external resistance r is from circumference to center

11. The magnetic flux ϕ linked with a conducting coil depends on time as $\phi = 4t^n + 6$, where n is a positive constant. The induced emf in the coil is e.
(1) If $0 < n < 1$, $e \neq 0$ and $|e|$ decreases with time.
(2) If $n = 1$, e is constant.
(3) If $n > 1$, $|e|$ increases with time.
(4) If $n > 1$, $|e|$ decreases with time.

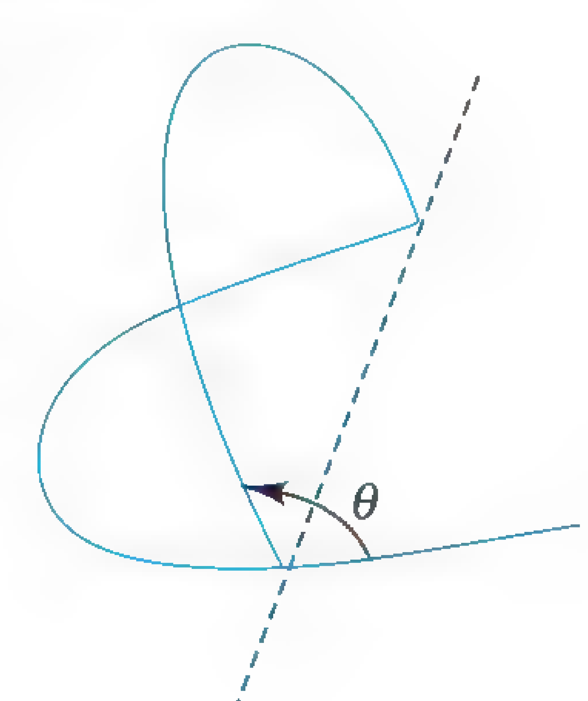
12. A circular loop of radius r, having N turns of a wire, is placed in a uniform and constant magnetic field B. The normal of the loop makes an angle θ with the magnetic field. Its normal rotates with an angular velocity ω such that the angle θ is constant. Choose the correct statement from the following.
(1) emf in the loop is $NB\omega r^2/2 \cos\theta$.
(2) emf induced in the loop is zero.
(3) emf must be induced as the loop crosses magnetic lines.
(4) emf must not be induced as flux does not change with time.

13. A conducting wire of length l and mass m can slide without friction on two parallel rails and is connected to capacitance C. The whole system lies in a magnetic field B and a constant force F is applied to the rod. Then



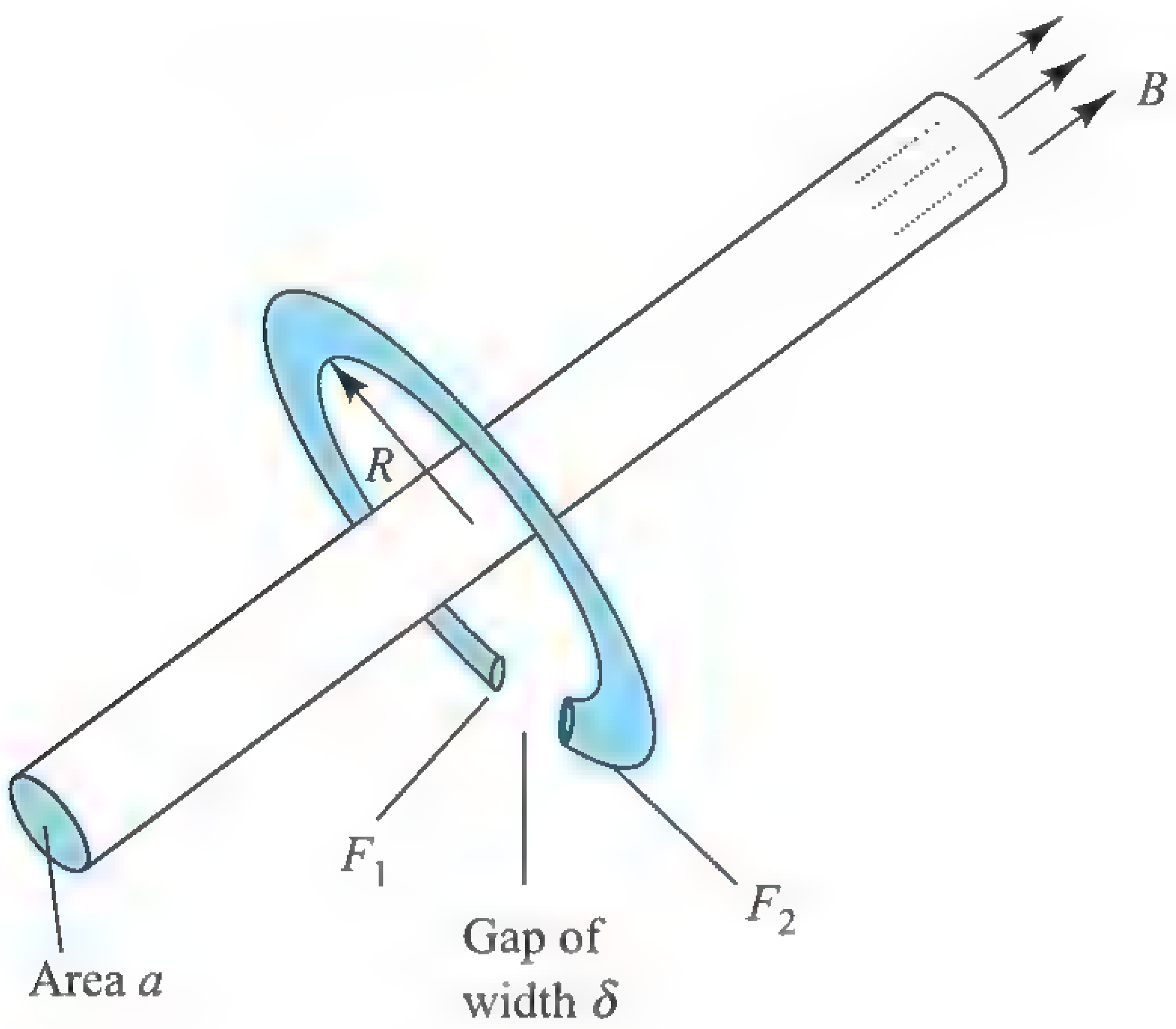
(1) the rod moves with constant velocity.
(2) the rod moves with an acceleration of $(F)/(m + B^2l^2C)$.
(3) there is constant charge on the capacitor.
(4) charge on the capacitor increases with time.

14. A uniform circular loop of radius a and resistance R is placed perpendicular to a uniform magnetic field B. One half of the loop is rotated about the diameter with angular velocity ω as shown in figure. Then, the current in the loop is



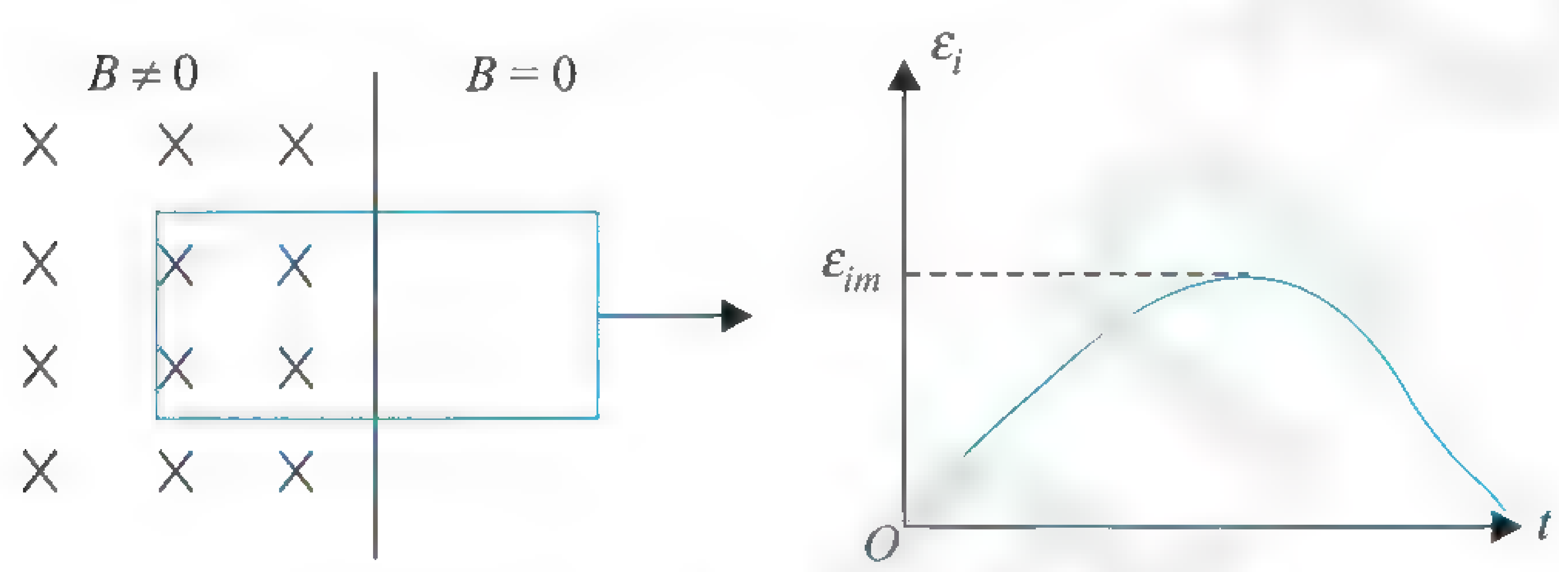
(1) zero, when θ is zero
(2) $\dfrac{\pi a^2 B\omega}{2R}$, when θ is zero
(3) zero, when $\theta = \pi/2$
(4) $\dfrac{\pi a^2 B\omega}{2R}$, when $\theta = \pi/2$

15. A highly conducting ring of radius R is perpendicular to and concentric with the axis of a long solenoid as shown in figure. The ring has a narrow gap of width δ in its circumference. The cross-sectional area of the solenoid is a. The solenoid has a uniform internal field of magnitude $B(t) = B_0 + \beta t$, where $\beta > 0$. Assume that no charge can flow across the gap, the face(s) accumulating an excess of positive charge is/are



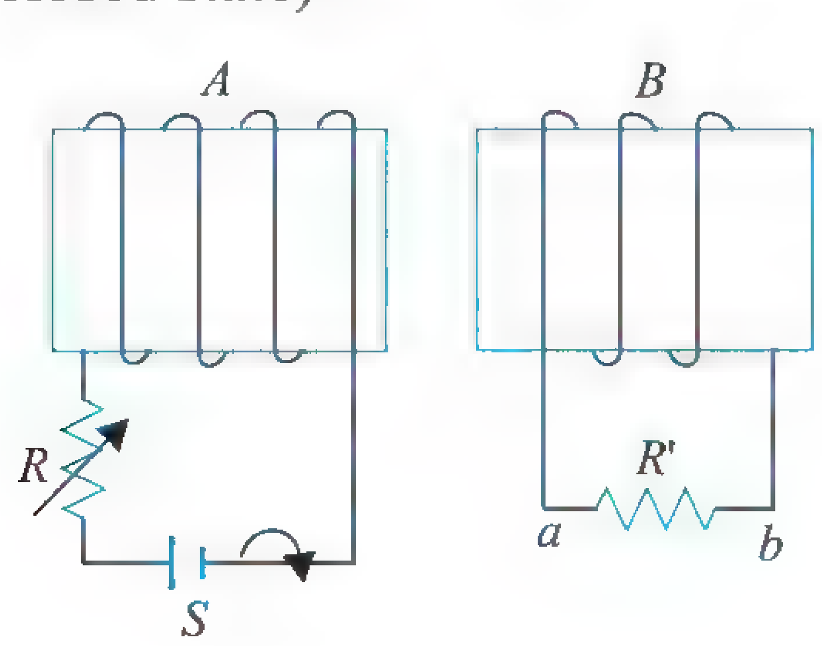
(1) F_1
(2) F_2
(3) F_1 and F_2 both
(4) difficult to conclude as data given are insufficient

16. A plane rectangular loop is placed in a magnetic field. The emf induced in the loop due to this field is ε_i whose maximum value is ε_{im}. The loop was pulled out of the magnetic field at a variable velocity. Assume that $\vec{B}$ is uniform and constant. ε_i is plotted against time t as shown in the graph. Which of the following are/is correct statement(s):



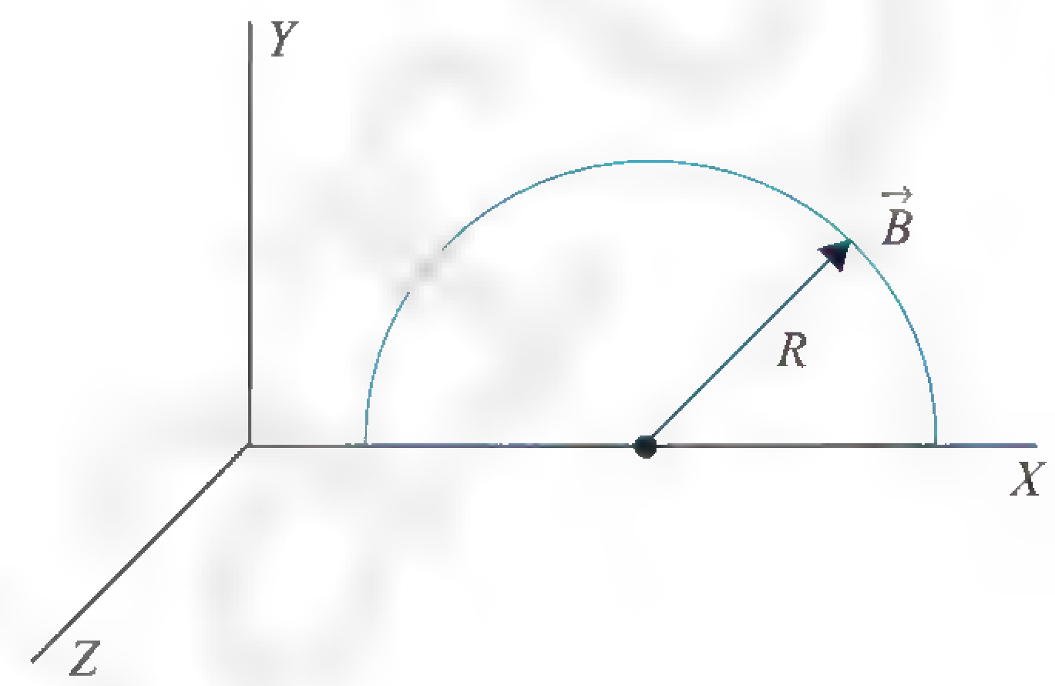
(1) ε_{im} is independent of rate of removal of coil from the field.
(2) The total charge that passes through any point of the loop in the process of complete removal of the loop does not depend on velocity of removal.
(3) The total area under the curve (ε_i vs t) is independent of rate of removal of coil from the field.
(4) The area under the curve is dependent on the rate of removal of the coil.

17. For the given electromagnetically coupled circuits: (S is initially in closed state)



(1) When switch S is opened, current in R' flows from a to b
(2) When switch S is opened, current in R' flows from b to a
(3) When coil B is brought closer to coil A (with S closed) current in R' flows from b to a
(4) When R is decreased (with S closed) then current in R' flows from b to a

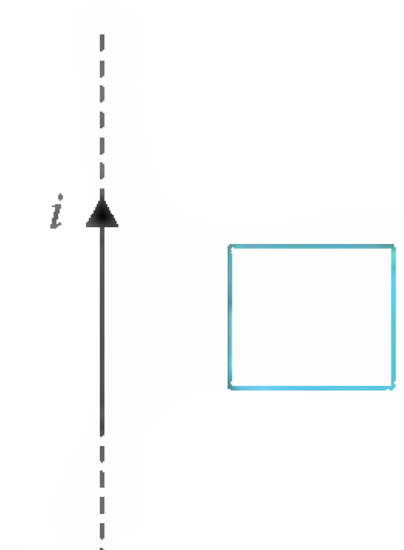
18. A semicircle conducting ring of radius R is placed in the xy plane, as shown in figure. A uniform magnetic field is set up along the x-axis. No emf, will be induced in the ring if



(1) it moves along the x-axis
(2) it moves along the y-axis
(3) it moves along the z-axis
(4) it remains stationary

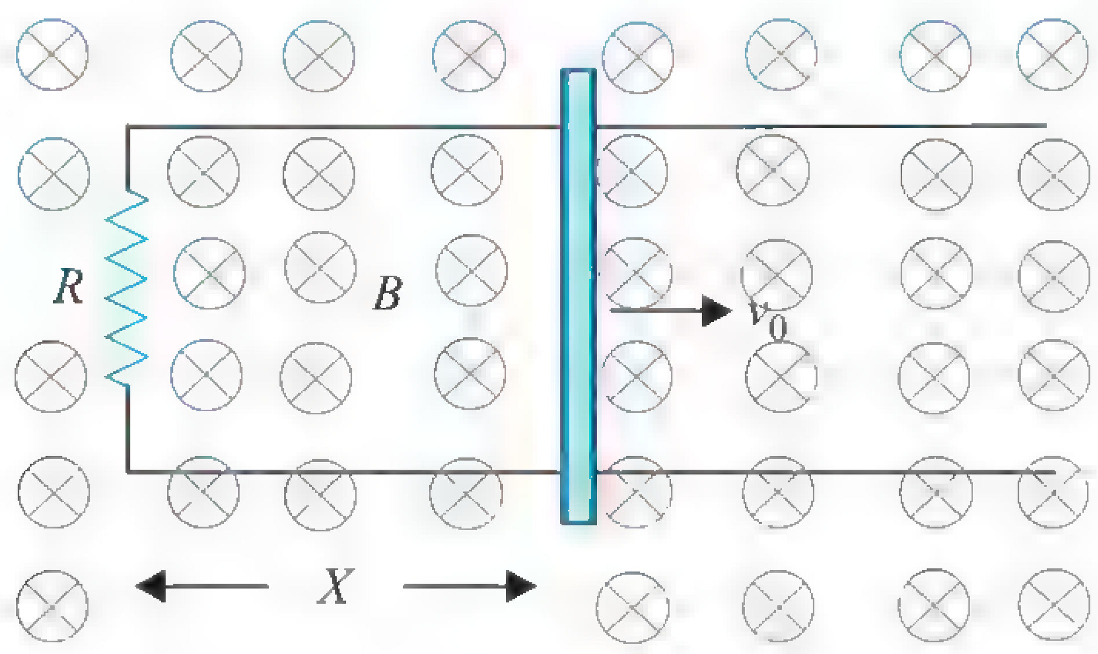
19. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?
(1) The south pole faces the ring and the magnet moves towards it.
(2) The north pole faces the ring and the magnet moves towards it.
(3) The south pole faces the ring and the magnet moves away from it.
(4) The north pole faces the ring and the magnet moves away from it.

20. A square conducting loop is placed in the neighbourhood of a coplanar long straight wire carrying a current i.



(1) If $\dfrac{di}{dt} = 0$, no current is induced in the loop
(2) If $\dfrac{di}{dt} > 0$, current in the loop is clockwise
(3) If $\dfrac{di}{dt} < 0$, current in the loop is anticlockwise
(4) If $\dfrac{di}{dt} > 0$, current in the loop is anticlockwise

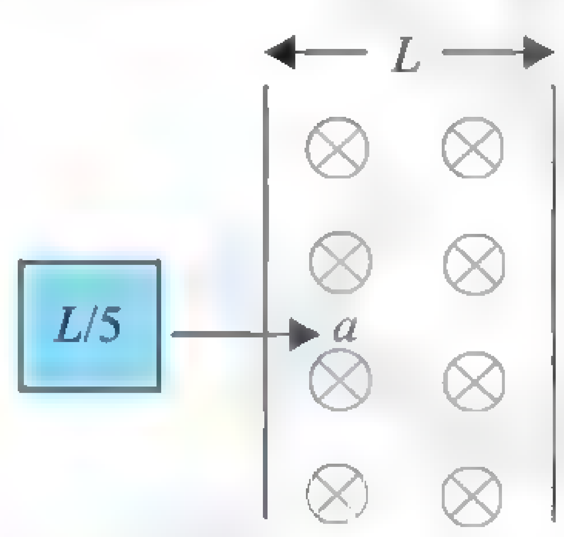
21. A conducting rod of length is moved at constant velocity 'v_0' on two parallel, conducting, smooth, fixed rails, that are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then which of the following is/are correct:



(1) The thermal power dissipated in the resistor is equal to rate of work done by external person pulling the rod.
(2) If applied external force is doubled then a part of external power increases the velocity of rod.
(3) Lenz's Law is not satisfied if the rod is accelerated by external force
(4) If resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

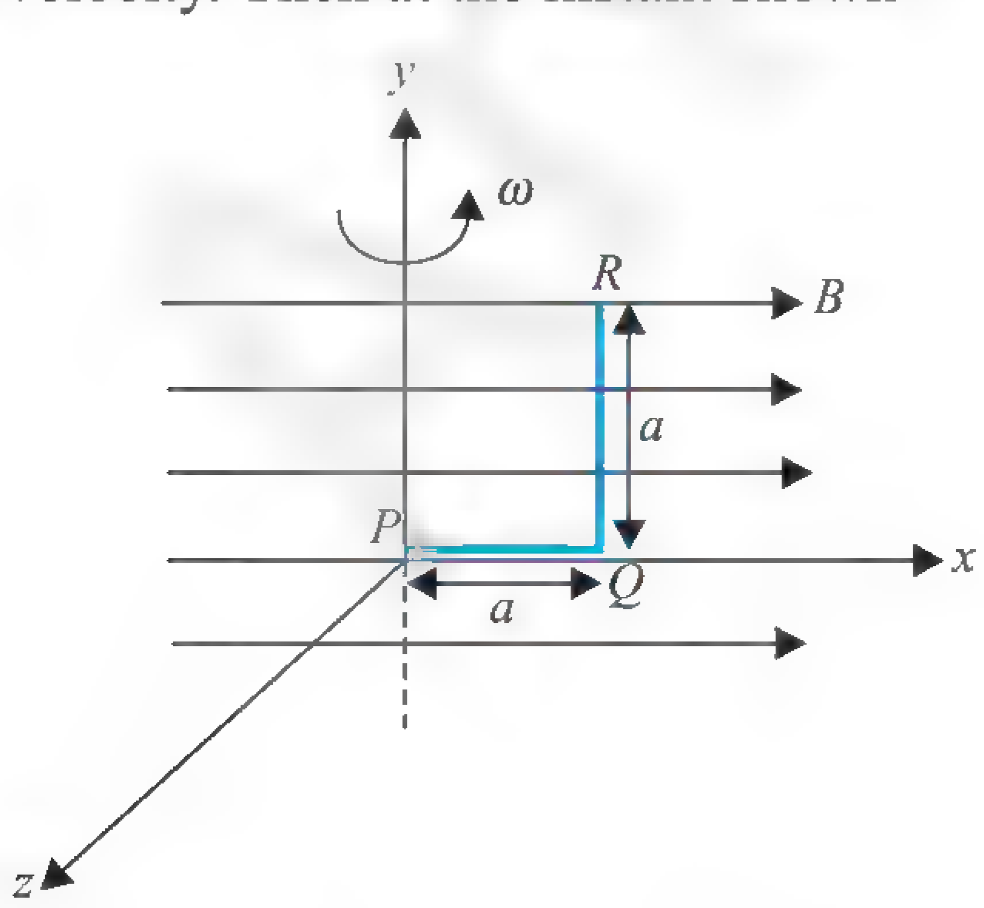
22. A rectangular coil 20 cm × 10 cm having 500 turns rotates in a magnetic field of 5×10^{-3} T with a frequency of 1200 rev min^{-1} about an axis perpendicular to the field.
(1) The maximum value of the induced emf is $2\pi/5$ volt.
(2) The instantaneous emf when the plane of the coil is perpendicular to the field is zero.
(3) The instantaneous emf when the plane of the coil makes an angle of 60° with the field is $\pi/5$ volt .
(4) The instantaneous emf when the plane of the coil makes an angle of 30° with the field is $\pi/10$ volt.

23. Uniform magnetic field $B = 5$ T is acting in the region of length $L = 5$ m as shown in figure. A square loop of side $L/5$ enters in it with constant acceleration $a = 1$ m s^{-2}. Resistance per unit length of the square frame is 1 Ω m^{-1}. At $t = 1$ s



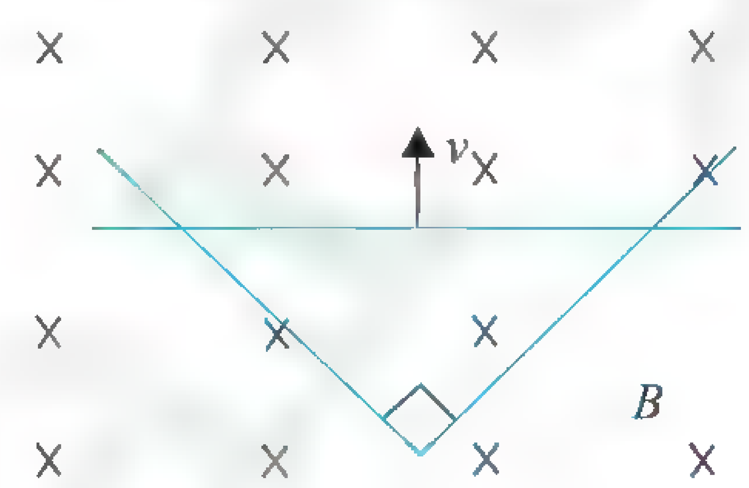
(1) Induced current in the square frame is anti-clockwise.
(2) Induced current in the frame is 1.25 A.
(3) Magnetic force on the frame is 6.25 N.
(4) Magnetic torque on the frame is zero

24. In a region there exists a magnetic field B_0 along positive x-axis. A metallic wire of length $2a$, one side along x-axis and one side parallel of y-axis is rotates about y-axis with a angular velocity. Then at the instant shown



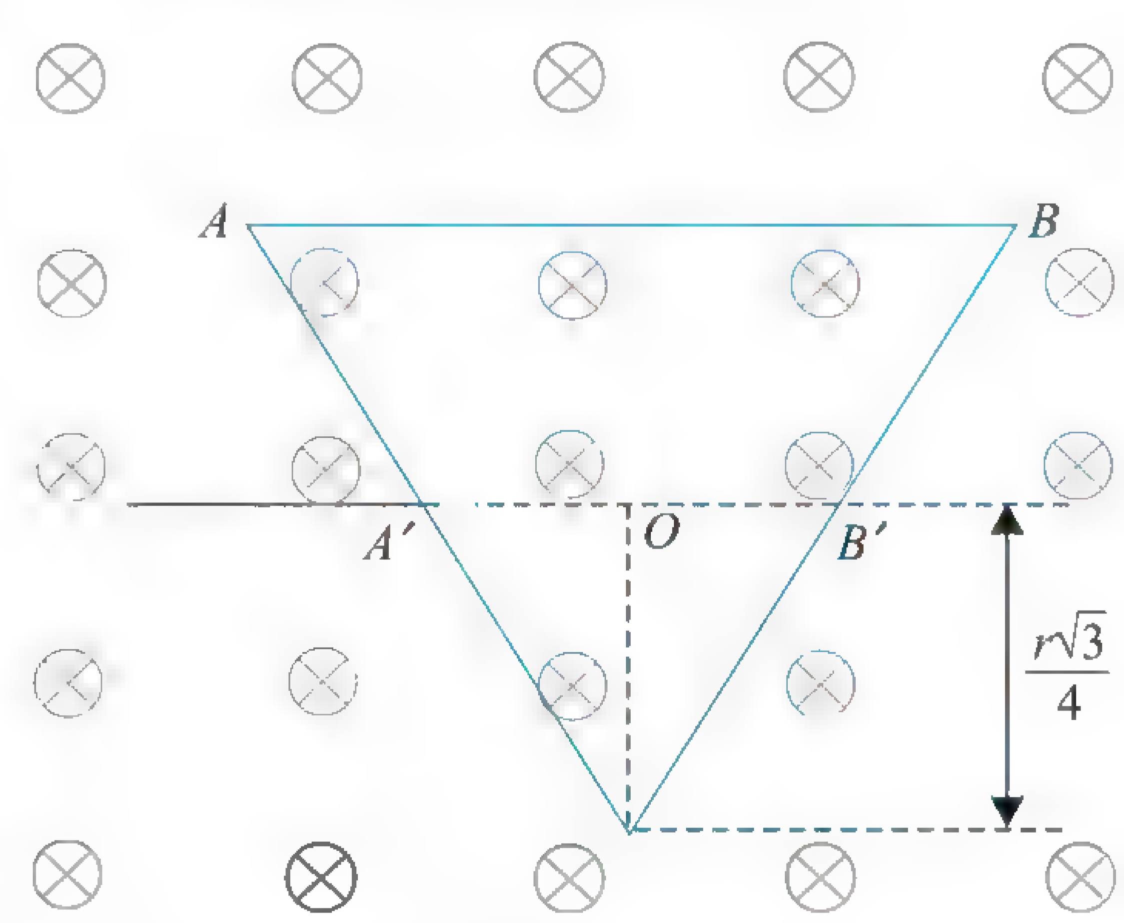
(1) Potential difference across PQ is 0
(2) Potential difference across PQ is $\frac{1}{2}B_0\omega a^2$
(3) Potential difference across QR is $\frac{1}{2}B_0\omega a^2$
(4) Potential difference across QR is $B_0\omega a^2$

25. Two straight conducting rails form a right angle where their ends are joined. A conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with constant velocity v along them as shown in figure. A magnetic field $\vec{B}$ is directed into the page. The induced emf in the circuit at any time t is proportional to



(1) t^2 (2) t
(3) v (4) v^2

26. ABC is an equilateral triangular frame of mass m and side r. It is at rest under the action of horizontal magnetic field B (as shown) and the gravitational field.



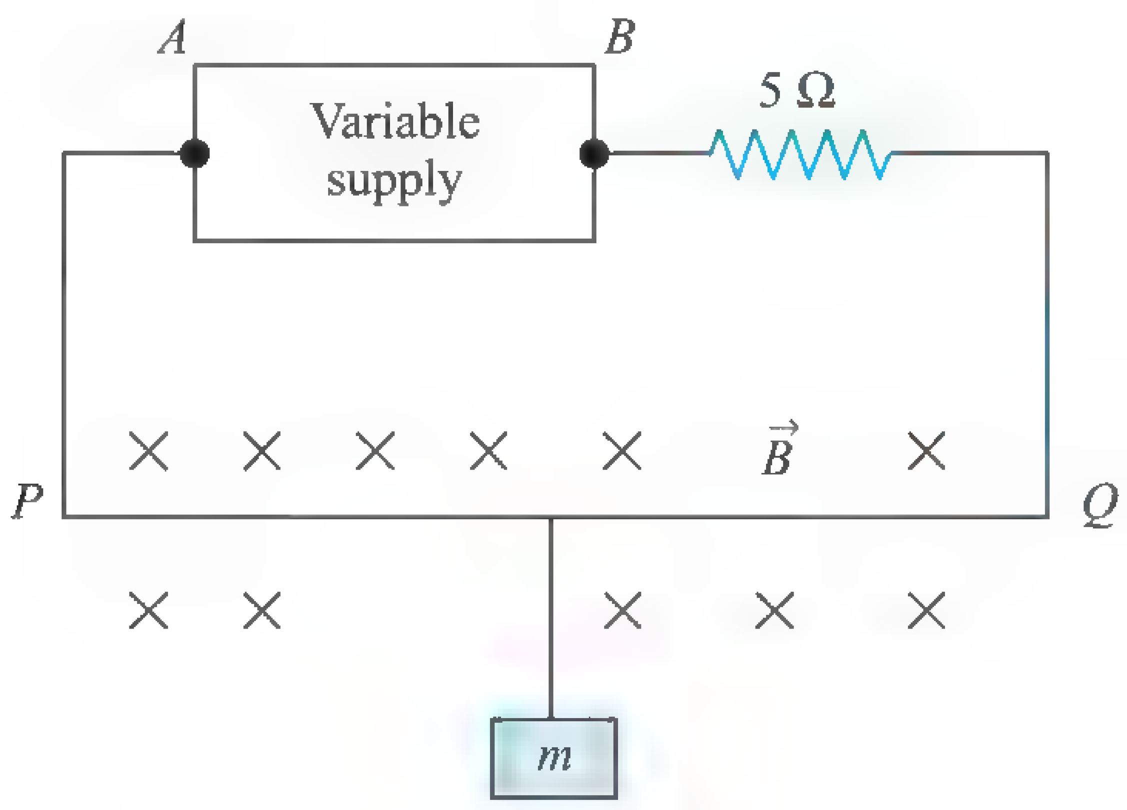
(1) The frame remains at rest if the current in the frame is $\frac{2mg}{rB}$.
(2) The frame remains at rest if the current in the frame is $\frac{2mg}{rB\sqrt{3}}$.
(3) The frame is in simple harmonic motion when frame is slightly displaced in its plane perpendicular to AB. The period of oscillation is $\pi\left[\frac{r\sqrt{3}}{g}\right]^{1/2}$.
(4) For same as in above option, the period of oscillation is $\pi\left[\frac{3r}{2g}\right]^{1/2}$.

Linked Comprehension Type

For Problems 1–2

A brillant student of physics developed a magnetic balance to weigh objects. The mass m to be measured is hung from the

center of the bar. Bar is kept in a uniform magnetic field of 1.5 T directed into the plane of the figure. Battery voltage can be adjusted to vary the current in the circuit. The horizontal bar shown is 60 cm long and is made of extremely light weight material. It is connected to the battery via a resistance. There is no tension in the supporting wires. The magnetic force only supports the hanging weight.

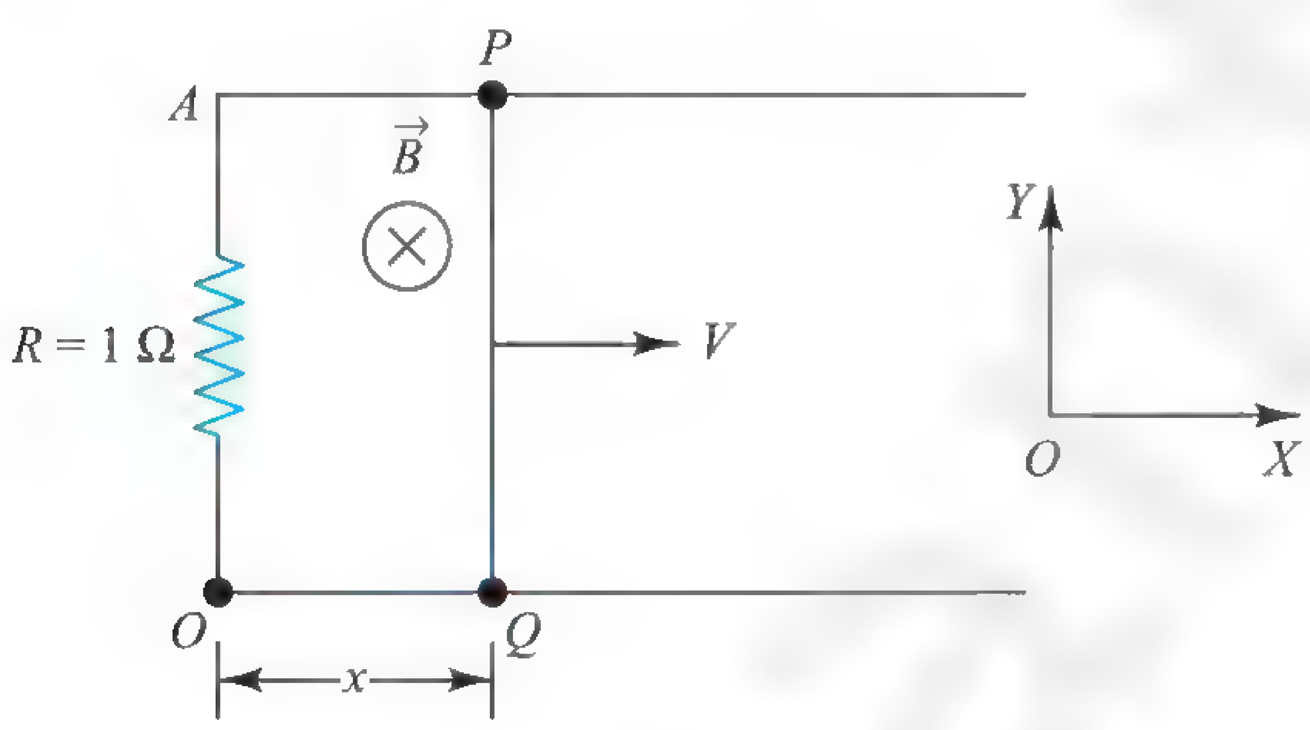


1. Which point of battery terminal is positive?
(1) A (2) B
(3) either A or B (4) cannot be found

2. If $V = 150$ V, what is the maximum mass m?
(1) 1.3 kg (2) 1.8 kg
(3) 2.2 kg (4) 2.7 kg

For Problems 3–5

Consider two parallel, conducting frictionless tracks kept in a gravity-free space as shown in figure. A movable conductor PQ, initially kept at OA, is given a velocity 10 m s^{-1} toward right. The space contains a magnetic field which depends upon the distance moved by conductor PQ from the OA line and given by



$$\vec{B} = cx(-\hat{k}) \;\; [c = \text{constant} = 1 \text{ SI unit}]$$

The mass of the conductor PQ is 1 kg and length of PQ is 1 m. Answer the following questions based on the above passage.

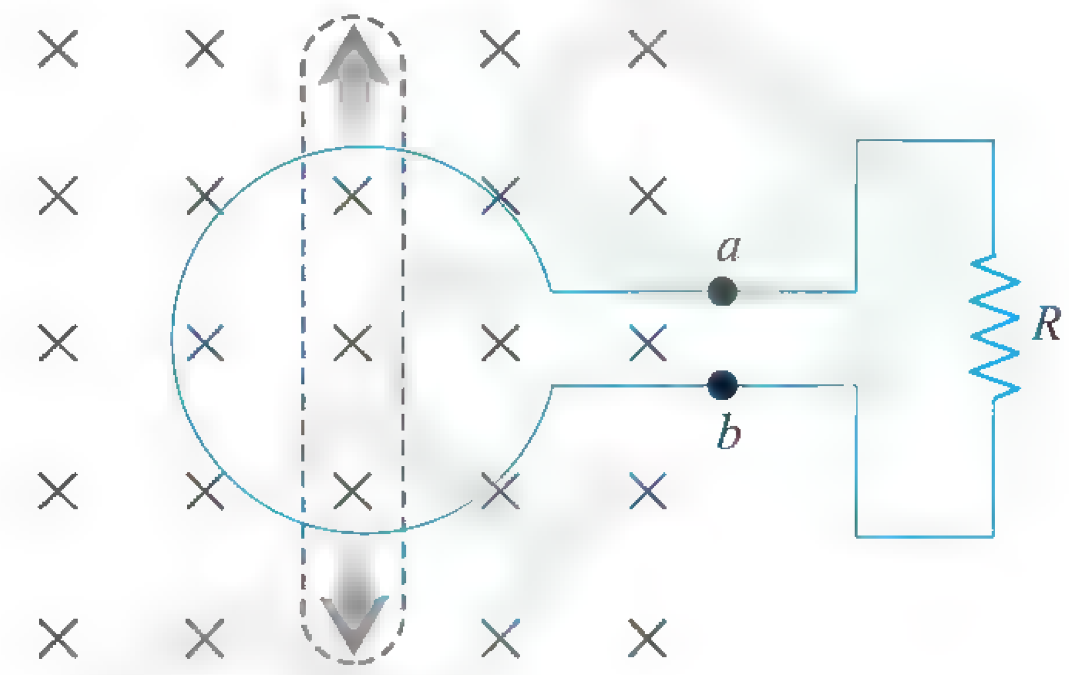
3. The distance travelled by the conductor when its speed is 5 m s^{-1} is
(1) $\left(\frac{15}{2}\right)^{1/3}$ (2) $\left(\frac{10}{3}\right)^{1/3}$
(3) $(10)^{1/3}$ (4) none of the above

4. The heat loss during the time interval $t = 0$ to time t seconds, when the speed of the conductor is 5 m s^{-1} is
(1) 50 J (2) 30 J
(3) 10 J (4) none of the above

5. The work done by magnetic force acting on the conductor PQ during its motion in the time interval $t = 0$ to $t = t$ seconds when the speed of conductor is 5 m s^{-1} is
(1) zero (2) 50 J
(3) 10 J (4) 30 J

For Problems 6–7

A flexible circular loop 20 cm in diameter lies in a magnetic field with magnitude 1.0 T, directed into the plane of the page as shown in figure. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.314 s.

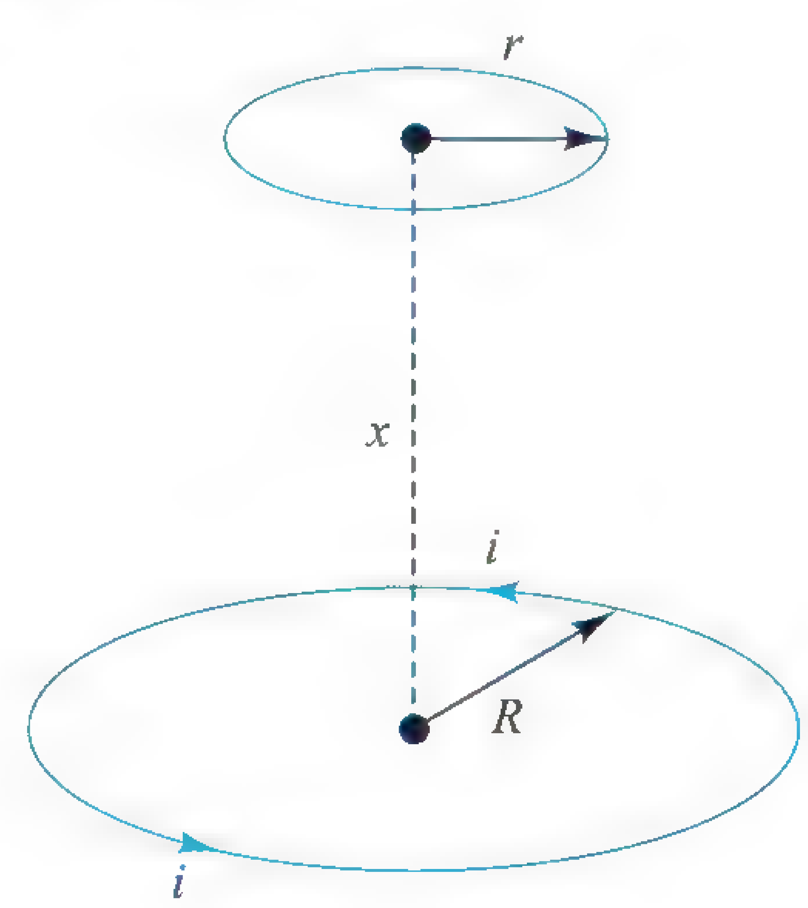


6. The average induced emf in the circuit is
(1) 0.2 V (2) 0.1 V
(3) 1 V (4) 10 V

7. If $R = 0.01\ \Omega$, the magnitude and direction of current flowing in the loop are
(1) 1 A, clockwise (2) 1 A, anticlockwise
(3) 10 A, clockwise (4) 10 A, anticlockwise

For Problems 8–9

Figure shows two parallel and coaxial loops. The smaller loop (radius r) is above the larger loop (radius R), by distance $x \gg R$. The magnetic field due to current i in the larger loop is nearly constant throughout the smaller loop. Suppose that x is increasing at a constant rate of $dx/dt = v$.

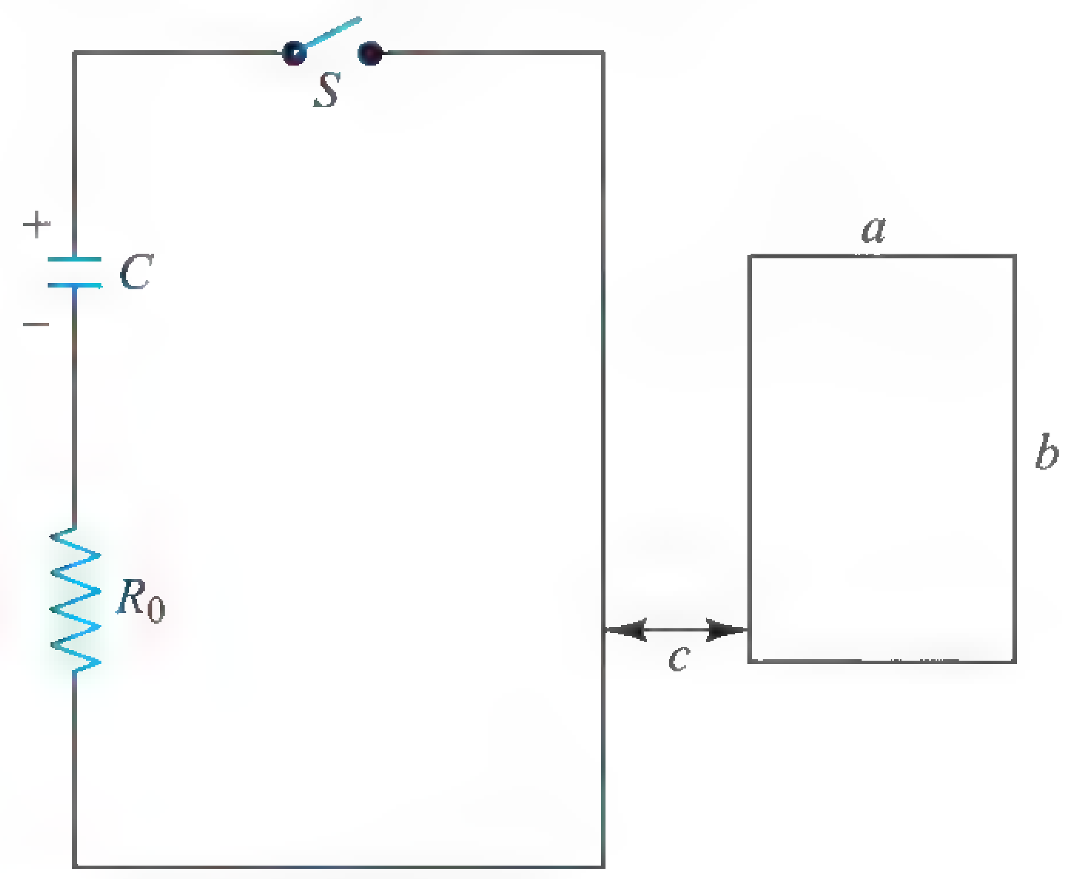


8. Determine the magnetic flux through the smaller loop as a function of x.
(1) $\dfrac{\mu_0 i R^2 \pi r^2}{x^3}$ (2) $\dfrac{\mu_0 i R^2 \pi r^2}{2x^3}$
(3) $\dfrac{2\mu_0 i R^2 \pi r^2}{x^3}$ (4) $\dfrac{\sqrt{2}\mu_0 i R^2 \pi r^2}{x^3}$

9. The induced emf in the smaller loop is
(1) $\dfrac{\mu_0 \pi i R^2 r^2}{x^4} v$ (2) $\dfrac{\mu_0 \pi i R^2 r^2}{2x^4} v$
(3) $\dfrac{3}{2} \dfrac{\mu_0 \pi i R^2 r^2}{x^4} v$ (4) $\dfrac{\mu_0 \pi i R^2 r^2}{3x^4} v$

For Problems 10–12

In the circuit shown in figure, the capacitor has capacitance $C = 20$ μF and is initially charged to 100 V with the polarity shown. The resistor R_0 has resistance 10 Ω. At time $t = 0$, the switch is closed. The smaller circuit is not connected in any way to the larger one. The wire of the smaller circuit has a resistance of 1.0 Ω m^{-1} and contains 25 loops. The larger circuit is a rectangle 2.0 m by 4.0 m, while the smaller one has dimensions $a = 10.0$ cm and $b = 20.0$ cm. The distance c is 5.0 cm. (The figure is not drawn to scale.) Both circuits are held stationary. Assume that only the wire nearest to the smaller circuit produces an appreciable magnetic field through it.



10. The current in the larger circuit 200 ms after closing S is

(1) $\frac{5}{e}$ A (2) $\frac{2}{e}$ A

(3) $\frac{15}{e}$ A (4) $\frac{10}{e}$ A

11. The current in the smaller circuit 200 μs after closing S is

(1) 54 μA (2) 10 μA

(3) 15 μA (4) 36 μA

12. The direction of current in the smaller circuit is

(1) clockwise

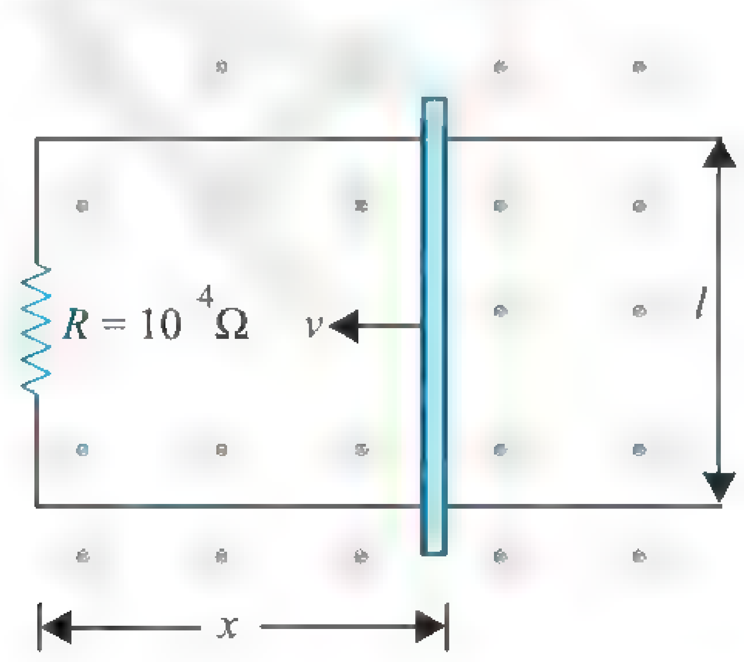
(2) anticlockwise

(3) always changes with time

(4) cannot be calculated

For Problems 13–14

A metal bar is moving with a velocity of $v = 5$ cms^{-1} over a U-shaped conductor. At $t = 0$, the external magnetic field is 0.1 T directed out of the page and is increasing at a rate of 0.2 T s^{-1}. Take $l = 5$ cm, and at $t = 0$, $x = 5$ cm.



13. The emf induced in the circuit is

(1) 125 μV (2) 250 μV

(3) 100 μV (4) 300 μV

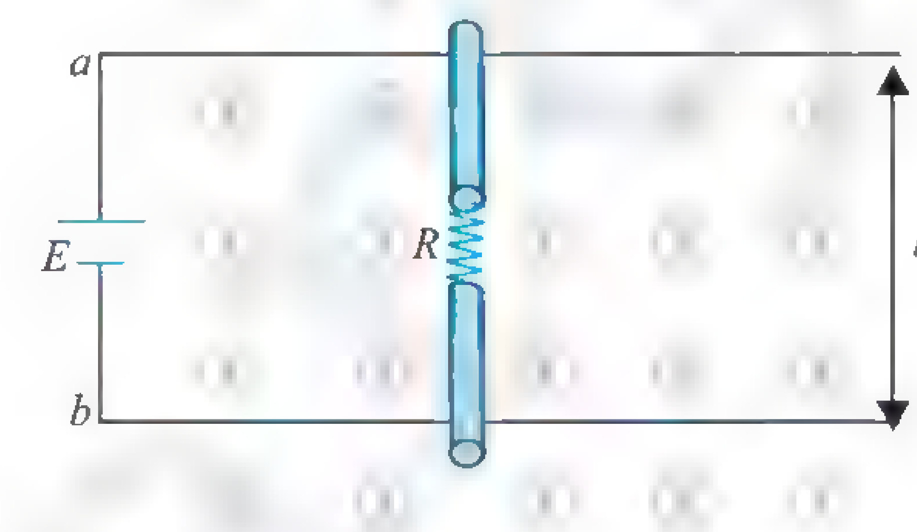
14. The current flowing in the circuit is

(1) 2.5 A (2) 5 A

(3) 1 A (4) 2 A

For Problems 15–17

In figure shown, the rod has a resistance R, the horizontal rails have negligible friction. A battery of emf E and negligible internal resistance is connected between points a and b. The rod is released from rest.



15. The velocity of the rod as function of time is

(1) $\frac{E}{Bl}(1 - e^{-t/\tau})$ (2) $\frac{E}{Bl}(1 + e^{-t/\tau})$

(3) $\frac{3}{2}\frac{E}{Bl}(1 - e^{-t/\tau})$ (4) $\frac{E}{2Bl}(1 - e^{-t/\tau})$

16. After some time, the rod will approach a terminal speed. Find an expression for it.

(1) $\frac{3}{2}\frac{E}{Bl}$ (2) $\frac{E}{2Bl}$

(3) $\frac{E}{Bl}$ (4) $\frac{2E}{Bl}$

17. The current when the rod attains its terminal speed is

(1) $\frac{2E}{R}$ (2) zero

(3) $\frac{3}{2}\frac{E}{R}$ (4) $\frac{E}{2R}$

For Problems 18–20

A fan operates at 200 V (dc) consuming 1000 W when running at full speed. Its internal wiring has resistance 1 Ω. When the fan runs at full speed, its speed becomes constant. This is because the torque due to magnetic field inside the fan is balanced by the torque due to air resistance on the blades of the fan and torque due to friction between the fixed part and the shaft of the fan. The electrical power going into the fan is spent (i) in the internal resistance as heat, call it P_1, (ii) in doing work against internal friction and air resistance producing heat, sound, etc., call it P_2. When the coil of fan rotates, an emf is also induced in the coil. This opposes the external emf applied to send the current into the fan. This emf is called back emf, call it e. Answer the following questions when the fan is running at full speed.

18. The current flowing into the fan and the value of back emf e is

(1) 200 A, 5 V (2) 5 A, 200 V

(3) 5 A, 195 V (4) 1 A, 0 V

19. The value of power P_1 is

(1) 1000 W (2) 975 W

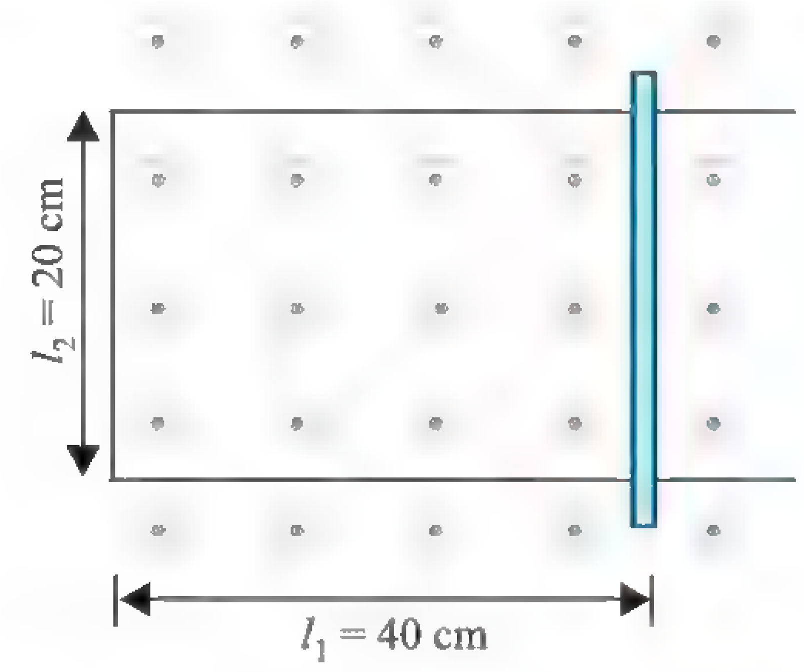
(3) 25 W (4) 200 W

20. The value of power P_2 is

(1) 10000 W (2) 975 W
(3) 25 W (4) 200 W

For Problems 21–23

Figure shows a conducting rod of negligible resistance that can slide on a smooth U-shaped rail made of wire of resistance 1 $\Omega\, m^{-1}$. Position of the conducting rod at $t = 0$ is shown. A time-dependent magnetic field $B = 2T$ (tesla) is switched on at $t = 0$.



21. The current in the loop at $t = 0$ due to induced emf is

(1) 0.16 A, clockwise (2) 0.08 A, clockwise
(3) 0.08 A, anticlockwise (4) zero

22. At $t = 0$, when the magnetic field is switched on, the conducting rod is moved to the left at a constant speed of 5 cm s^{-1} by some external means. The rod moves perpendicular to the rail. At $t = 2$ s, induced emf has magnitude

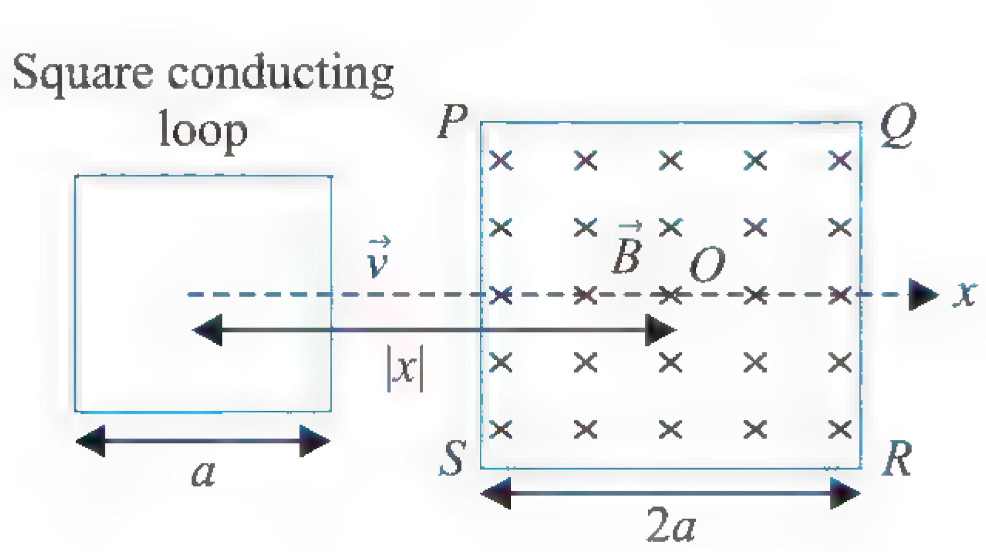
(1) 0.12 V (2) 0.08 V
(3) 0.04 V (4) 0.02 V

23. Following situation of the previous question, the magnitude of the force required to move the conducting rod at a constant speed of 5 cm s^{-1} at the same instant $t = 2$ s is equal to

(1) 0.16 N (2) 0.12 N
(3) 0.08 N (4) 0.06 N

For Problems 24–26

PQRS is a square region of side $2a$ in the plane of paper. A uniform magnetic field B, directed perpendicular to the plane of paper and into its plane is confined within this square region. A square loop of side 'a' and made of a conducting wire of resistance R is moved at a constant velocity $\vec{v}$ from left to right in the plane of paper as shown. Obviously, the square loop will enter the magnetic field at some time and then leave it after some time. During the motion of loop, whenever magnetic flux through it changes, emf will be induced resulting in induced current. Let the motion of the square loop be along x-axis and let us measure x coordinate of the centre of square loop from the centre of the square magnetic field region (taken as origin). Thus, x coordinate will be positive if the centre of square loop is to the right of the origin O (centre of magnetic field) and negative if centre is to the left.



24. For $x = -9a/5$, magnitude of induced current and its direction as seen from above will be:

(1) Bav, clockwise (2) Bav/R, clockwise
(3) zero (4) Bav/R, anticlockwise

25. External force required to maintain constant velocity of the loop for $x = -\frac{9}{5}a$ will be

(1) $B^2a^2v^2$ to the right (2) $\frac{B^2a^2v^2}{R}$ to the right

(3) $\frac{B^2a^2v^2}{R}$ to the left (4) zero

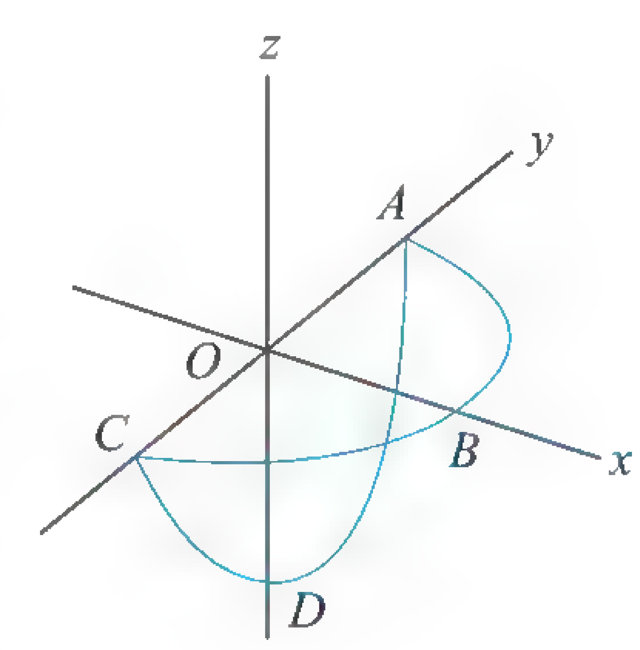
26. For $x = -a/4$,

(i) magnetic flux through the loop,
(ii) induced current in the loop and
(iii) external force required to maintain constant velocity of the loop, will be

(1) (i) Ba^2 (ii) $\frac{Bav}{2R}$ (iii) $\frac{B^2a^2v^2}{4R^2}$
(2) (i) Ba^2 (ii) zero (iii) zero
(3) (i) Ba^2 (ii) $\frac{Bav}{2R}$ (iii) zero
(4) (i) zero (ii) zero (iii) zero

For Problems 27–29

ABCDA is a closed loop of conducting wire consisting of two semicircular sections, the part *ABC* lying in the *XY* plane and the part *CDA* lying in the *YZ* plane, both the parts having the centre at origin *O* (see the diagram). This loop is placed in a uniform field $\vec{B}$ which varies with time. Radius of the semicircular section is 0.5 m.



27. If $\vec{B}$ be directed along $(+\vec{i} - \vec{k})$ and decreases at the rate of 10^{-2} Tesla/second, the magnitude and the sense of the induced emf in the loop, as seen along the magnetic field direction will be

(1) zero

(2) $\frac{5\pi}{4\sqrt{2}}$ mV in the counterclockwise sense

(3) $\frac{5\pi}{4\sqrt{2}}$ mV in the clockwise sense

(4) $\frac{5\pi}{2\sqrt{2}}$ mV in the clockwise sense

28. If $\vec{B}$ be directed along the vector $(\vec{i} - \vec{j})$ and decreases at the rate of 10^{-2} Tesla/second. The magnitude and sense of the induced emf in the loop as seen along the positive x-axis will be

(1) zero

(2) $\frac{5\pi}{4\sqrt{2}}$ mV in the counterclockwise sense

(3) $\frac{5\pi}{4\sqrt{2}}$ mV in the clockwise sense

(4) $\frac{5\pi}{2\sqrt{2}}$ mV in the clockwise sense

29. If $\vec{B}$ be directed along the $+z$ axis and increases at the rate of 10^{-2} Tesla/second, the magnitude and sense of the induced emf in the loop as seen along the field direction will be

(1) zero

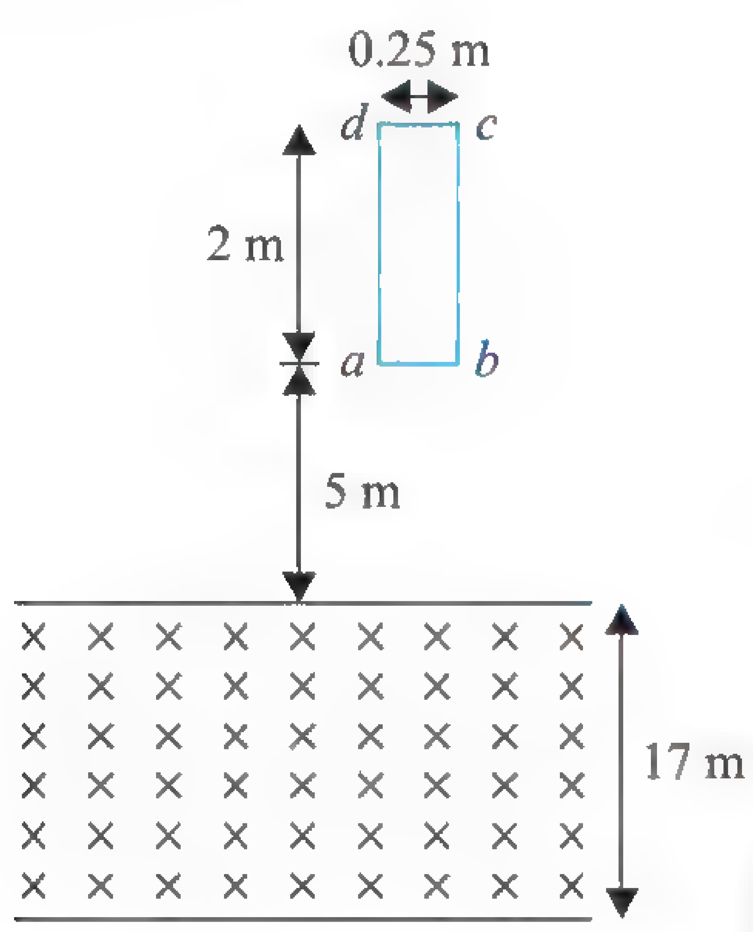
(2) $\frac{5\pi}{4}$ mV in the counterclockwise sense

(3) $\frac{5\pi}{4\sqrt{2}}$ mV in the clockwise sense

(4) $\frac{5\pi}{2\sqrt{2}}$ mV in the counter clockwise sense

For Problems 30–31

A rectangular wire frame of dimensions (0.25 m × 2.0 m) and mass 0.5 kg falls from a height 5m above a region occupied by uniform magnetic field of magnetic induction 1 T. The resistance of the wire frame is 1/8 Ω. Find



30. time taken to completely enter into the field is

(1) 0.2 s (2) 1 s

(3) 2.2 s (4) $\sqrt{\frac{1}{5}}$ s

31. time taken by the wire frame when it just starts coming out of the magnetic field.

(1) 0.2 s (2) 1 s

(3) 2.2 s (4) $\sqrt{\frac{1}{5}}$ s

For Problems 32–33

A stationary circular loop of radius a is located in a magnetic field which varies with time from $t = 0$ to $t = T$ according to law $B = B_0 t(T - t)$. If plane of loop is normal to the direction of field and resistance of the loop is R, calculate

32. amount of heat generated in the loop during this inter-val.

(1) $\frac{\pi^2 a^4 B_0^2 T^3}{3R}$ (2) $\frac{\pi^2 a^4 B_0^2 T^3}{R}$

(3) $\frac{3\pi^2 a^4 B_0^2 T^3}{R}$ (4) None of these

33. magnitude of charge flown through the loop from instant $t = 0$ to the instant when current reverses its direction.

(1) $\frac{\pi a^2 B_0 T^2}{R}$ (2) $\frac{\pi a^2 B_0 T^2}{4R}$

(3) $\frac{4\pi a^2 B_0 T^2}{R}$ (4) None of these

For Problems 34–35

A circular ring of radius a is made from a wire having resistance λ per unit length. The ring is mounted on a car such that ring remains vertical. The car moves along a horizontal circle of radius R and completes n revolutions per minute. The horizontal component of earth's magnetic field be H,

34. The emf induced in the ring is

(1) $\frac{\pi^2 a^2 nH}{10}\cos\left(\frac{\pi nt}{30}\right)$ (2) $\frac{\pi^2 a^2 nH}{30}\cos\left(\frac{\pi nt}{30}\right)$

(3) $\frac{\pi^2 a^2 nH}{30}\sin\left(\frac{\pi nt}{30}\right)$ (4) None of these

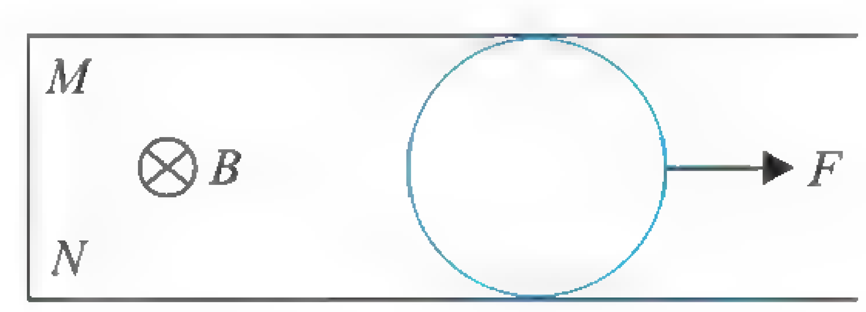
35. Calculate average rate at which heat is produced in the ring.

(1) $\frac{\pi^3 a^3 H^2 n^2}{1800\lambda}$ J s^{-1} (2) $\frac{\pi^3 a^3 H^2 n^2}{3600\lambda}$ J s^{-1}

(3) $\frac{\pi^3 a^3 H^2 n^2}{1200\lambda}$ J s^{-1} (4) None of these

For Problems 36–37

Two long parallel conducting horizontal rails are connected by a conducting wire at one end. A uniform magnetic field B (directed vertically downwards) exists in the region of space.



A light uniform ring of dia-meter d which is practically equal to separation between the rails is placed over the rails as shown in figure. If resistance of ring be λ per unit length,

36. The current in the wire MN is

(1) $\frac{2Bv}{\pi\lambda}$ from M to N (2) $\frac{Bv}{\pi\lambda}$ from M to N

(3) $\frac{2Bv}{\pi\lambda}$ from N to M (4) None of these

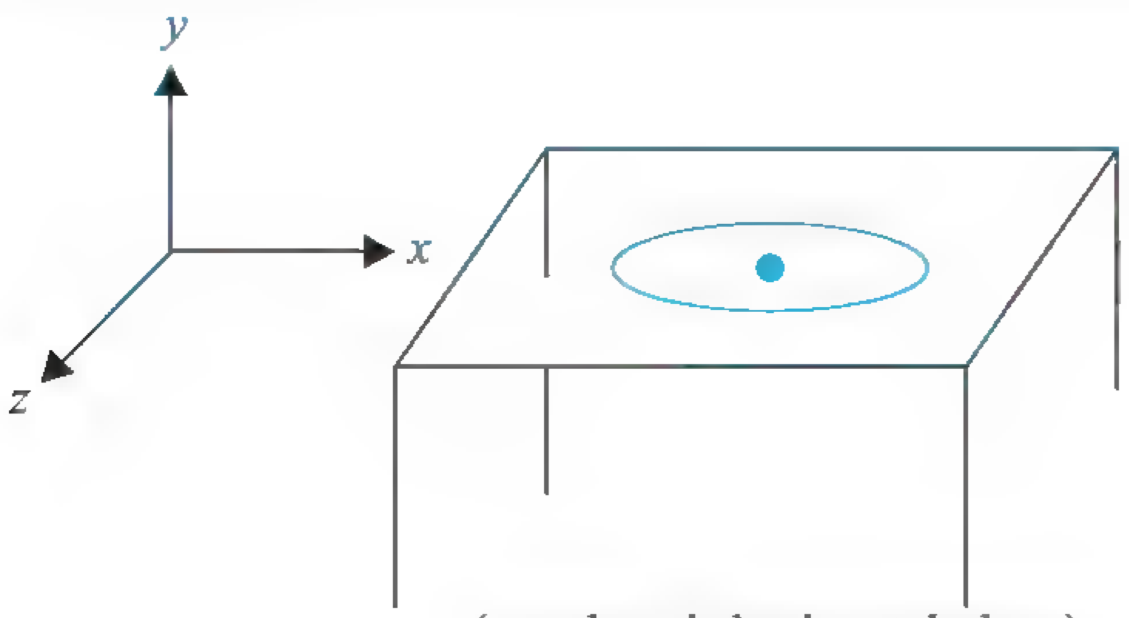
37. The force required to pull the ring with uniform velocity v is

(1) $\frac{4B^2 vd}{3\pi\lambda}$ (2) $\frac{3B^2 vd}{4\pi\lambda}$

(3) $\frac{2B^2 vd}{3\pi\lambda}$ (4) $\frac{4B^2 vd}{\pi\lambda}$

For Problems 38–41

A uniform conducting ring of mass π kg and radius 1 m is kept on smooth horizontal table. A uniform but time varying magnetic field $B = (\hat{i} + t^2\hat{j})$ T is present in the region, where t is time in seconds. Resistance of ring is 2 Ω. Then,



(x-z plane is horizontal plane)

38. Net magnetic field (in Newton) on conducting ring as function of time is

(1) $2\pi^2 t$ (2) $2\pi^2 t^2$

(3) $2\pi^2 t^3$ (4) zero

39. Time (in second) at which ring start toppling is

(1) $\frac{10}{\pi}$ (2) $\frac{20}{\pi}$

(3) $\frac{5}{\pi}$ (4) $\frac{25}{\pi}$

40. Heat generated (in kJ) through the ring till the instant when ring start toppling is

(1) $\frac{1}{3\pi}$ (2) $\frac{2}{\pi}$

(3) $\frac{2}{3\pi}$ (4) $\frac{1}{\pi}$

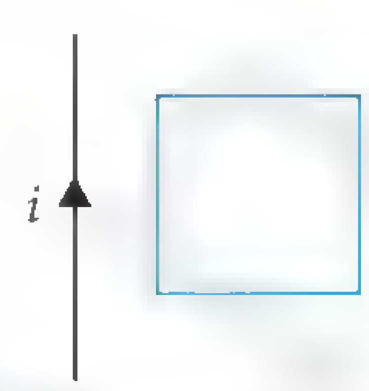
41. Induced electric field (in volt/metre) at the circumference of ring at the instant ring start toppling is

(1) $\frac{10}{\pi}$ (2) $\frac{20}{\pi}$

(3) $\frac{5}{\pi}$ (4) $\frac{25}{\pi}$

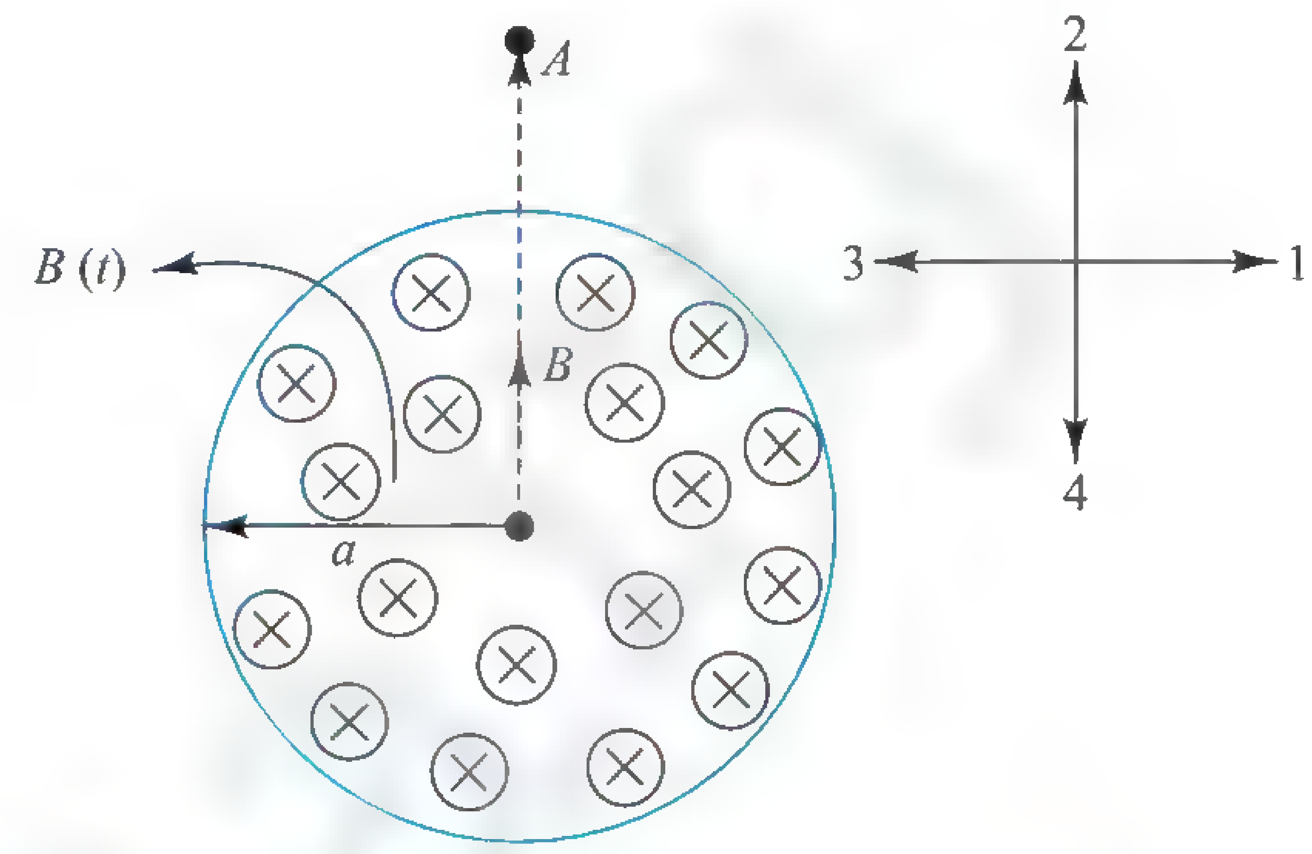
Matrix Match Type

1. A square loop is placed near a long straight current carrying wire as shown in figure. Match the following table.



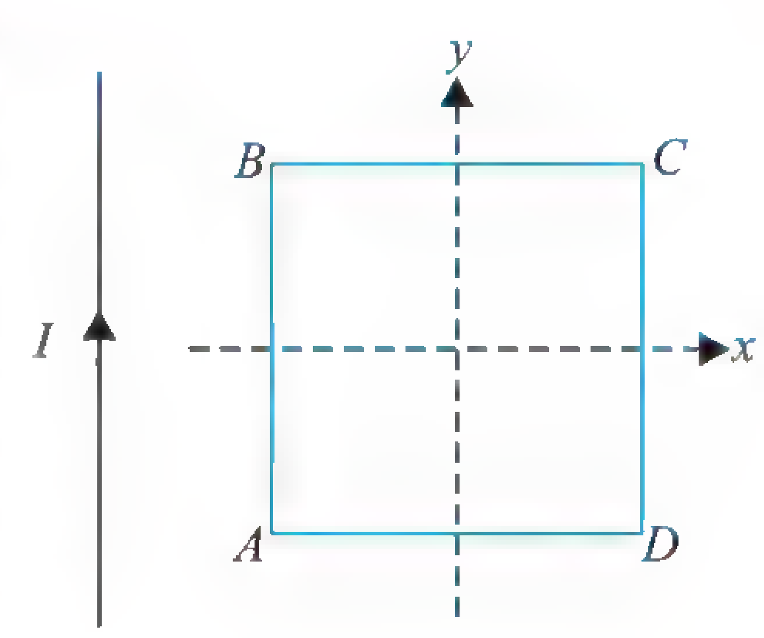
	Column I		Column II
i.	If current is increased,	**a.**	induced current in loop is clockwise
ii.	If current is decreased,	**b.**	induced current in loop is anticlockwise
iii.	If loop is moved away from the wire,	**c.**	wire will attract the loop
iv.	If loop is moved towards the wire,	**d.**	wire will repel the loop

2. A uniform but time varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper, as shown in figure. The magnetic field decreases at a constant rate inside the region. If r is the distance from the axis of the cylindrical region, then match column I with column II.



	Column I		Column II
i.	Induced electric field at point A	**a.**	Directed along 3
ii.	Induced electric field at point B	**b.**	Directed along 1
iii.	Force on an electron placed at point A	**c.**	Increases as r
iv.	Force on an electron placed at point B	**d.**	Decreases as $1/r$

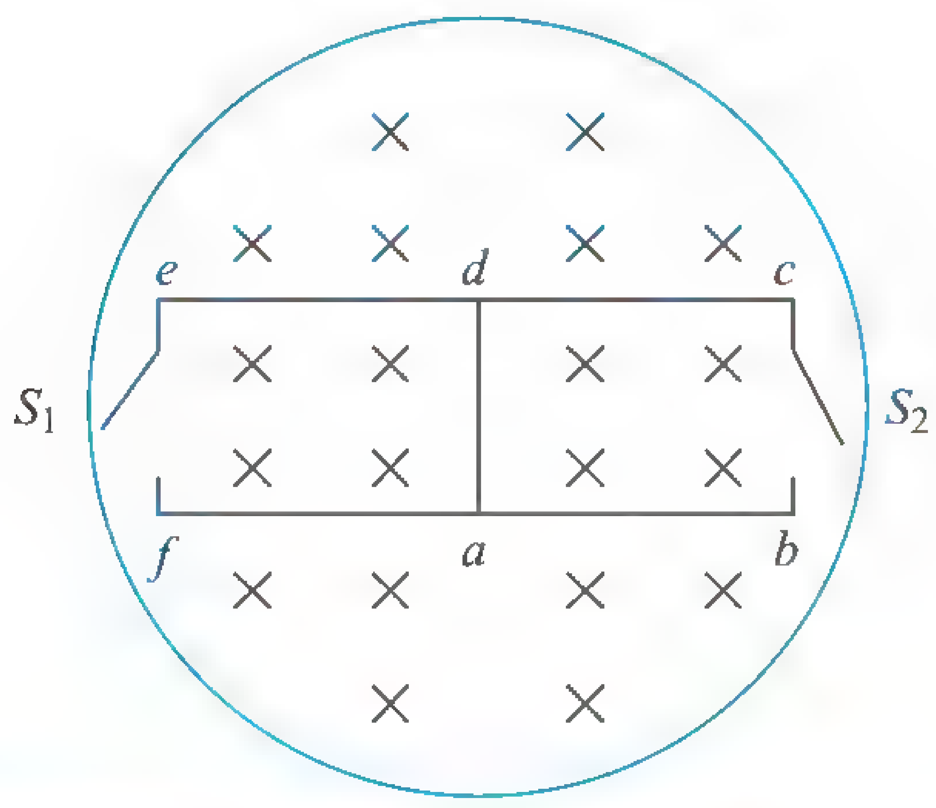
3. A long current carrying wire and a loop made of conducting wire are placed in x–y plane, such that the long wire is parallel to y-axis. Column I is regarding some changes made in the position of loop and Column II indicates the resulting effects.



Match the columns.

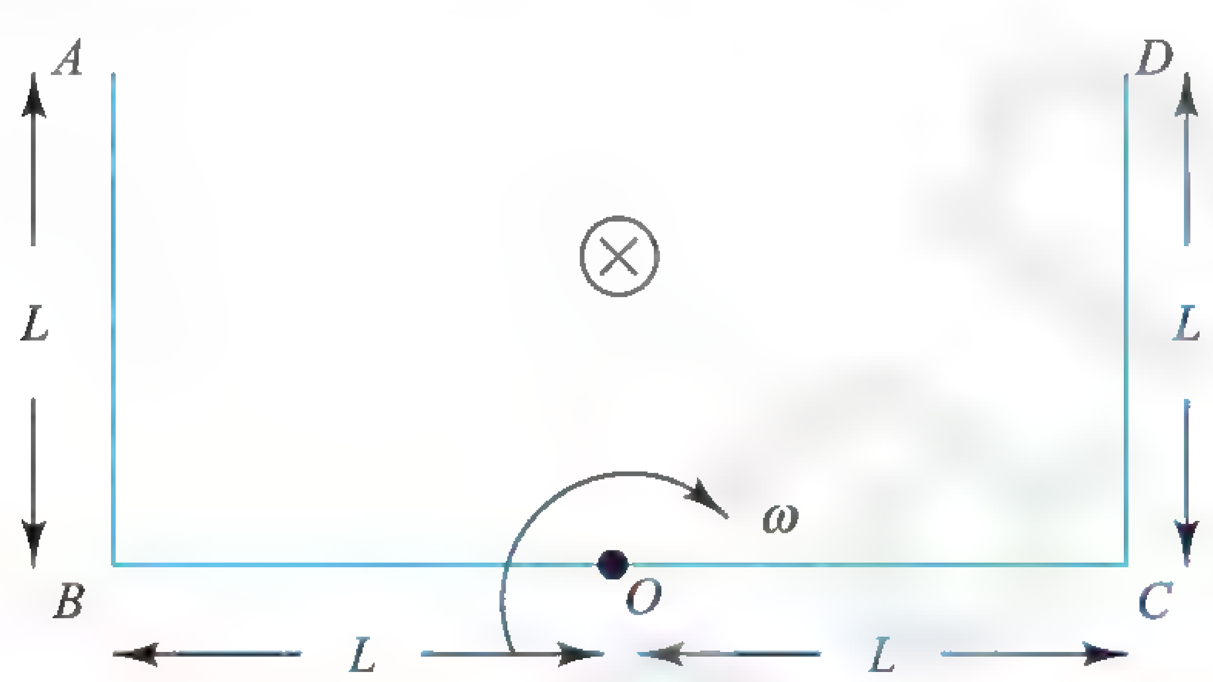
	Column I		Column II
i.	If loop is moved away from the wire while keeping in x-y plane,	**a.**	current is induced in the loop in anticlockwise direction.
ii.	If loop is moved toward the wire while keeping in x-y plane	**b.**	current is induced in the loop in clockwise direction.
iii.	If loop is rotated about x-axis, then just after this	**c.**	no emf is induced in the loop.
iv.	If loop is rotated about y-axis, then just after this	**d.**	the wire will attract or repel the loop.

4. The magnetic field in the cylindrical region shown in figure increases at a constant rate of 10.0 mT s^{-1}. Each side of the square loop *abcd* and *defa* has a length of 2.00 cm and a resistance of 2.00 Ω. Correctly match the current in the wire *ad* in four different situations as listed in column I with the values given in column II.



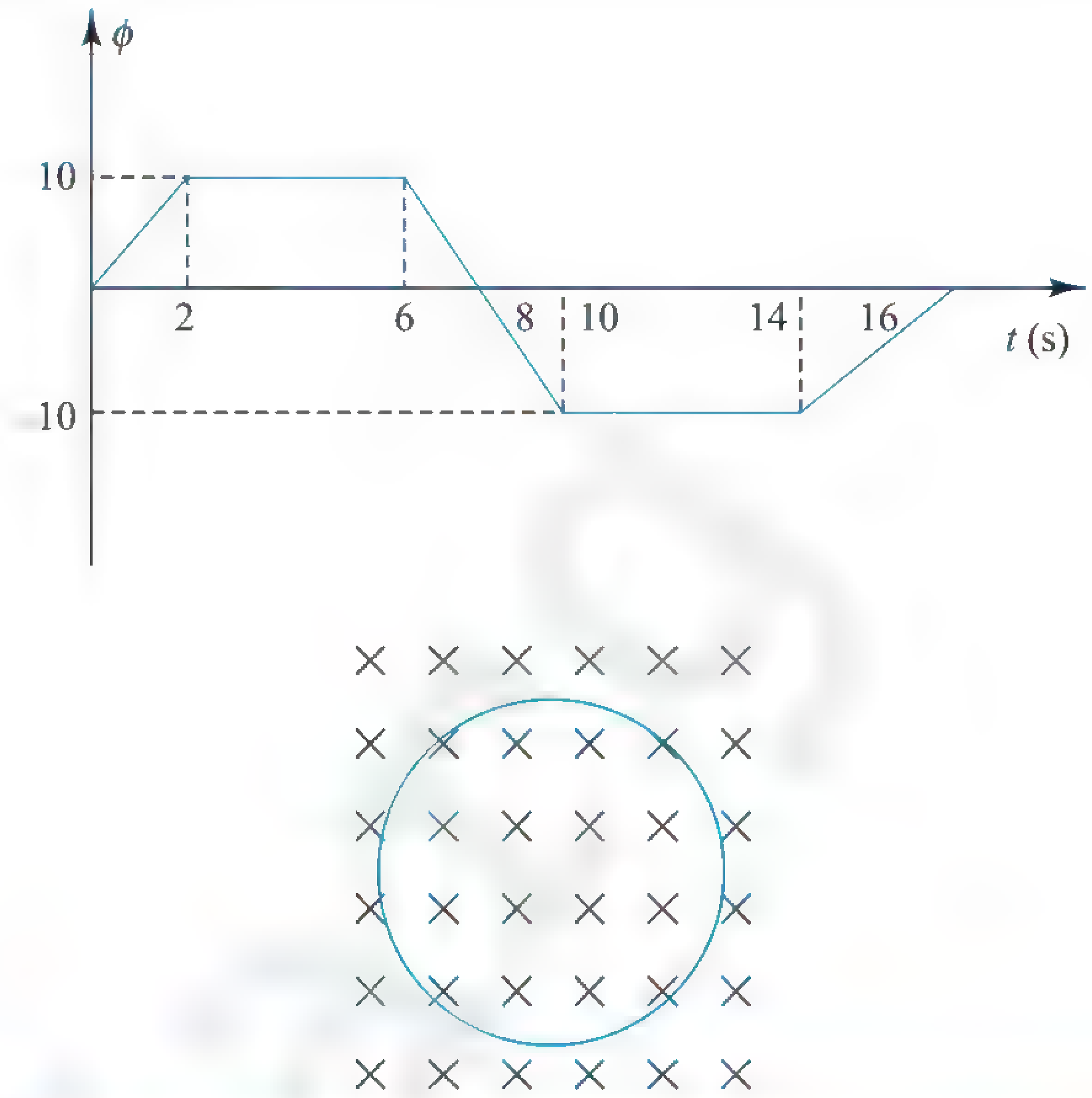
	Column I		Column II
i.	The switch S_1 is closed but S_2 is open	**a.**	5×10^{-7} A, *d* to *a*
ii.	S_1 is open but S_2 is closed	**b.**	5×10^{-7} A, *a* to *d*
iii.	both S_1 and S_2 are open	**c.**	2.5×10^{-8} A, *d* to *a*
iv.	both S_1 and S_2 are closed	**d.**	no current flows

5. A frame *ABCD* is rotating with an angular velocity ω about an axis passing through point *O* perpendicular to the plane of paper as shown in the figure. A uniform magnetic field $\vec{B}$ is applied into the plane of the paper in the region as in the figure. Match the following.



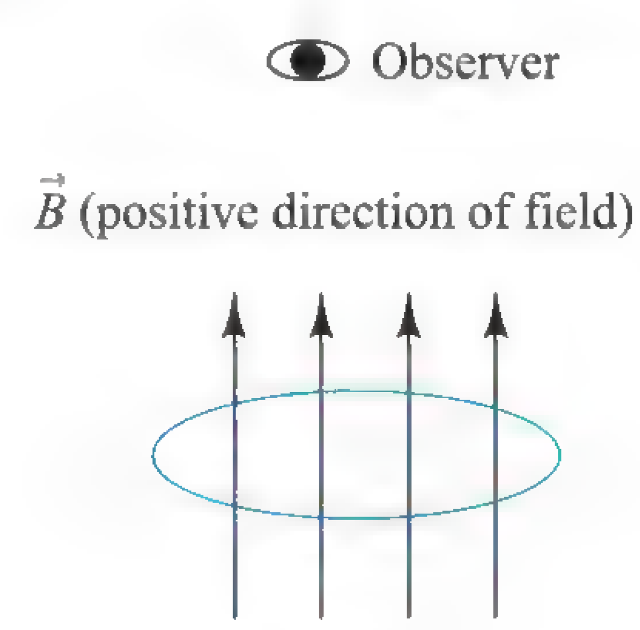
	Column I		Column II
i.	Potential difference between *A* and *O* is	**a.**	zero
ii.	Potential difference between *O* and *D* is	**b.**	$\frac{B\omega L^2}{2}$
iii.	Potential difference between *C* and *D* is	**c.**	$B\omega L^2$
iv.	Potential difference between *A* and *D* is	**d.**	constant

6. Magnetic flux in a circular coil of resistance 10 Ω changes with time as shown in figure. Cross indicates a direction perpendicular to paper inward. Match the following.

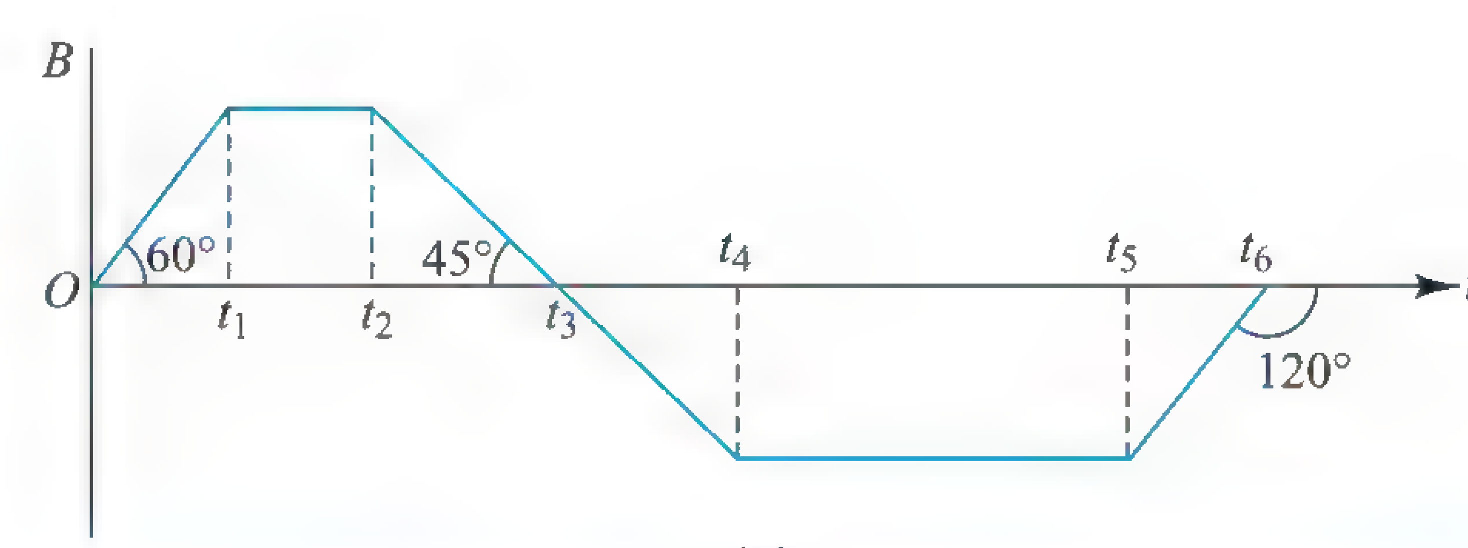


	Column I		Column II
i.	At 1 s, induced current is	**a.**	Clockwise
ii.	At 5 s, induced current is	**b.**	Anticlockwise
iii.	At 9 s, induced current is	**c.**	Zero
iv.	At 15 s, induced current is	**d.**	2 A

7. A conducting loop is held in a magnetic field such that the field is oriented perpendicular to the area of the loop as shown in Fig. (a). At any instant, magnetic flux density over the entire area has the same value but it varies with time as shown in Fig. (b).



(a)



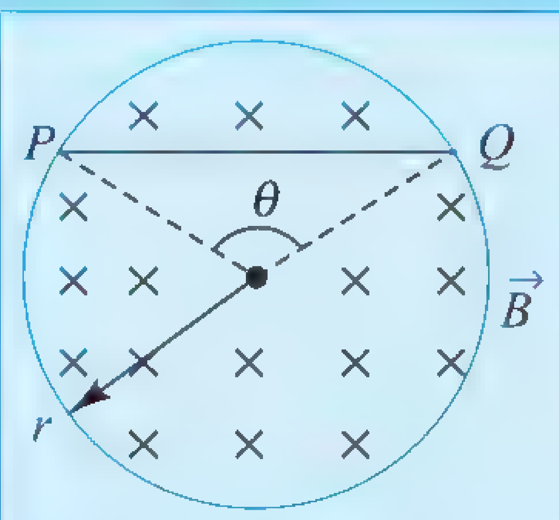
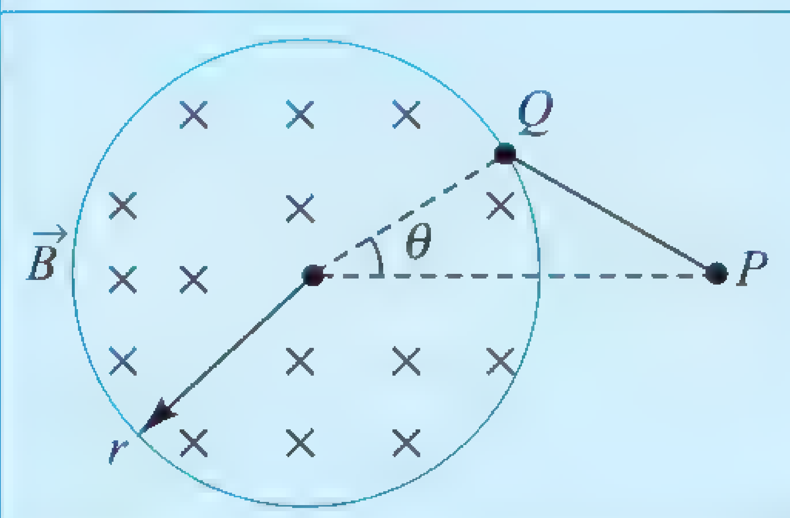
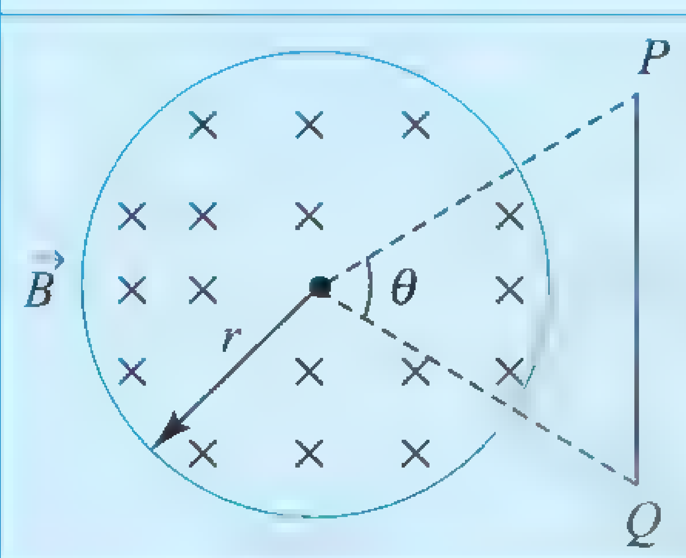
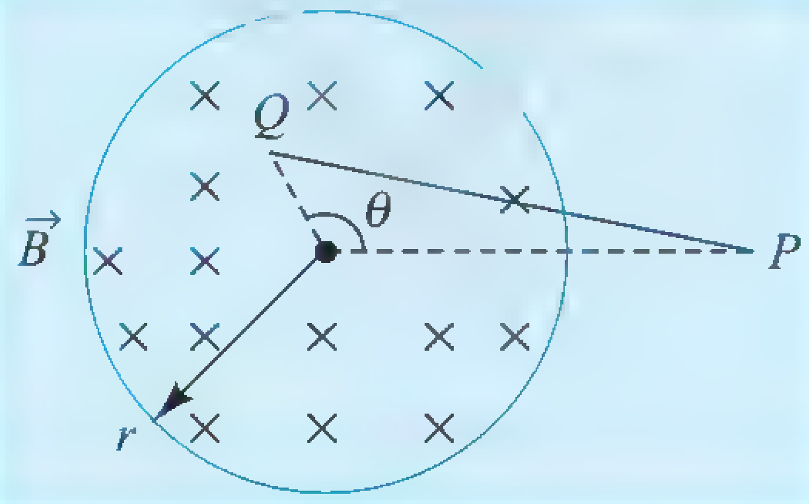
(b)

	Column I		Column II
i.	Induced current in the coil is in the clockwise sense	**a.**	For $t_2 < t < t_3$
ii.	Induced current in the coil is in the anticlockwise sense	**b.**	For $t_3 < t < t_4$
iii.	Induced current is zero	**c.**	For $t_5 < t < t_6$
iv.	Induced current is maximum	**d.**	For $t_4 < t < t_5$

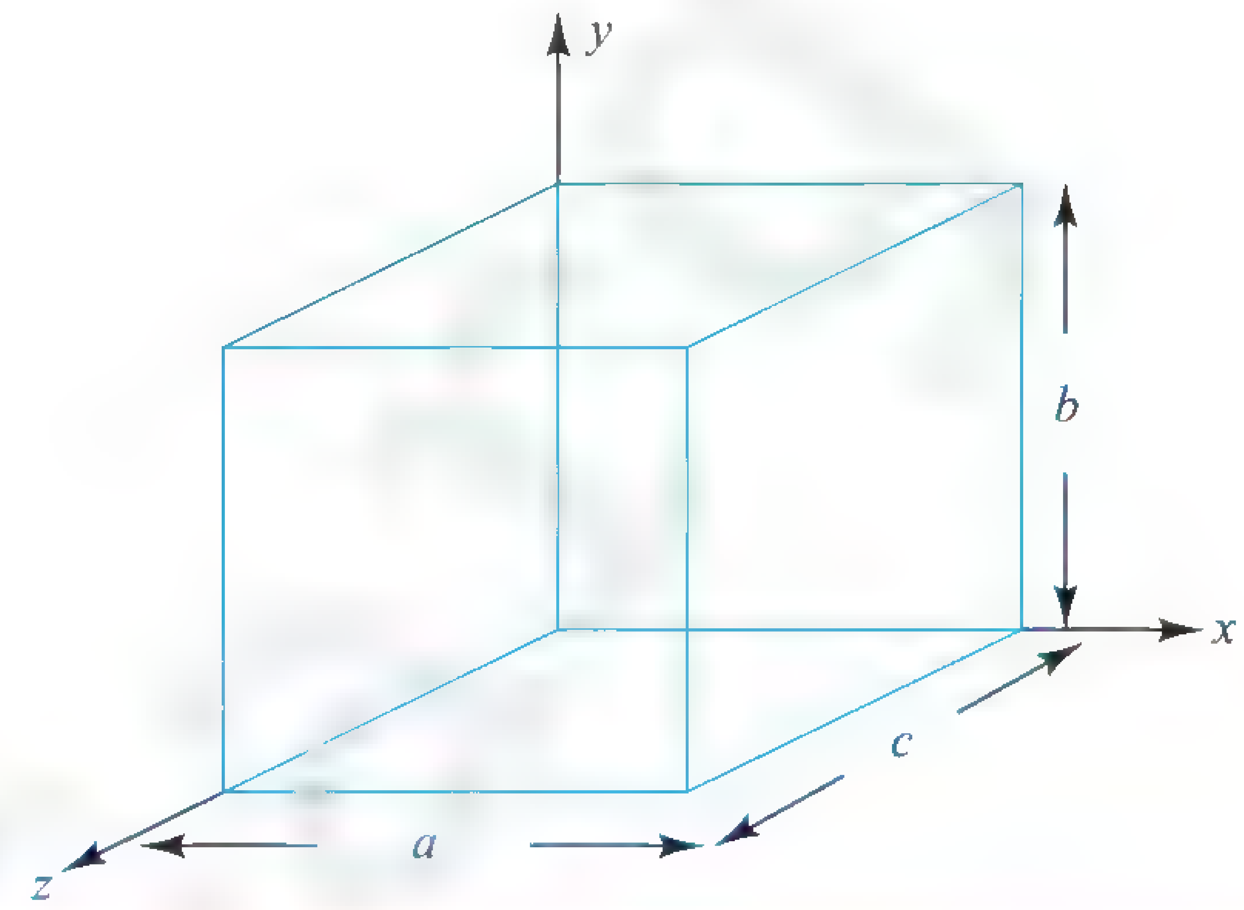
8. Column I gives situations involving a charged particle which may be realized under the condition given in column II. Match the situations in column I with the conditions in column II.

	Column I		Column II
i.	Increase in speed of a charged particle	**a.**	Electric field uniform in space and constant in time
ii.	Exert a force on an electron initially at rest	**b.**	Magnetic field uniform in space and constant in time
iii.	Move a charged particle in a circle with uniform speed	**c.**	Magnetic field uniform in space but varying with time
iv.	Accelerate a moving charged particle	**d.**	Magnetic field non-uniform in space but constant with time

9. Column I shows the cylindrical region of radius r where a downward magnetic field $\vec{B}$ exists, where $\vec{B}$ is increasing at the rate of dB/dt. A rod PQ is placed in different citation as shown. Match the column I with the correct statement in column II regarding the induced emf in rod.

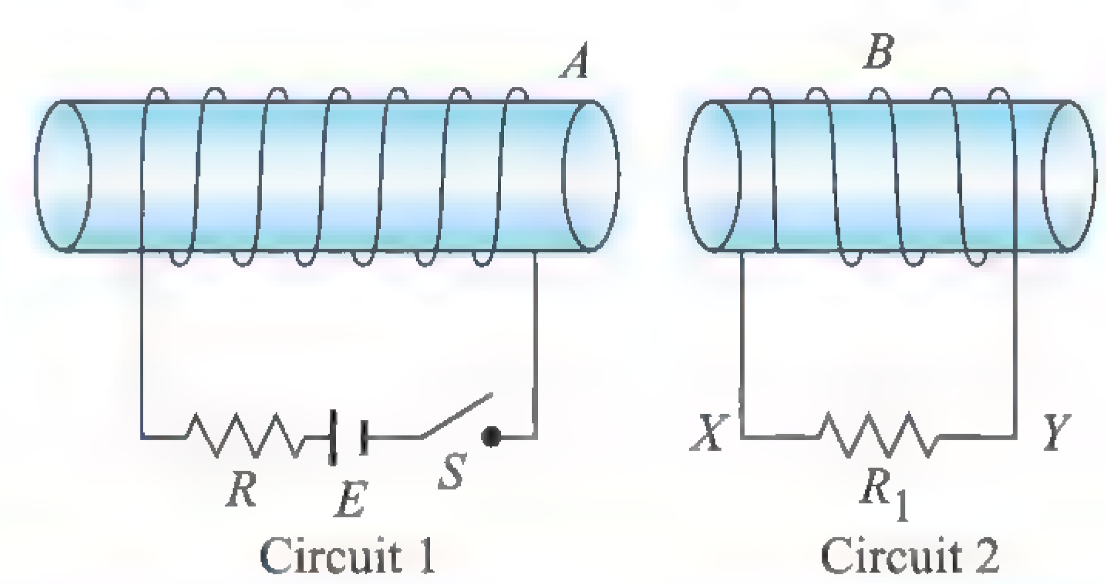
	Column I		Column II
i.		**a.**	Induced emf in rod PQ is $\frac{1}{2}r^2\theta\frac{dB}{dt}$.
ii.		**b.**	Induced emf in rod PQ is less than $\frac{1}{2}r^2\theta\frac{dB}{dt}$.
iii.		**c.**	End P is positive with respect to point Q.
iv.		**d.**	End Q is positive with respect to point P.

10. Figure shows a metallic solid block, placed in a way so that its faces are parallel to the coordinate axes. Edge lengths along axes x, y, and z are a, b, and c, respectively. The block is in a region of uniform magnetic field of magnitude 30 mT. One of the edge lengths of the block is 25 cm. The block is moved at 4 ms^{-1} parallel to each axis and in turn, the resulting potential difference V that appears across the block is measured. When the motion is parallel to the y-axis, V = 24 mV; with the motion parallel to the z-axis, V = 36 mV; with the motion parallel to the x-axis, V = 0. Using the given information, correctly match the dimensions of the block with the values given.



	Column I		Column II
i.	a	**a.**	20 cm
ii.	b	**b.**	24 cm
iii.	c	**c.**	25 cm
iv.	$(bc)/a$	**d.**	30 cm

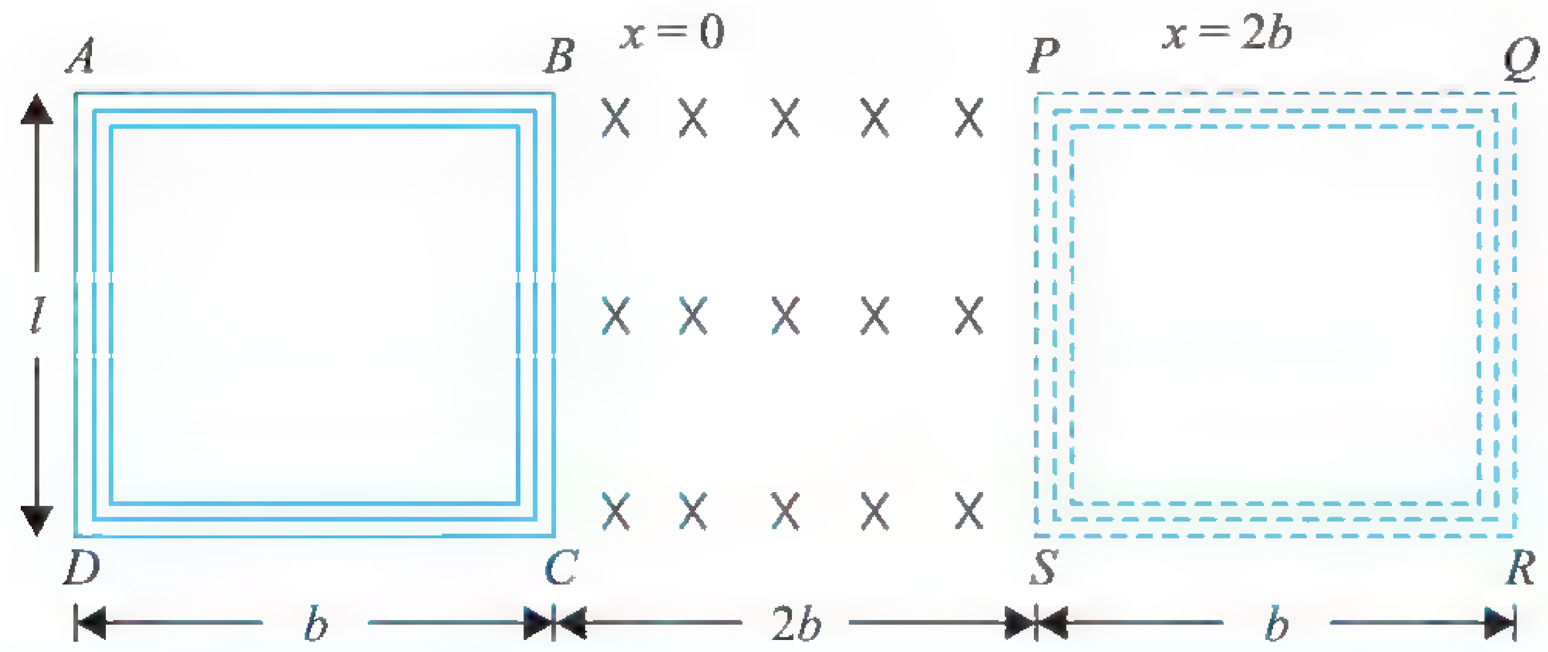
11. In the circuit shown in figure, two coils are arranged as shown. In Column I some operations which are carried out in circuit 1 are mentioned and in Column II its effects. Match the entries of Column I with Column II. Take heating effect of current also into the consideration.



	Column I		Column II
i.	Switch S is closed [exclude small switching time]	**a.**	current in R_1 is from X to Y
ii.	Switch S is closed for long time then it is opened. For this the transition time is	**b.**	current in R_1 is from Y to X
iii.	If coil A is brought nearer to B while switch S is kept closed, then	**c.**	no current is flowing through R_1.
iv.	The battery of constant emf is replaced by a varying emf, switch S is closed.	**d.**	current in R_1 can be from X to Y or from Y to X.

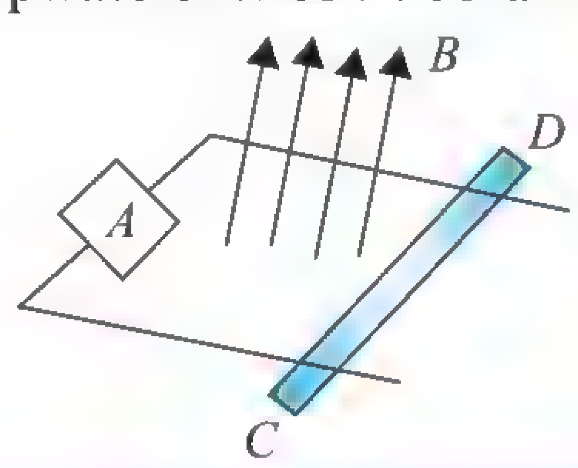
12. A rectangular loop of wire with dimensions l and b has N turns and a total resistance R. The loop is moved with constant velocity such that one side shifts from AB to PQ

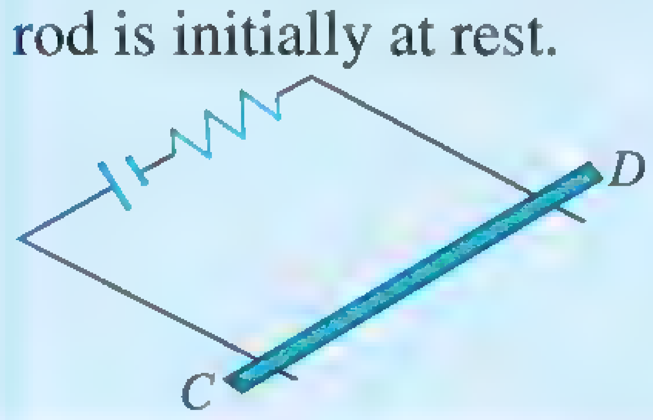
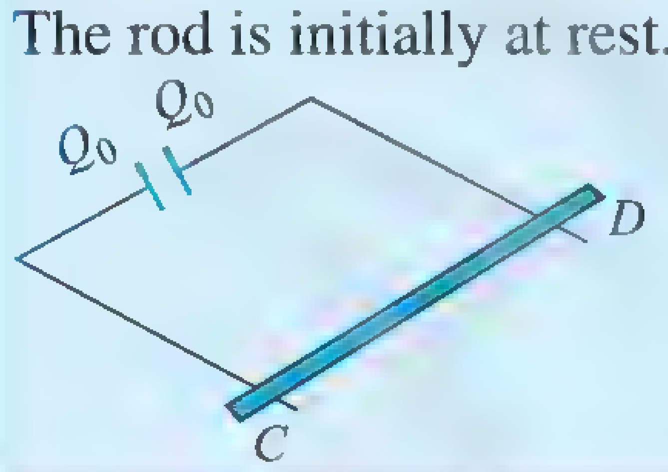
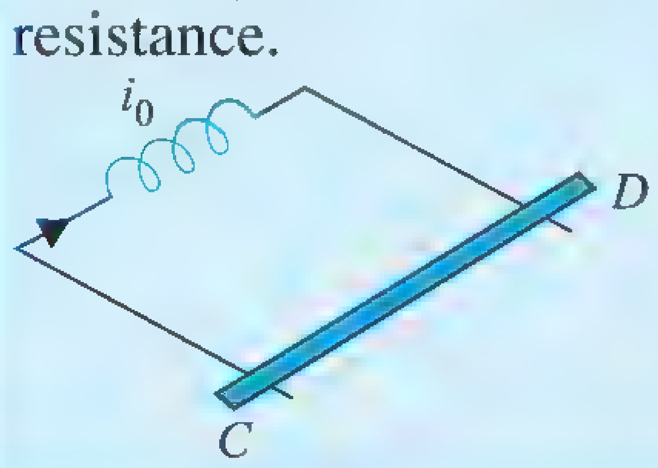
in a region of uniform magnetic field as shown. Let x be the distance of AB from PQ.



	Column I		Column II
i.	$0 < x < b$, $BNlx$	**a.**	$BvlN$
ii.	$b < x < 2b$, $BNlb$	**b.**	0
iii.	$2b < x < 3b$, $BNl(3b - x)$	**c.**	$-BvlN$
iv.	$x > 3b$	**d.**	BvN

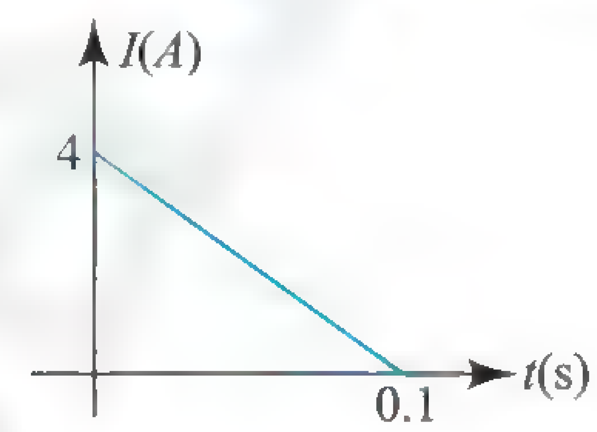
13. A frictionless conducting resistance less rod of mass m straddles the two rails of a track as shown in figure. The rails are also resistanceless. Width of the track is l and it is sufficiently long. Entire arrangement lies in horizontal plane. A homogeneous magnetic field B is present perpendicular to the plane of track. For the situation suggested in column I match the appropriate entries in column II.



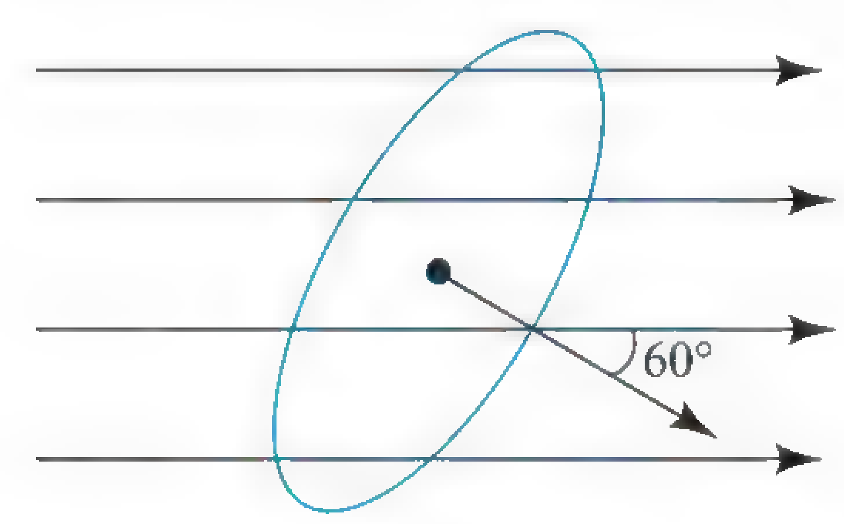
	Column I		Column II
i.	A is a battery of emf V and internal resistance R. The rod is initially at rest. 	**a.**	Energy is dissipated during the motion of the rod.
ii.	A is a charged capacitor. The system has no resistance. The rod is initially at rest. 	**b.**	The rod moves with a constant velocity after a long time.
iii.	A is an inductor with initial current i_0. It is having no resistance. 	**c.**	After a certain time interval, rod will change its direction of motion.
iv.	A is a resistance. The rod is projected to the right with a velocity V_0.	**d.**	If a constant force is applied on the rod to the right, it can immediately move with a constant velocity.

Numerical Value Type

1. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result, an induced current is developed in it, which varies with time as shown in figure. Find the magnitude of the change in flux through the coil in weber.

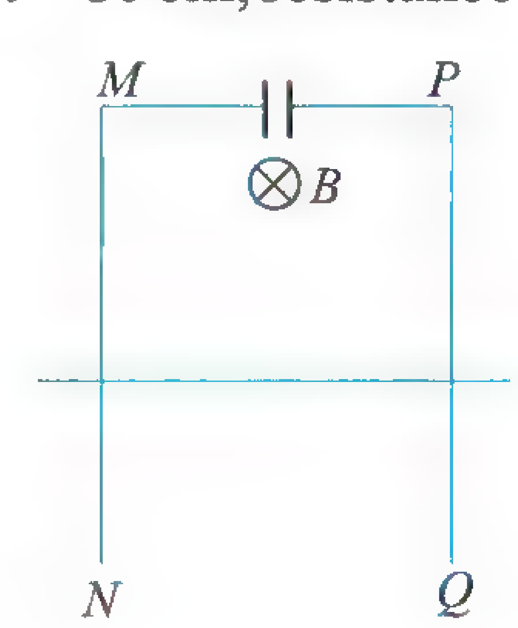


2. A circular coil of wire consists of exactly 100 turns with a total resistance 0.20 Ω. The area of the coil is 100 cm². The coil is kept in a uniform magnetic field B as shown in figure. The magnetic field is increased at a constant rate of 2 T s^{-1}. Find the induced current in the coil in A.

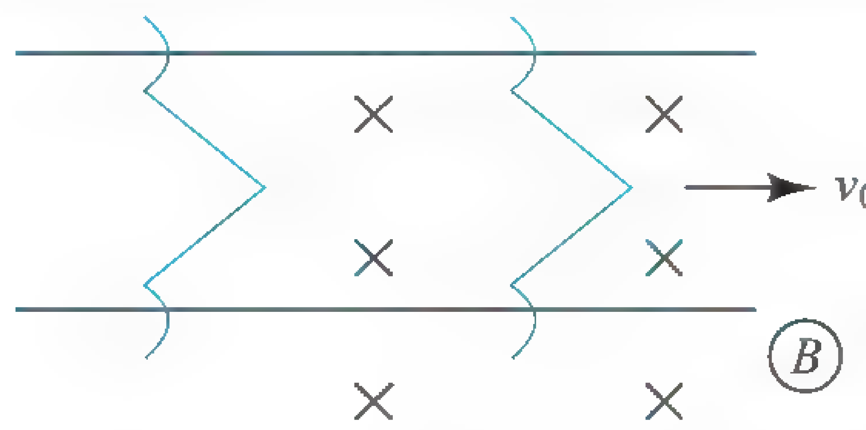


3. A long coaxial cable consists of two thin-walled conducting cylinders with inner radius 2 cm and outer radius 8 cm. The inner cylinder carries a steady current 0.1 A, and the outer cylinder provides the return path for that current. The current produces a magnetic field between the two cylinders. Find the energy stored in the magnetic field for length 5 m of the cable. Express answer in nJ (use ln 2 = 0.7).

4. Figure shows a conducting rod of length $l = 10$ cm, resistance R and mass m = 100 mg moving vertically downward due to gravity. Other parts are kept fixed. Magnetic field is B = 1 T. MN and PQ are vertical, smooth, conducting rails. The capacitance of the capacitor is C = 10 mF. The rod is released from rest. Find the maximum current (in mA) in the circuit.

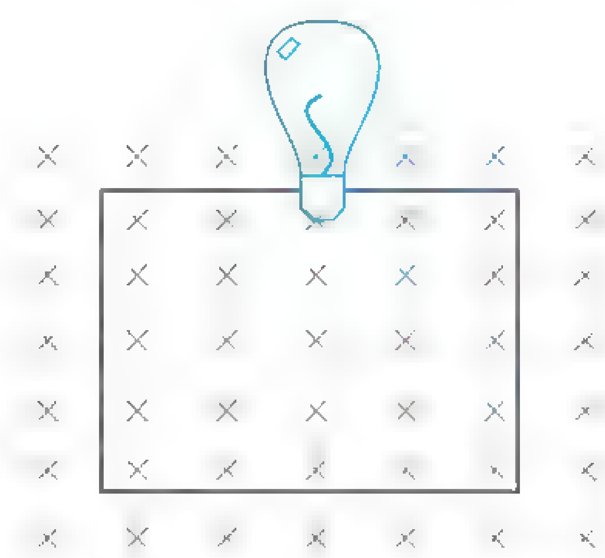


5. In figure, there are two sliders and they can slide on two frictionless parallel wires in uniform magnetic field B, which is present everywhere. The mass of each slider is m, resistance R and initially these are at rest. Now, if one slider is given a velocity v_0 = 16 m s^{-1}, what will be the velocity (in m s^{-1}) of other slider after long time? (neglect the self-induction)

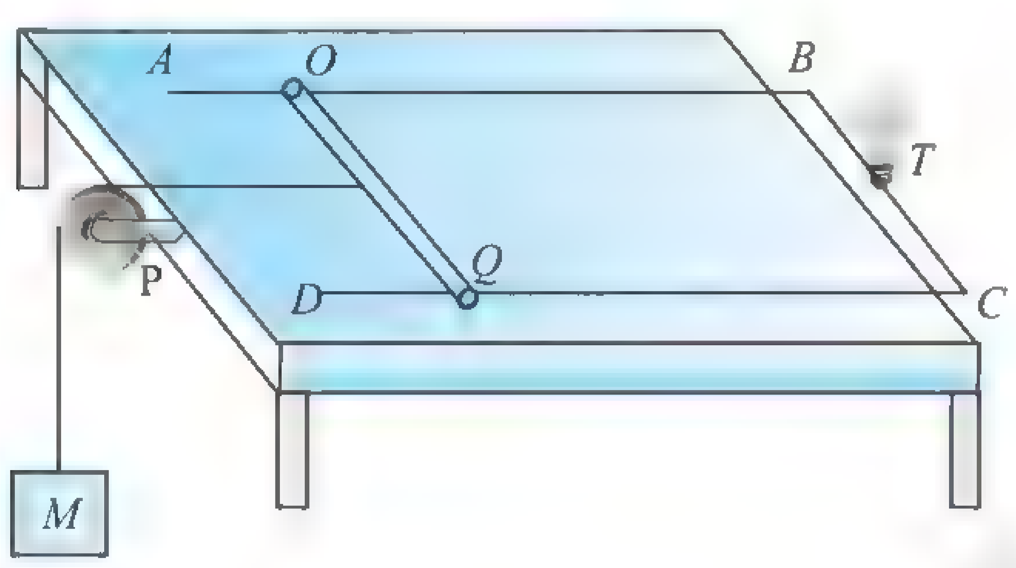


6. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 7 T.

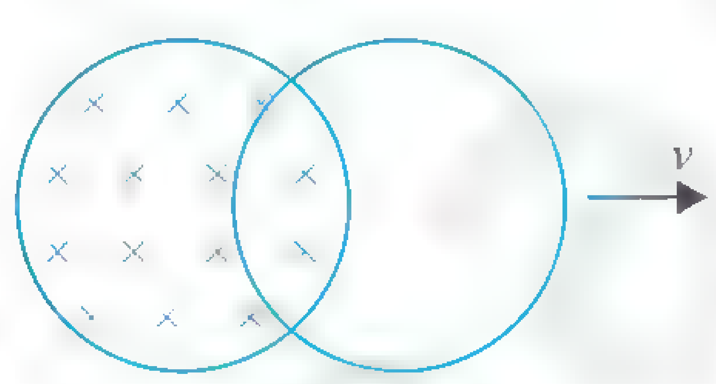
A 10 V light bulb is in a series with the loop as shown in figure. The magnetic field is decreasing steadily to zero over a time interval Δt. For what value of Δt (in ms), the bulb will shine with full brightness?



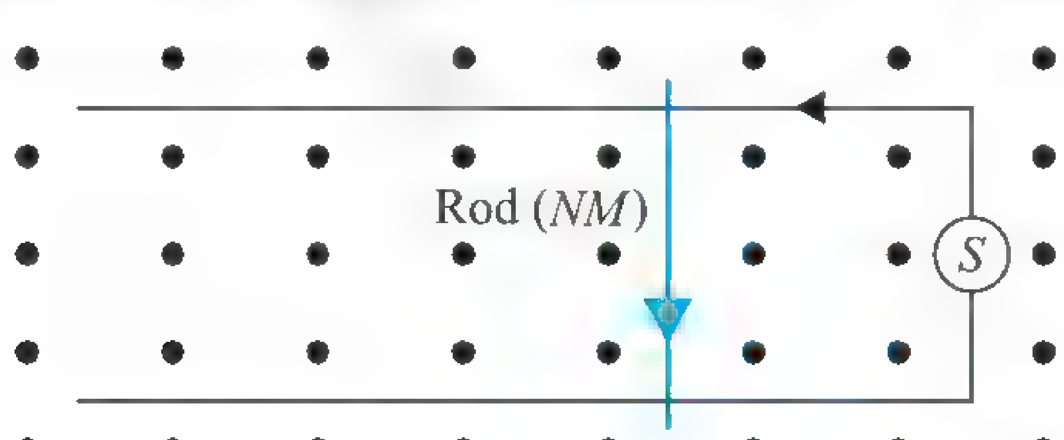
7. In figure, $ABCD$ is a fixed smooth conducting frame in horizontal plane. T is a bulb of power 100 W, P is a smooth pulley and OQ is a conducting rod. Neglect the self-inductance of the loop and resistance of any part other than the bulb. The mass M is moving down with constant velocity 10 m s^{-1}. Bulb lights at its rated power due to induced emf in the loop due to earth's magnetic field. Find the mass M (in kg) of the block. (g = 10 m s^{-2})



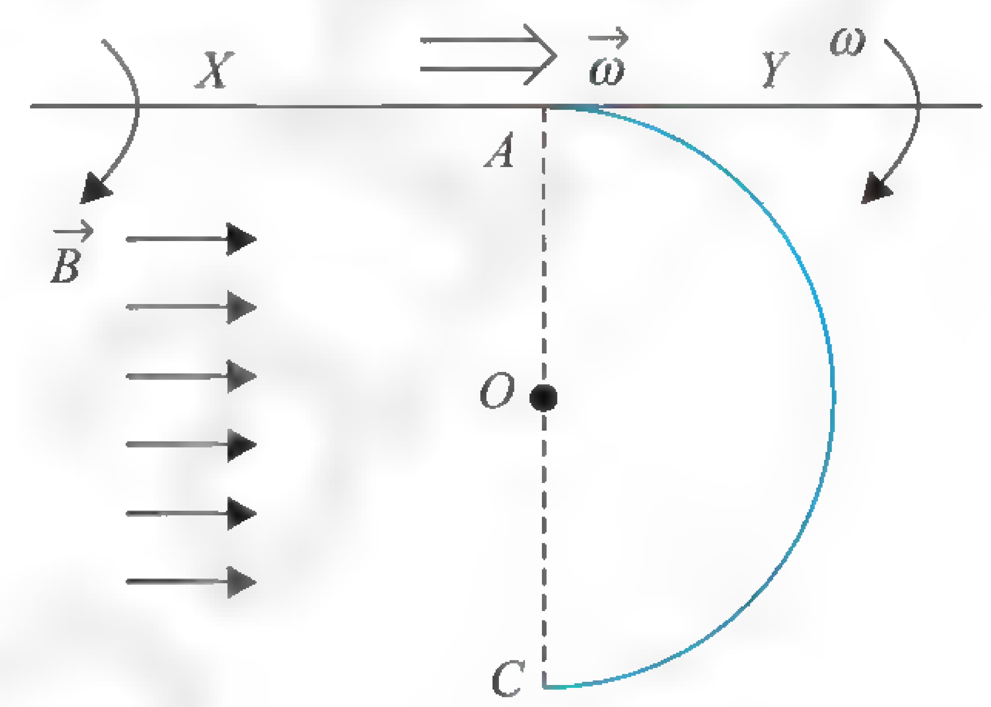
8. A uniform magnetic field B = 0.5 T exists in a circular region of radius R = 5 m. A loop of radius R = 5 m encloses the magnetic field at t = 0 and then pulled at uniform speed v = 2 ms^{-1} in the plane of the paper. Find the induced emf (in V) in the loop at time t = 3 s.



9. A conducting rod of mass 200 g and length 10 cm can slide without friction on two long, horizontal rails. A uniform magnetic field of magnitude 5 mT exists in the region as shown. A source S is used to maintain a constant current 2 A through the rod. If motion of the rod starts from the rest, find its speed (in cm/s) after 10 s from the start of the motion.



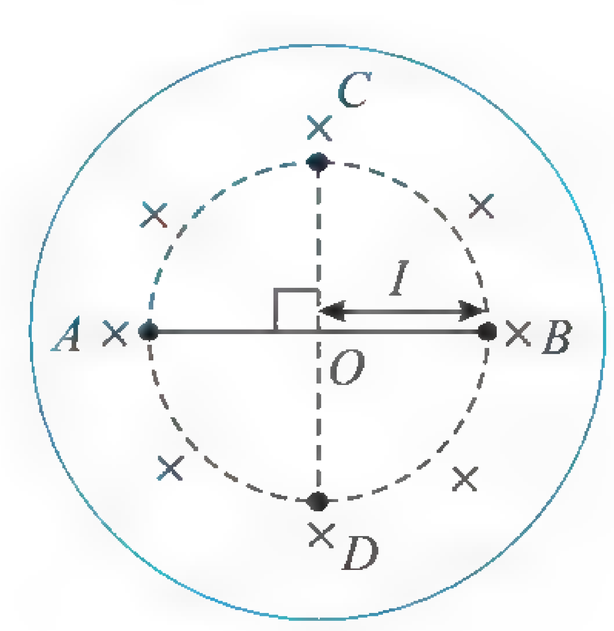
10. A thin wire AC shaped as a semi-circle of diameter d rotates with a constant angular frequency ω in a uniform magnetic field of induction $\vec{B}$. The vector $\vec{\omega}$ is parallel to $\vec{B}$ and the rotation axis XY passes through the end A of the wire and is perpendicular to the diameter AC (see figure). $\vec{B}$ is directed left to right in the plane of the paper. The value of the line integral $I = \int \vec{E} \cdot d\vec{r}$ taken along the wire from the point A to the point C will be $\frac{\omega B d^2}{?}$. What integer should be inserted in place of question mark.



11. A copper wire of length 2 m placed perpendicular to the plane of magnetic field $\vec{B} = (2\hat{i} + 4\hat{j})T$. If it moves with velocity $(4\hat{i} + 6\hat{j} + 8\hat{k})$m/sec. Calculate the magnitude of dynamic emf (in volt)across its ends.

12. Two concentric coplanar circular loops made of wire, resistance per unit length $10^{-4}\ \Omega m^{-1}$, have diameters 0.2 m and 2 m. A time-varying potential difference (4 + 2.5t) volt is applied to the larger loop. Calculate the current in the smaller loop (in A).

13. A massless non-conducting rod AB of length $2l$ is placed in uniform time varying magnetic field confined in a cylindrical region of radius ($R > l$) as shown in the figure. The center of the rod coincides with the centre of the cylindrical region. The rod can freely rotate in the plane of the figure about an axis coinciding with the axis of the cylinder. Two particles, each of mass m and charge q are attached to the ends A and B of the rod. The time varying magnetic field in this cylindrical region is given by $B = B_0\left[1 - \frac{t}{2}\right]$ where B_0 is a constant. The field is switched on at time t = 0. Consider: B_0 = 100T, l = 4 cm, $\frac{q}{m} = \frac{4\pi}{100}$C/kg. Calculate the time (in sec) in which the rod will reach position CD shown in the figure for the first time. Will end A be at C or D at this instant?

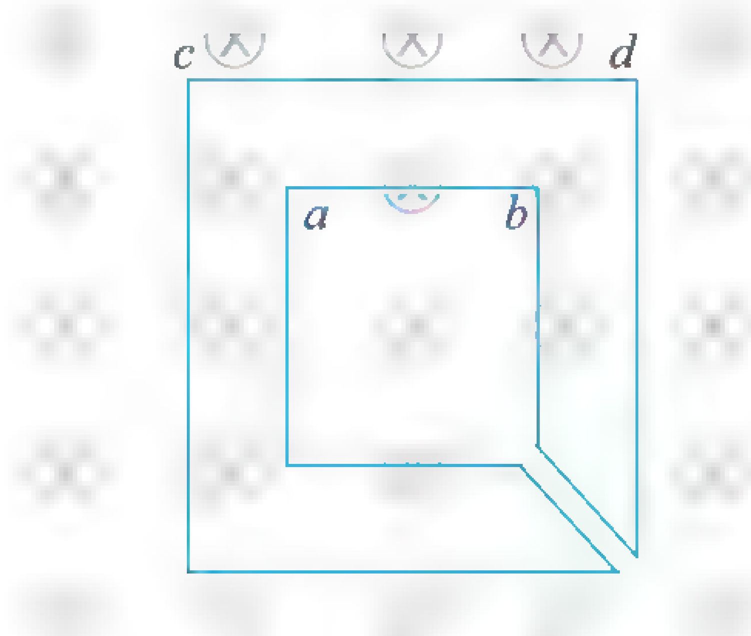


Archives

JEE Advanced

Single Correct Answer Type

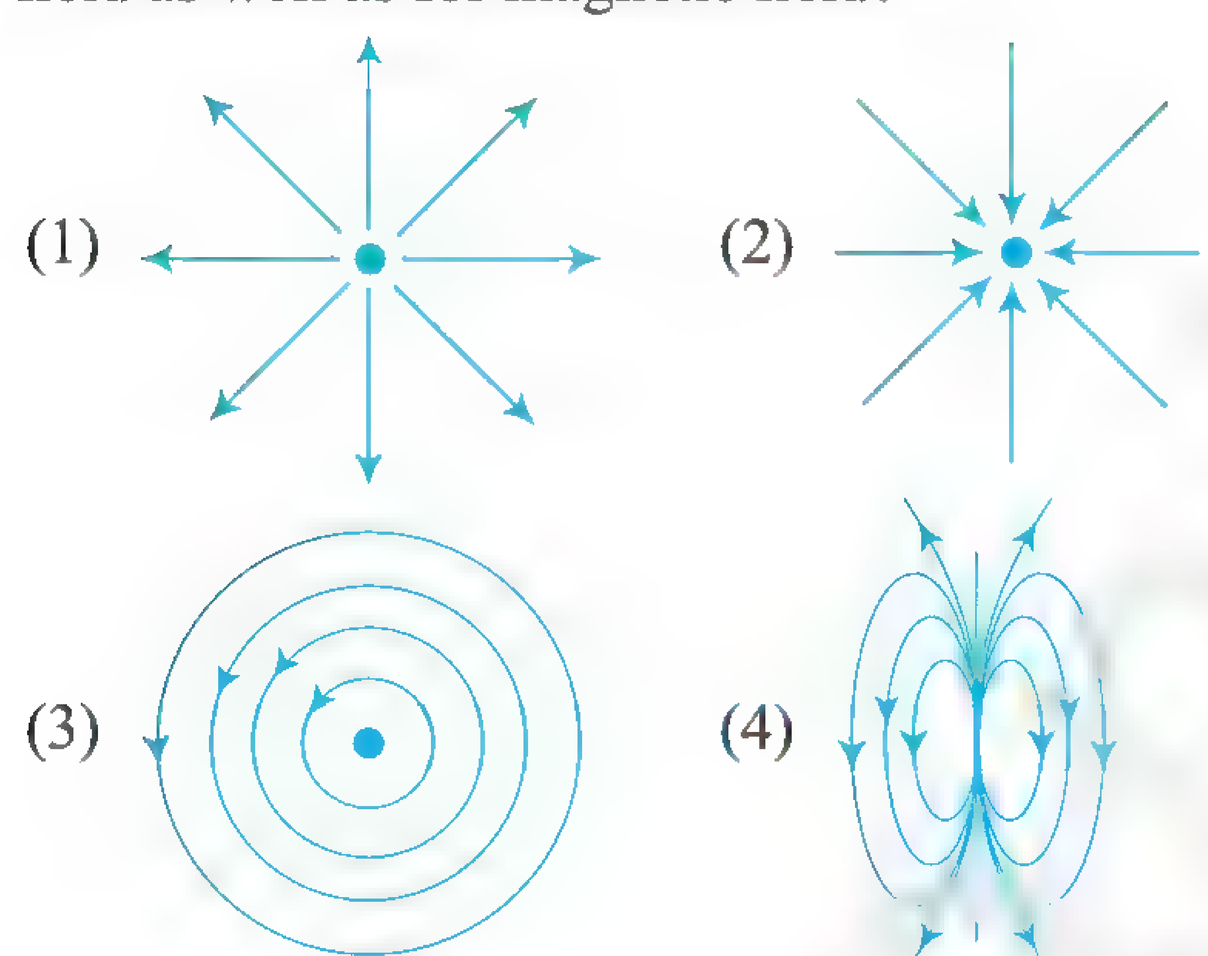
1. Figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then,



(1) $I_1 > I_2$
(2) $I_1 < I_2$
(3) I_1 is in the direction ba and I_2 is in the direction cd
(4) I_1 is in the direction ab and I_2 is in the direction dc

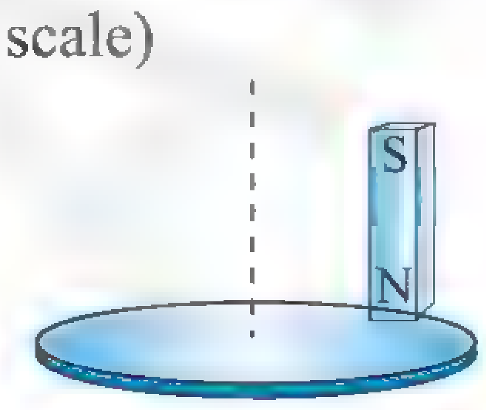
(IIT-JEE 2009)

2. Which of the field patterns given below is valid for electric field as well as for magnetic field?



(IIT-JEE 2011)

3. A light disc made of aluminum (a nonmagnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (figure is schematic and not drawn to scale)

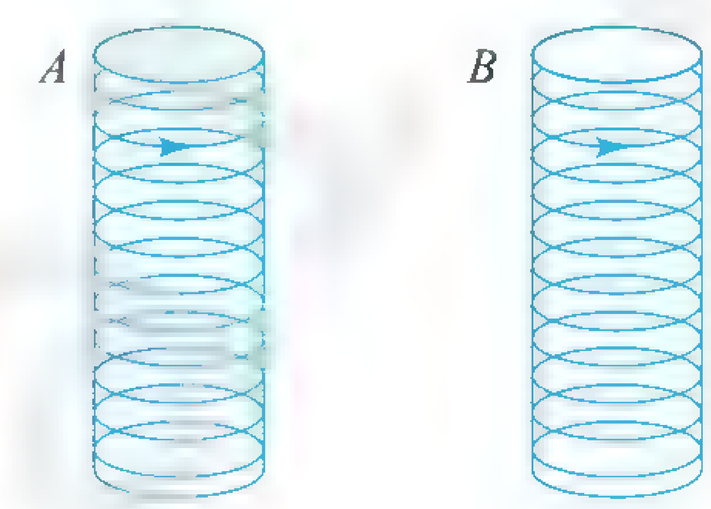


(1) rotate in the direction opposite to the direction of magnet's motion
(2) rotate in the same direction as the direction of magnet's motion
(3) not rotate and its temperature will remain unchanged
(4) not rotate but its temperature will slowly rise

(JEE Advanced 2020)

Multiple Correct Answers Type

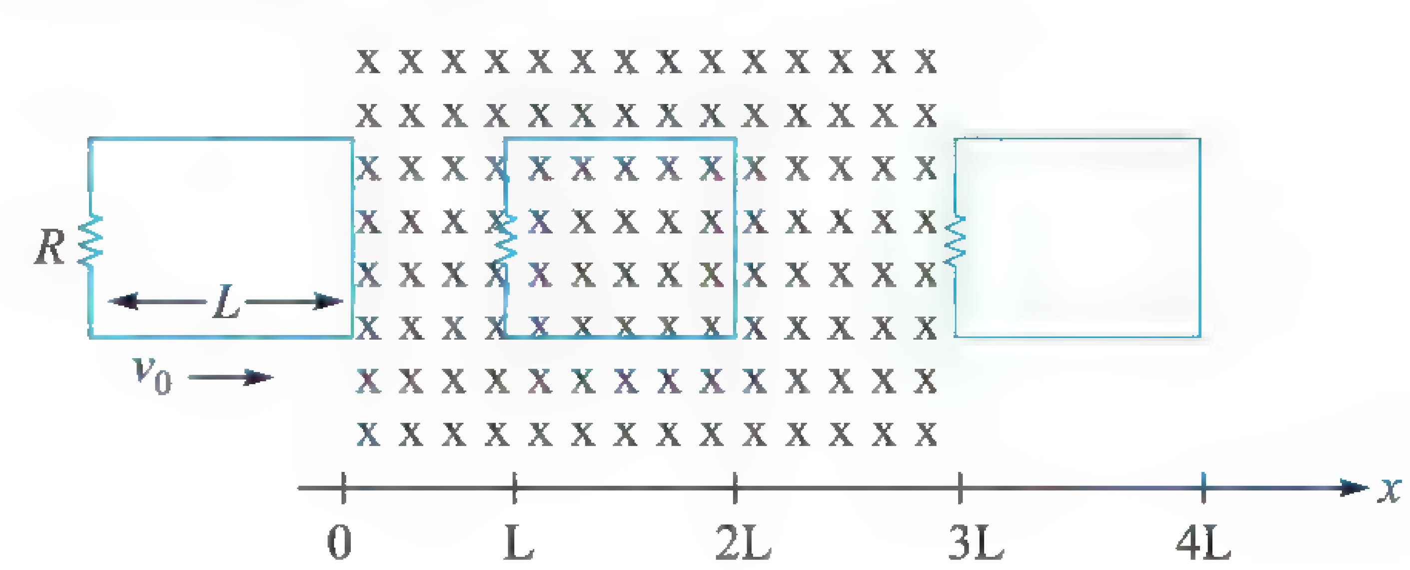
1. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)

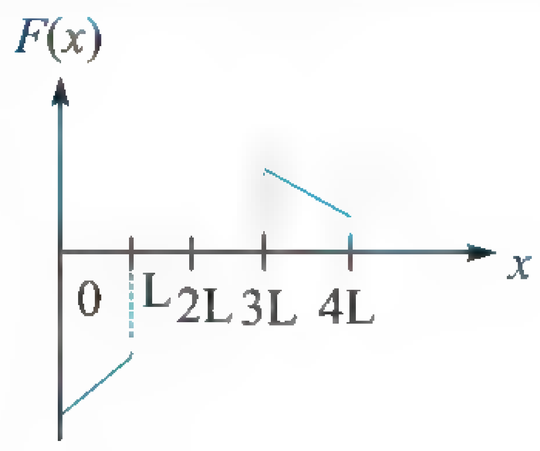


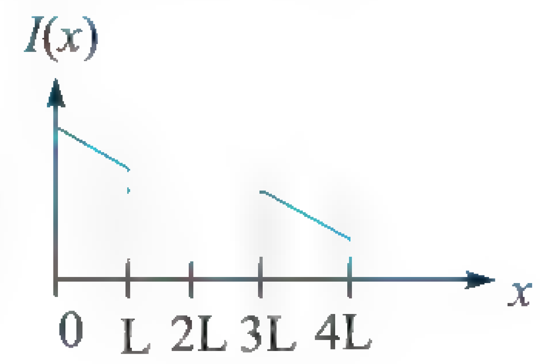
(1) $\rho_A > \rho_B$ and $m_A = m_B$
(2) $\rho_A < \rho_B$ and $m_A = m_B$
(3) $\rho_A > \rho_B$ and $m_A > m_B$
(4) $\rho_A < \rho_B$ and $m_A < m_B$ **(IIT-JEE 2009)**

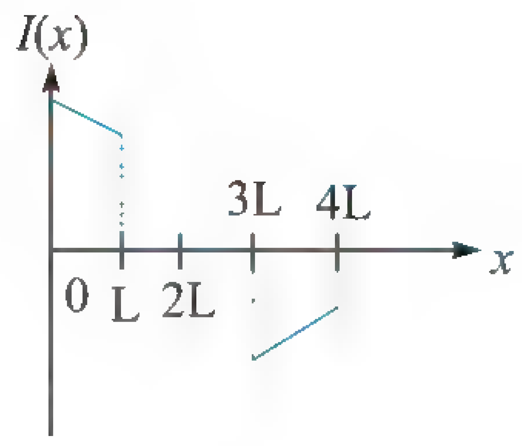
2. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is/are
The current induced in the loop is
(1) The emf induced in the loop is zero if the current is constant
(2) The emf induced in the loop is finite if the current is constant
(3) The emf induced in the loop is zero if the current decreases at a steady rate.
(4) The emf induced in the loop is finite if the current decreases at a steady rate. **(IIT-JEE 2012)**

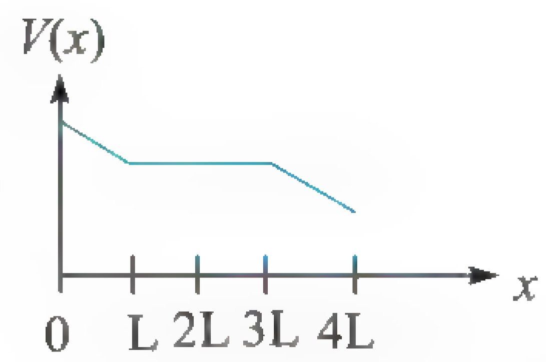
3. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper; as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.



(1) 

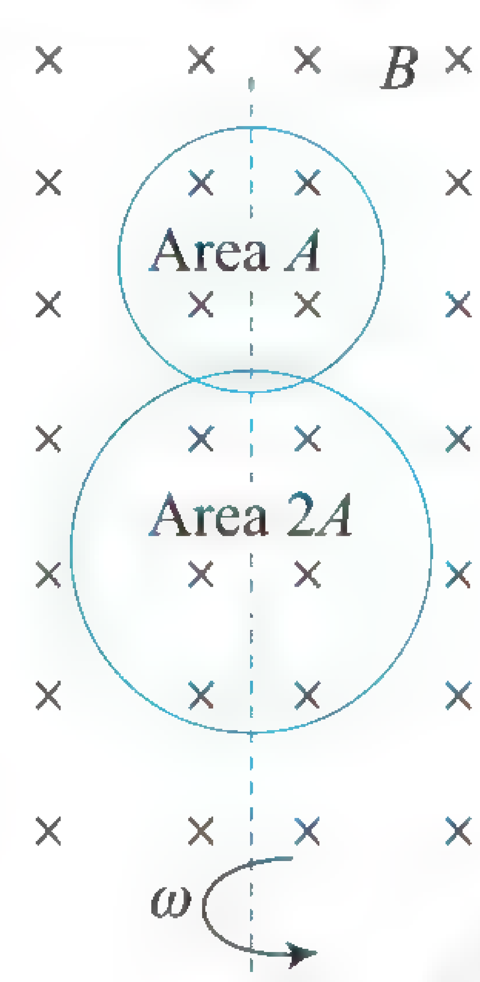
(2) 

(3) 

(4) 

(JEE Advanced 2016)

4. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(1) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

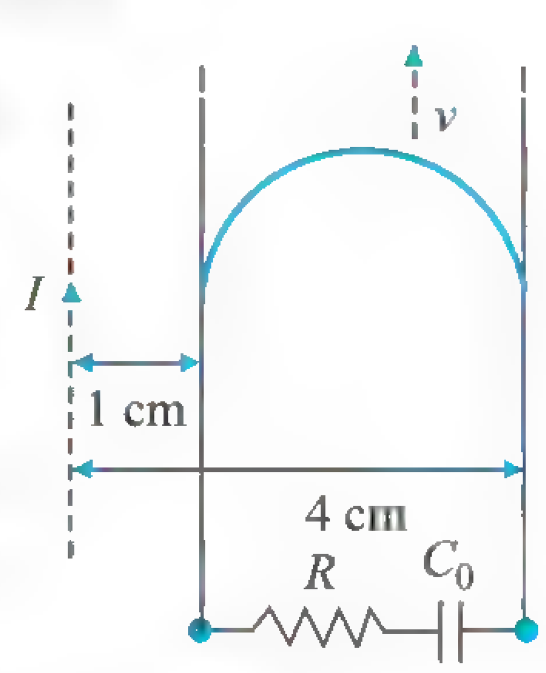
(2) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(3) The net emf induced due to both the loops is proportional to cos ωt

(4) The emf induced in the loop is proportional to the sum of the areas of the two loops

(JEE Advanced 2017)

5. A long straight wire carries a current, I = 2 ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time $t = 0$, the rod starts moving on the rails with a speed $v = 3.0$ m/s (see the figure).



A resistor $R = 1.4\ \Omega$ and a capacitor $C_0 = 5.0\ \mu$F are connected in series between the rails. At time $t = 0$, C_0 is uncharged. Which of the following statement(s) is(are) correct ? [$\mu_0 = 4\ \pi \times 10^{7}$ SI units. Take ln 2 = 0.7]

(1) Maximum current through R is 1.2×10^{6} ampere

(2) Maximum current through R is 3.8×10^{-6} ampere

(3) Maximum charge on capacitor C_0 is 8.4×10^{12} coulomb

(4) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

(JEE Advanced 2021)

Linked Comprehension Type

For Problems 1–2

Point Q is moving in a circular orbit of radius R in the x–y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $Q\omega/2\pi$. uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

(JEE Advanced 2013)

1. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change, is

(1) $\dfrac{BR}{4}$ (2) $\dfrac{BR}{2}$

(3) BR (4) $2BR$

2. The change in the magnetic dipole moment associated with the orbit, at the end of time interval of the magnetic field change, is

(1) $-\gamma BQR^2$ (2) $-\gamma \dfrac{BQR^2}{2}$

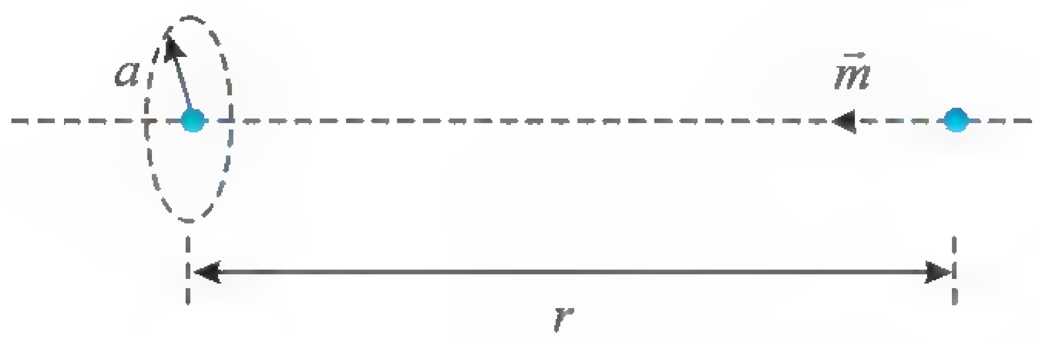
(3) $\gamma \dfrac{BQR^2}{2}$ (4) γBQR^2

For Problems 3–4

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its center at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r (>> a) from the center of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\dfrac{\mu_0}{2\pi}\dfrac{m}{r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the

common axis, with their north poles facing each other, is $\frac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



(JEE Advanced 2021)

3. When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to

(1) $\frac{m}{r^3}$ (2) $\frac{m^2}{r^2}$

(3) $\frac{m}{r^2}$ (4) $\frac{m^2}{r}$

4. The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given process is proportional to

(1) $\frac{m}{r^5}$ (2) $\frac{m^2}{r^5}$

(3) $\frac{m^2}{r^6}$ (4) $\frac{m^2}{r^7}$

Answers Key

EXERCISES

Single Correct Answer Type

1. (1)	**2.** (2)	**3.** (2)	**4.** (3)	**5.** (2)
6. (1)	**7.** (4)	**8.** (4)	**9.** (4)	**10.** (4)
11. (4)	**12.** (3)	**13.** (2)	**14.** (1)	**15.** (2)
16. (3)	**17.** (2)	**18.** (1)	**19.** (3)	**20.** (1)
21. (3)	**22.** (3)	**23.** (1)	**24.** (1)	**25.** (2)
26. (1)	**27.** (1)	**28.** (2)	**29.** (1)	**30.** (2)
31. (1)	**32.** (4)	**33.** (4)	**34.** (1)	**35.** (3)
36. (4)	**37.** (3)	**38.** (1)	**39.** (2)	**40.** (4)
41. (4)	**42.** (1)	**43.** (1)	**44.** (1)	**45.** (4)
46. (4)	**47.** (4)	**48.** (4)	**49.** (2)	**50.** (2)
51. (3)	**52.** (3)	**53.** (4)	**54.** (4)	**55.** (4)
56. (4)	**57.** (4)	**58.** (3)	**59.** (3)	**60.** (1)
61. (4)	**62.** (1)	**63.** (4)	**64.** (3)	**65.** (3)
66. (3)	**67.** (3)	**68.** (4)	**69.** (2)	**70.** (4)
71. (2)	**72.** (4)	**73.** (1)	**74.** (3)	**75.** (4)
76. (3)	**77.** (1)	**78.** (1)	**79.** (2)	**80.** (3)
81. (2)	**82.** (2)	**83.** (4)	**84.** (3)	**85.** (2)
86. (1)				

Multiple Correct Answers Type

1. (1),(2)	**2.** (1),(2),(3),(4)	**3.** (2),(3)
4. (1),(3)	**5.** (1),(3)	**6.** (1),(3),(4)
7. (1),(2),(3),(4)	**8.** (2),(3)	**9.** (1),(2),(3),(4)
10. (2),(4)	**11.** (1),(2),(3)	**12.** (2),(4)
13. (2),(4)	**14.** (1),(4)	**15.** (1),(4)
16. (2),(3)	**17.** (1),(3),(4)	**18.** (1),(2),(3),(4)
19. (2),(3)	**20.** (1),(4)	**21.** (1),(2),(4)
22. (1),(2),(3)	**23.** (1),(2),(3),(4)	**24.** (1),(4)
25. (2),(4)	**26.** (1),(3)	

Linked Comprehension Type

1. (1)	**2.** (4)	**3.** (1)	**4.** (4)	**5.** (1)
6. (2)	**7.** (3)	**8.** (2)	**9.** (3)	**10.** (4)
11. (1)	**12.** (2)	**13.** (2)	**14.** (1)	**15.** (1)
16. (3)	**17.** (2)	**18.** (3)	**19.** (3)	**20.** (2)
21. (1)	**22.** (2)	**23.** (3)	**24.** (3)	**25.** (4)
26. (3)	**27.** (4)	**28.** (3)	**29.** (2)	**30.** (1)
31. (3)	**32.** (1)	**33.** (2)	**34.** (3)	**35.** (2)
36. (4)	**37.** (4)	**38.** (4)	**39.** (1)	**40.** (3)
41. (1)				

Matrix Match Type

1. i. → b., d.; ii. → a., c.; iii. → a., c.; iv. → b., d.
2. i. → b., d.; ii. → b., c.; iii. → a., d.; iv. → a., c.
3. i. → b., d.; ii. → a., d.; iii. → c.; iv. → c.
4. i. → b.; ii. → a.; iii. → d.; iv. → d.
5. i. → c., d.; ii. → c., d.; iii. → b., d.; iv. → a., d.
6. i. → b.; ii. → c.; iii. → a.; iv. → b.
7. i. → c.; ii. → a., b.; iii. → d.; iv. → c.
8. i. → a., c.; ii. → a., c.; iii. → b., d.; iv. → a., b., c., d.
9. i. → b., c.; ii. → a., d.; iii. → a., c.; iv. → b., d.
10. i. → c.; ii. → d.; iii. → a.; iv. → b.
11. i. → a., d.; ii. → a., d.; iii. → d.; iv. → d.
12. i. → c.; ii. → b.; iii. → a.; iv. → b.
13. i. → a., b.; ii. → a., b.; iii. → c.; iv. → a., d.

Numerical Value Type

1. (2)	**2.** (5)	**3.** (7)	**4.** (5)	**5.** (8)
6. (7)	**7.** (1)	**8.** (8)	**9.** (5)	**10.** (4)
11. (8)	**12.** (1.25)	**13.** (1)		

ARCHIVES

JEE Advanced

Single Correct Answer Type

1. (4) **2.** (3) **3.** (2)

Multiple Correct Answers Type

1. (2),(4) **2.** (1),(3) **3.** (3),(4) **4.** (1),(2) **5.** (1),(3)

Linked Comprehension Type

1. (2) **2.** (2) **3.** (1) **4.** (3)

Chapter 4

Concept Application Exercises

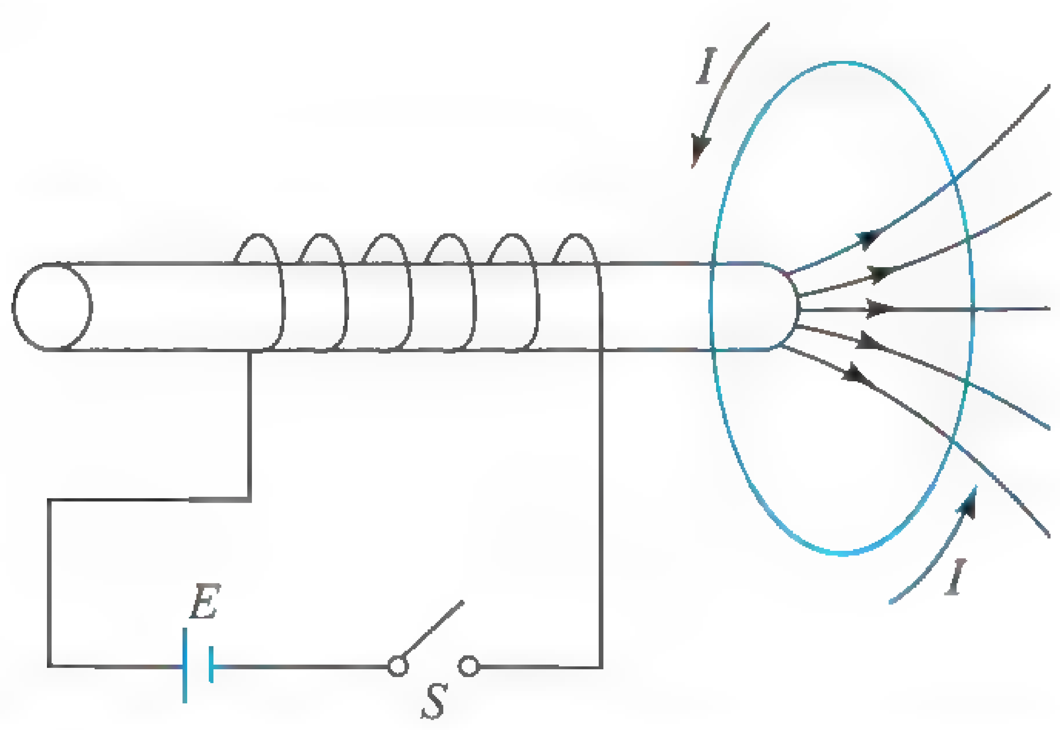
Exercise 4.1

1. (a) $\vec{A} = \ell^2 \hat{i}$

$$\phi = \vec{B}.\vec{A} = (5\hat{i} + 4\hat{j} + 3\hat{k}).(2.5 \times 10^{-2})^2 \hat{i} = 3.125 \times 10^{-3}\ \text{Wb}$$

(b) As magnetic field is uniform no net flux through the closed structure (zero)

2. (a) As the switch is closed, the situation changes from one in which no magnetic flux passes through the ring to the one in which flux passes through the ring in the direction shown. The induced current flows in the anticlockwise sense as seen from the side on which the solenoid lies.



(b) When the switch has been closed for a long time, no change in the magnetic flux through the loop occurs; hence, no current is induced in the loop.

(c) When the switch is opened, flux decreases in the direction shown above. Hence current will be opposite to the direction of situation (a), i.e., in clockwise direction.

3. (a) Induced current should be in anticlockwise direction, so that upper face becomes north pole.

(b) Induced current should be clockwise so that upper face becomes south pole.

(c) Flux is increasing in the inward direction, so induced current should be in anticlockwise direction. It will oppose the increase of flux.

(d) Flux is decreasing in upward direction, so induced current will be in anticlockwise direction. It will oppose the decrease in flux.

4. (a) When the switch is opened, the magnetic field to the right decreases. Therefore, the second coil's induced current produces its own field to the right. That means the current must pass through the resistor from point a to point b.

(b) If coil B is moved closer to coil A, more flux passes through it towards the right. Therefore, the induced current must produce its own magnetic field to the left to oppose the increased flux. That means the current must pass through the resistor from point b to point a.

(c) If the variable resistor R is decreased, then more current flows through coil A, and so a stronger magnetic field is produced, leading to more flux to the right through coil B. Therefore, the induced current must produce its own magnetic field to the left to oppose the increased flux. That means the current must pass through the resistor from point b to point a.

5. (a) When current is passing from a to b and is increasing, the magnetic field becomes stronger to the left, so the induced field points towards right, and the induced current must flow from right to left through the resistor.

(b) If the current passes from b to a, and is decreasing, then there is less magnetic field pointing right, so the induced field points towards right, and the induced current must flow from right to left through the resistor.

(c) If current passes from b to a and is increasing, then there is more magnetic field pointing right, so the induced field points left, and the induced current must flow from left to right through the resistor.

6. (a) Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is clockwise.

(b) The current reaches a constant value so Φ_B is constant $d\Phi_B/dt = 0$ and there is no induced current.

(c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.

7. $\phi = BA$

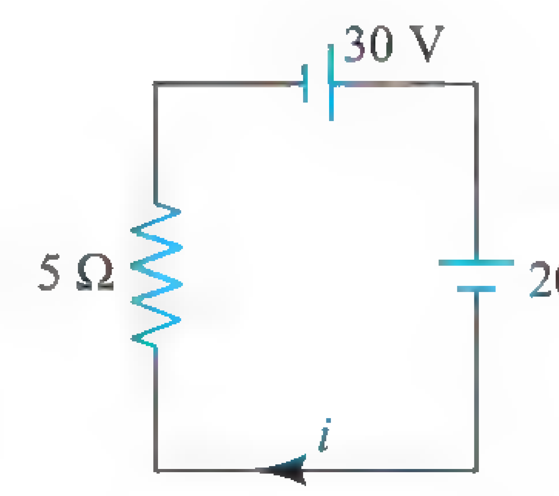
$$\text{emf} = A\frac{dB}{dt} = 2 \times 10 = 20\ \text{V}$$

$\therefore\ i = 20/5 = 4$ A.

From lenz's law, direction of current will be anticlockwise.

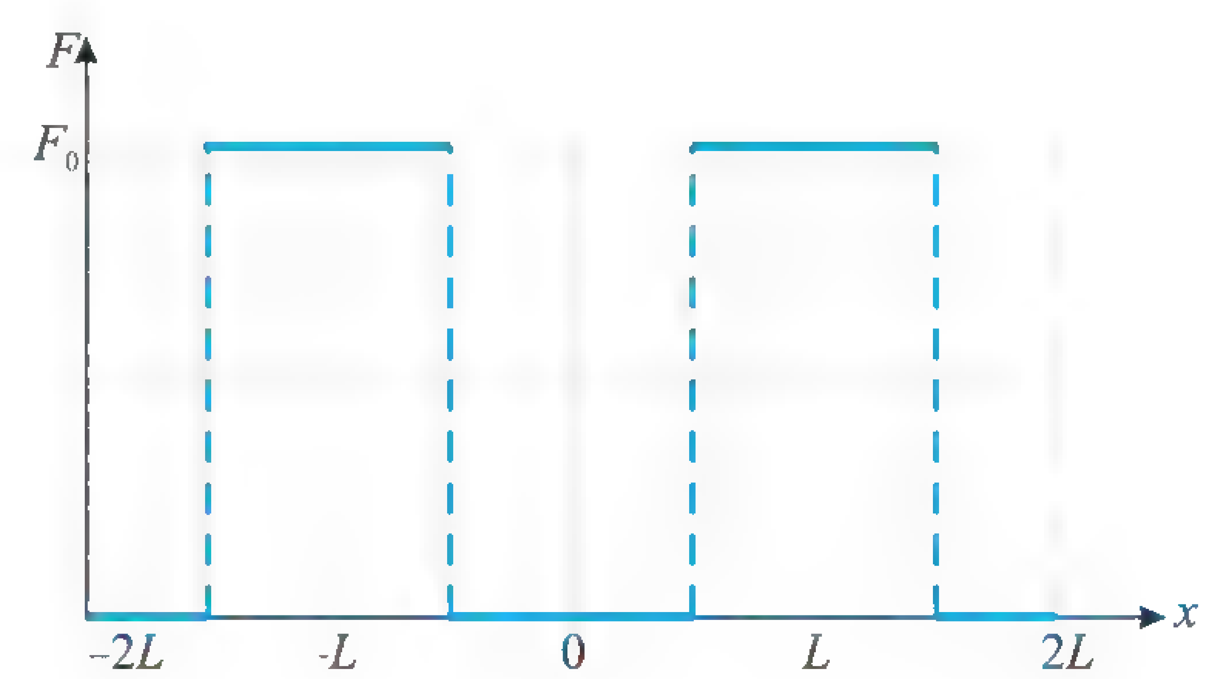
8. Induce emf = 20 V

The equivalent circuit can be drawn as

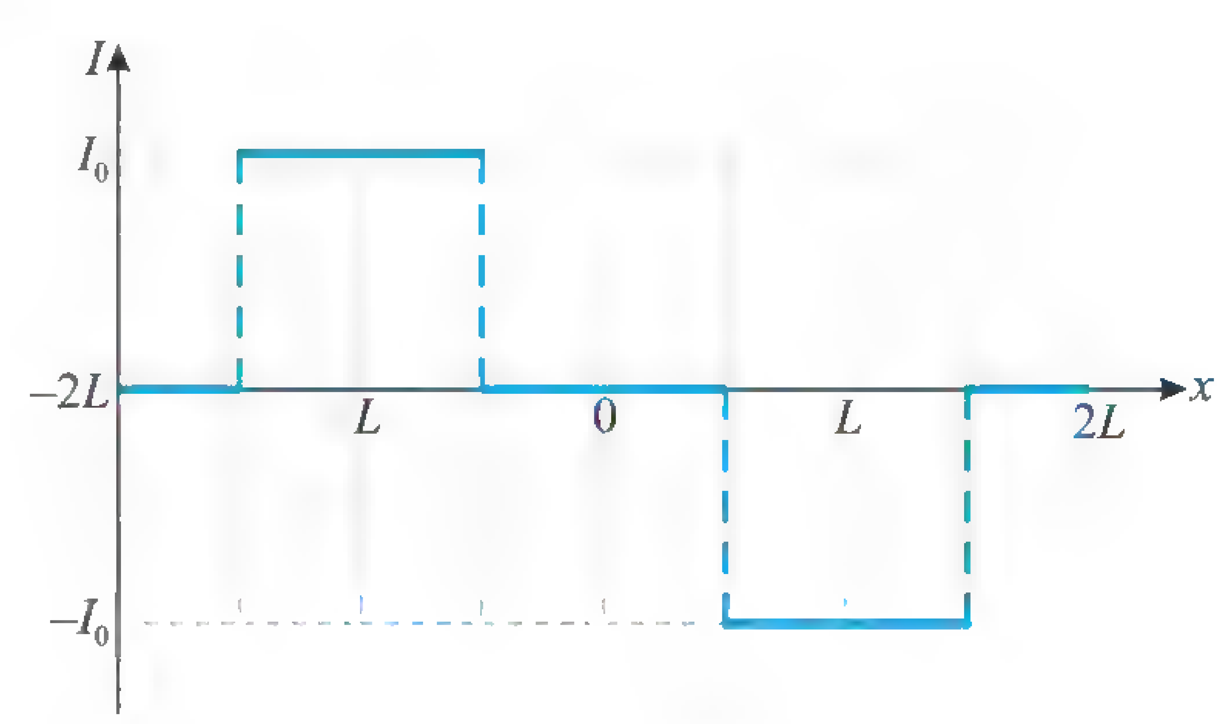


$$\text{current } i = \frac{30-20}{5} = 2\ \text{A clockwise.}$$

9. (a)



(b)



Here, $\varepsilon = vBL = I_0 R \ \Rightarrow\ I_0 = \dfrac{vBL}{R}$

and $F_0 = I_0 LB = \dfrac{vB^2L^2}{R}$

10. $$e_{av} = \frac{\Delta\phi}{\Delta t} = \frac{B(\text{change in area})}{\Delta t} = \frac{0.3\left[2\left(\frac{\sqrt{3}}{4}L^2\right)\right]}{0.10} = 5.84\ \text{V}$$

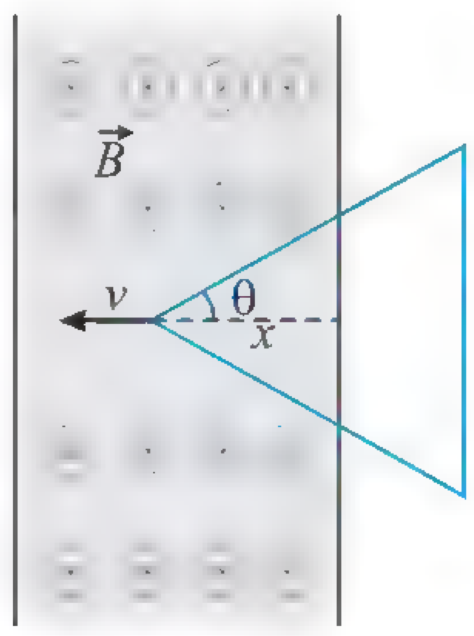
11. Area $A = a^2 = (0.20)^2 = 0.04\text{ m}^2$

$$\phi = AB = 15 \times 10^{-6} \times 0.04 = 6 \times 10^{-7}\text{ Wb}$$

$$q = \frac{\Delta\phi}{R} = \frac{6 \times 10^{-7}}{0.50} = 12 \times 10^{-7}\text{ C} = 1.2\ \mu\text{C}$$

12. (a) The area exposed to the magnetic field,

$$A = (2)\left(\frac{1}{2}x\right)(x\tan\theta) = x^2\tan\theta$$



The magnitude of induced EMF at any time '*t*',

$$|\varepsilon| = B\frac{dA}{dt} = 2Bx\tan\theta \cdot \frac{dx}{dt}, \text{ where } \frac{dx}{dt} = v$$

or $\quad |\varepsilon| = 2Bvx\tan\theta = 2(Bv^2\ \tan\theta)t$

(b) Induced current $i = \dfrac{|\varepsilon|}{R} = \dfrac{2Bv^2\tan\theta}{R}t$,

From Lenz law current in the loop will be in counter clockwise sense.

13. Flux: $\phi = B \cdot \pi r^2$

Induced emf: $\varepsilon = \left|\dfrac{d\phi}{dt}\right| = 2\pi Br\dfrac{dr}{dt} = 2\pi Brn$

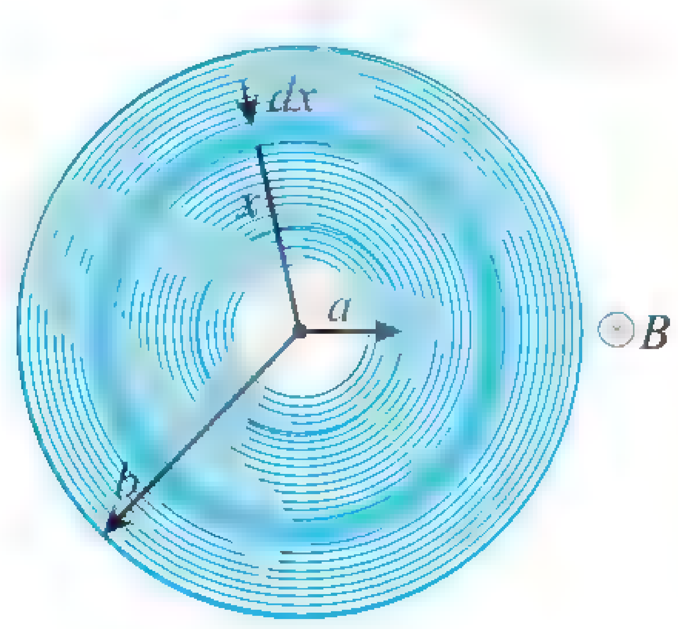
Induced current: $I = \dfrac{\varepsilon}{R} = \dfrac{2\pi Brn}{R}$

Rate of work done = Rate of heat dissipated in Joule heating

$$= I^2R = \frac{4\pi^2B^2r^2n^2}{R}$$

14. Consider a circular strip of radius x and width dx.

Number of turns in the strip $dN = \dfrac{Ndx}{(b-a)}$



Flux linked with a circular loop of radius x is $\phi = B \cdot \pi x^2$

$$\Rightarrow \quad \frac{d\phi}{dt} = \pi x^2\frac{dB}{dt}$$

Emf induced in coil of width dx will be

$$d\varepsilon = (dN)\left(\frac{d\phi}{dt}\right) = \frac{N\,dx}{b-a} \cdot \pi x^2\frac{dB}{dt}$$

$\therefore$ Emf induced in the complete coil is

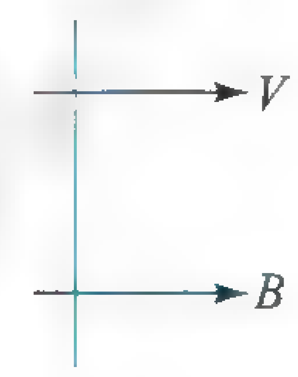
$$\varepsilon = \int d\varepsilon = \frac{\pi N}{(b-a)}\frac{dB}{dt}\int_a^b x^2dx$$

$$= \frac{\pi N}{(b-a)}\frac{dB}{dt} \cdot \frac{(b^3-a^3)}{3} = \frac{\pi N\alpha}{3}(a^2+b^2+ab)$$

Exercise 4.2

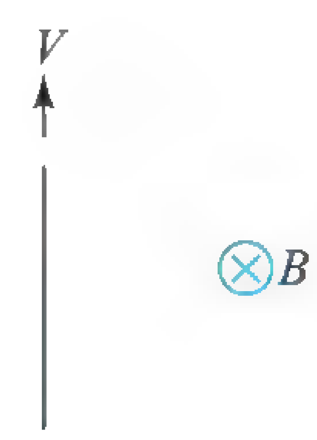
1. In loop (a), no emf will be induced because there is no flux change. In loop (b) emf will be induced because the coil is moving in a region of decreasing magnetic field in inward direction. Therefore, to oppose the decrease in flux in inward direction, current will be induced such that its magnetic field will be inwards. For this, direction of current should be clockwise.

2. (a)



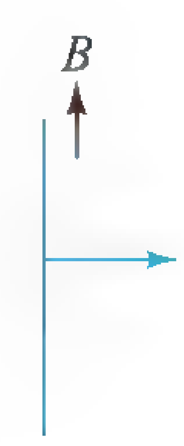
emf $= 0$ as $\vec{B} \parallel \vec{v}$

(b)



emf $= 0$ as $\vec{\ell} \parallel \vec{v}$

(c)



emf $= 0$ as $\vec{\ell} \parallel \vec{B}$

3. Consider rod AB, which is a part of the coil. emf induced in the rod $= BLv$.

Suppose the emf induced in part ACB is E, as shown.

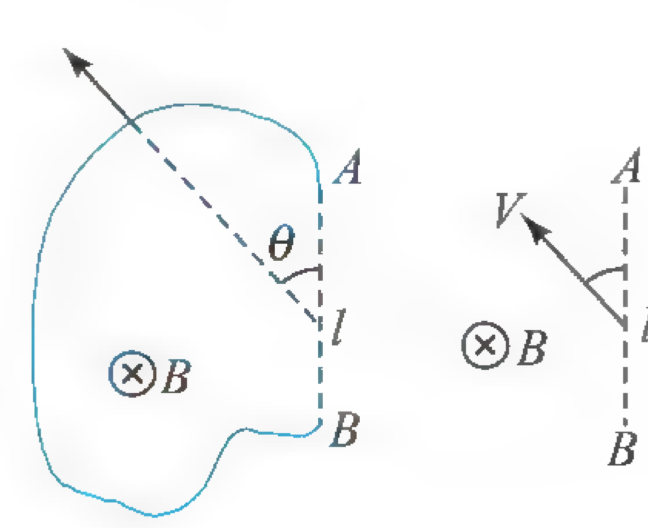
Since the emf in the coil is zero, emf (in ACB) + emf (in BA) = 0

or $\quad -E + vBL = 0$

or $\quad E = vBL$

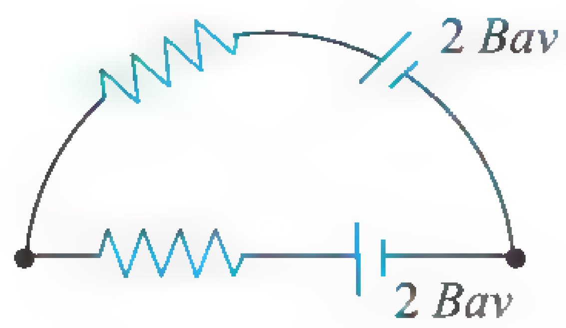
Thus, emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also, the equivalent emf between A and B is BLv (here the two emf's are in parallel).

4. The same emf will be induced in the straight imaginary wire joining A and B, which is $Bv\ell\sin\theta$.

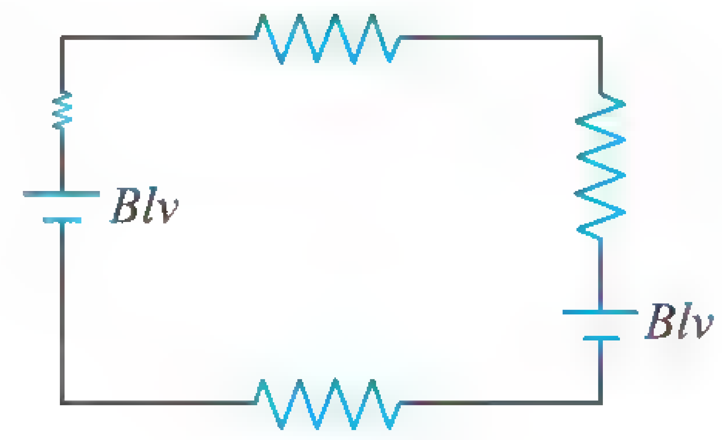


5. Induced emf $= 0$, because V is parallel to diameter.

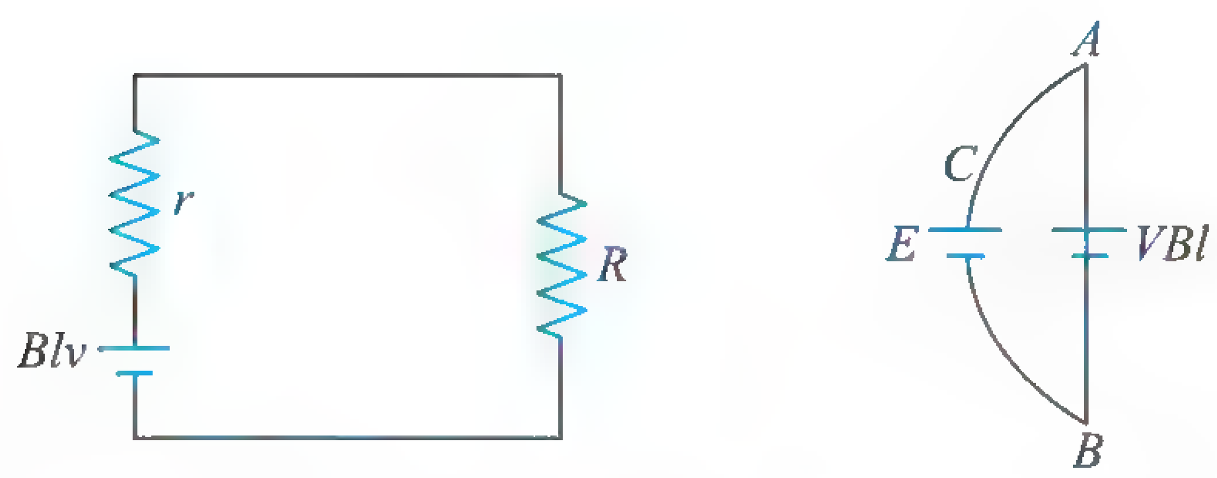
6. Induced emf = $2Bav$



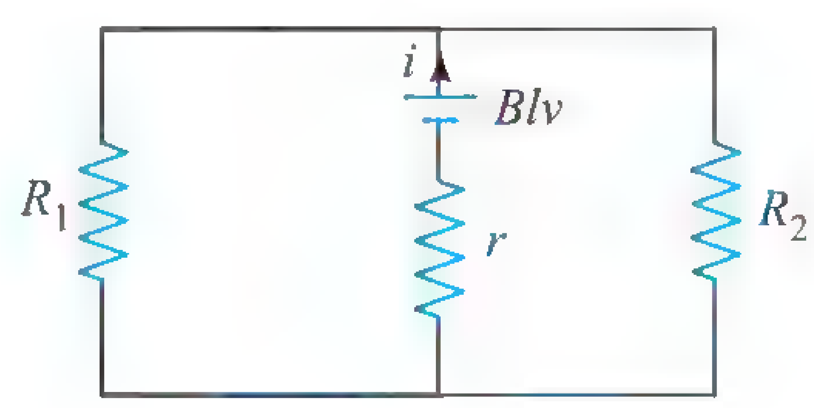
7. Net induced emf is zero.



8.

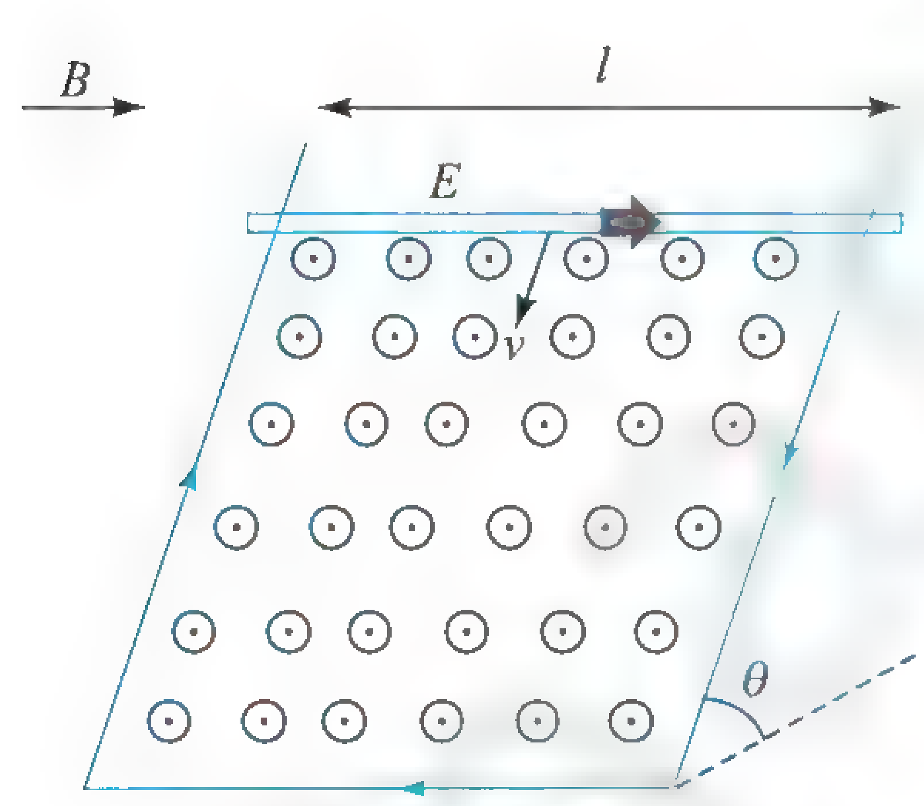


9. The given circuit is equivalent to the following diagram:

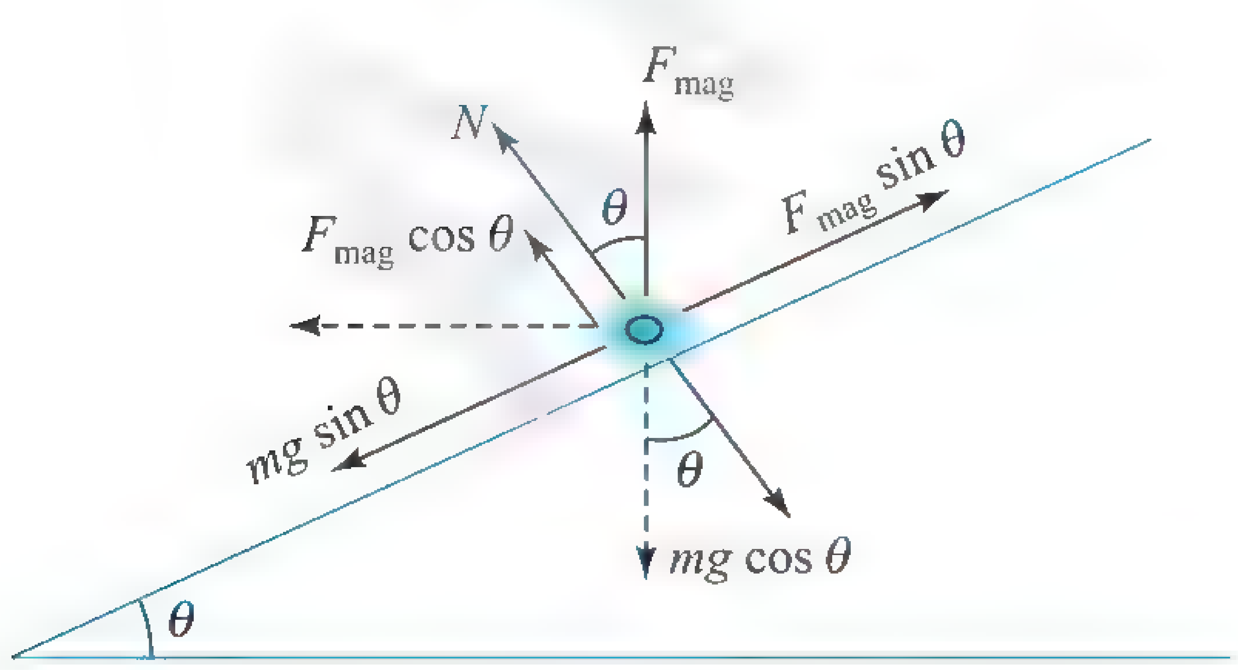


$$i = \frac{B\ell v}{r + \frac{R_1R_2}{R_1 + R_2}}$$

10.



Conducting wire



The emf induced in the rod,

$$E = (B \sin\theta)lv = Blv \sin\theta$$

The current in the circuit $I = \dfrac{E}{R} = \dfrac{Blv\sin\theta}{R}$

Magnetic force acting on the sliding rod, $F_{mag} = IlB$

$$F_{mag} = \left(\frac{Blv\sin\theta}{R}\right)lB = \frac{B^2l^2v\sin\theta}{R}$$

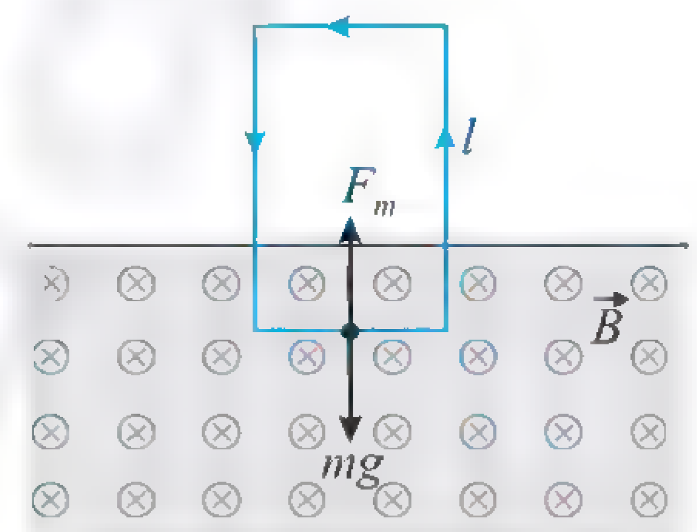
If the rod is sliding with constant velocity,

$$F_{mag} \sin\theta = mg \sin\theta$$

$$\Rightarrow \quad \frac{B^2l^2v\sin^2\theta}{R} = mg \sin\theta$$

$$\Rightarrow \quad B = \sqrt{\frac{mgR}{l^2v\sin\theta}}$$

11. For the loop to move with constant velocity v in the magnetic field, the net force acting on it must be zero. Hence, magnetic force nullifies the effect of gravity.

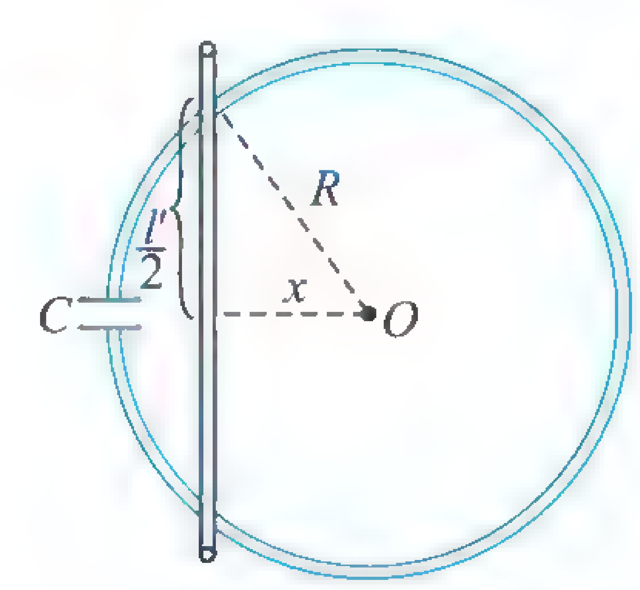


Then, $F_m = mg$; Substituting $F_m = \dfrac{B^2l^2v}{R}$, we obtain $v = \dfrac{mgR}{B^2l^2}$.

This is the terminal speed. This velocity is attained by the loop after falling through a height $h = \dfrac{v^2}{2g}$ in gravity.

Substitution of v yields $h = \dfrac{m^2gR^2}{2B^4l^4}$

12. (a) The length of the rod in the magnetic field , $l' = 2\sqrt{R^2 - x^2}$



Then, the induced emf, $\varepsilon = Bl'v = 2Bv\sqrt{R^2 - x^2}$

The change stored in the capacitor is $q = C\varepsilon = 2BCv\sqrt{R^2 - x^2}$

The current in the circuit is

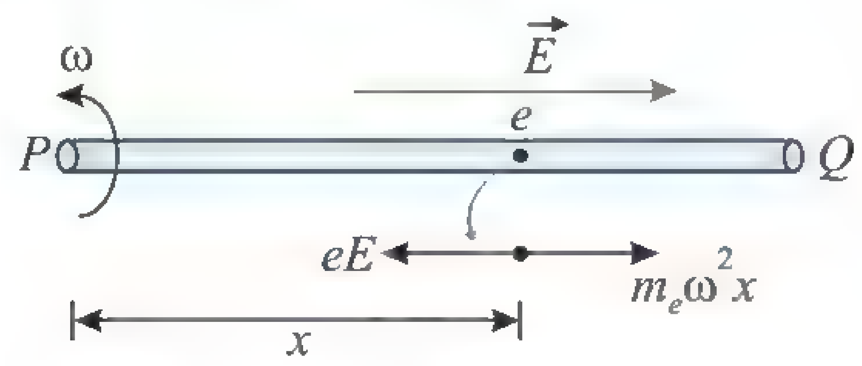
$$i = \left|\frac{dq}{dt}\right| = \left|2BCv\frac{d}{dt}\left(\sqrt{R^2 - x^2}\right)\right| = \left|2BCv\left\{\frac{-2x\left(\frac{dx}{dt}\right)}{2\sqrt{R^2 - x^2}}\right\}\right|$$

$$i\Big|_{x=\frac{R}{2}} = \frac{BCv^2x}{\sqrt{R^2 - x^2}}\Bigg|_{x=\frac{R}{2}} = \frac{2BCv^2}{\sqrt{3}R}$$

(b) $i = \dfrac{\varepsilon}{R_0} = \dfrac{2Bv}{R_0}\sqrt{R^2 - (R - vt)^2}$

Exercise 4.3

1. Flux passing through the ring, $\phi = BA$ is constant here, therefore, emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.
2. The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces and a steady state will be reached. In the steady state, $m_e\omega^2 x = eE$.



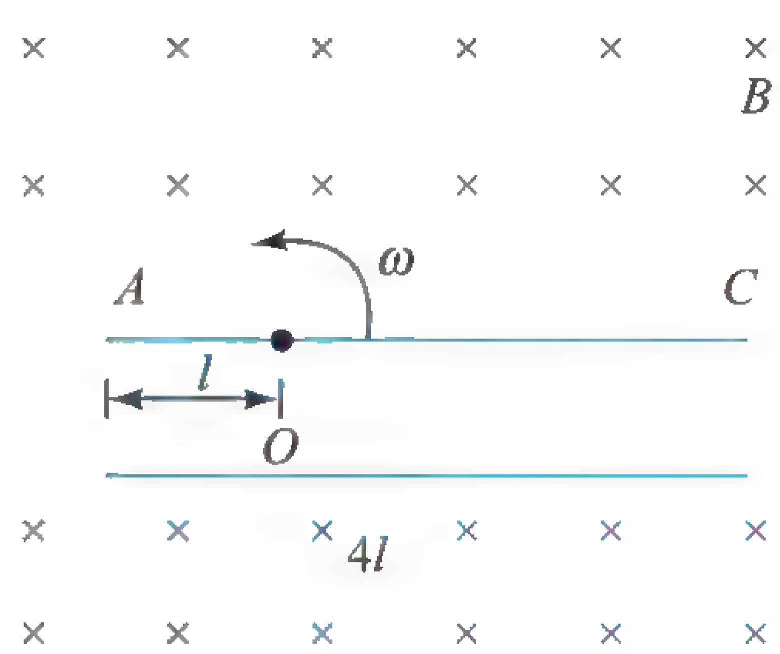
$$V_P - V_Q = \int_{x=0}^{x=\ell} \vec{E}\cdot d\vec{x} = \int_0^{\ell} \frac{m_e\omega^2 x}{e}\,dx = \frac{m_e\omega^2\ell^2}{2e}$$

3. Rotational emf will be induced due to rotation of the rod.

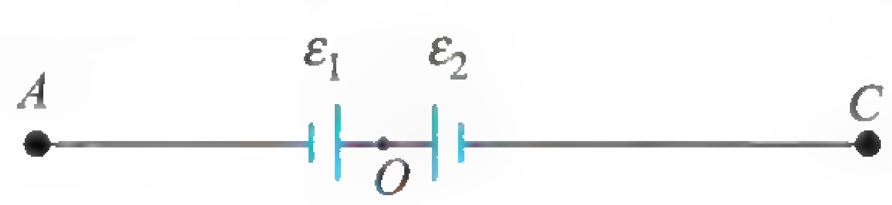
Emf of part OA is

$$\varepsilon_1 = \frac{1}{2}Bl^2\omega$$

Emf of part OC is $\varepsilon_2 = \dfrac{9}{2}Bl^2\omega$



The circuit can be separated as



$$V_A + \varepsilon_1 - \varepsilon_2 = V_C$$

$$V_A - V_C = \varepsilon_2 - \varepsilon_1 = \frac{9}{2}Bl^2\omega - \frac{1}{2}Bl^2\omega = 4\,Bl^2\omega$$

$$V_A - V_C = 4\,B\omega l^2$$

4. At any time t, $\phi = BA\cos\theta = BA\cos\omega t$

Now, induced emf in the loop

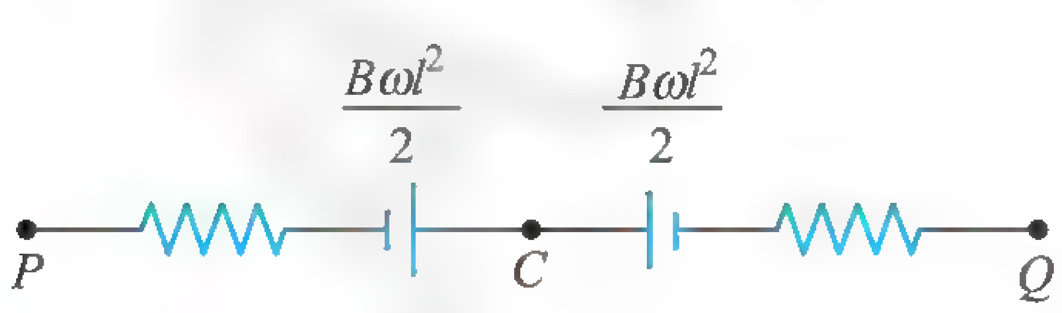
$$e = -\frac{d\phi}{dt} = BA\,\omega\sin\omega t$$

5. $E_{PQ} = \dfrac{1}{2}B\omega(2\ell)^2 = 100\text{ V} \Rightarrow B\omega\ell^2 = 50\text{ V}$

$$E_{PM} = \frac{1}{2}B\omega(\ell)^2 = 25\text{ V}$$

$$E_{MQ} = E_{PQ} - E_{PM} = 100 - 25 = 75\text{ V}$$

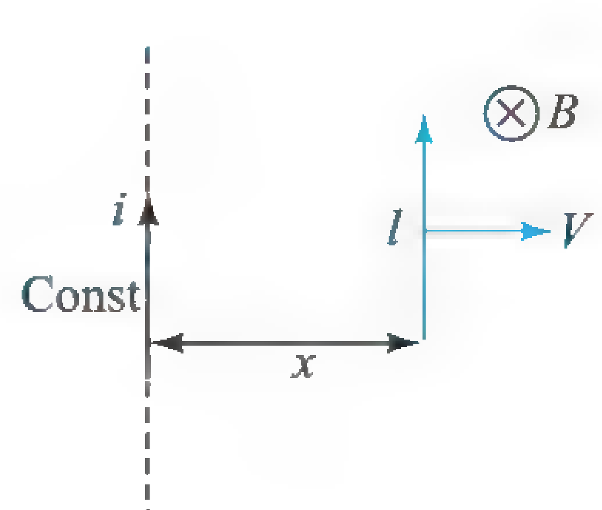
6. $\text{emf}_{PQ} = 0$; $\text{emf}_{PC} = \dfrac{B\omega\ell^2}{2}$



7. $E = Blv = \dfrac{\mu_0 ilv}{2\pi x}$

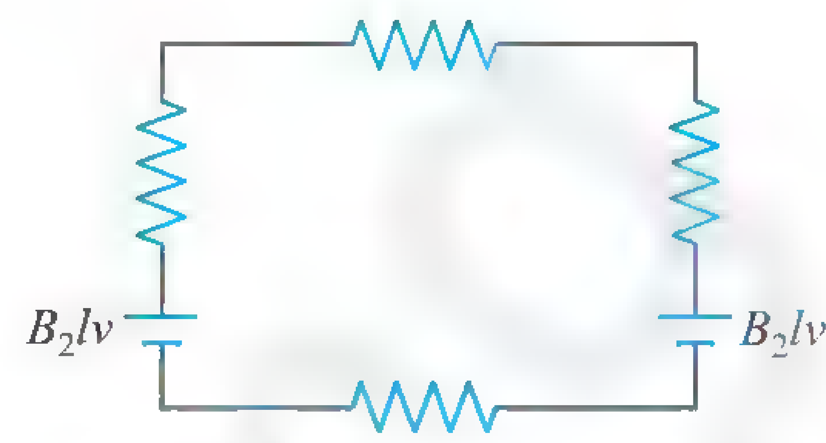
Alternatively:

emf is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is $lv\,dt$. The magnetic field lines cut in dt time $= Blv\,dt = \dfrac{\mu_0 i l v\,dt}{2\pi x}$.



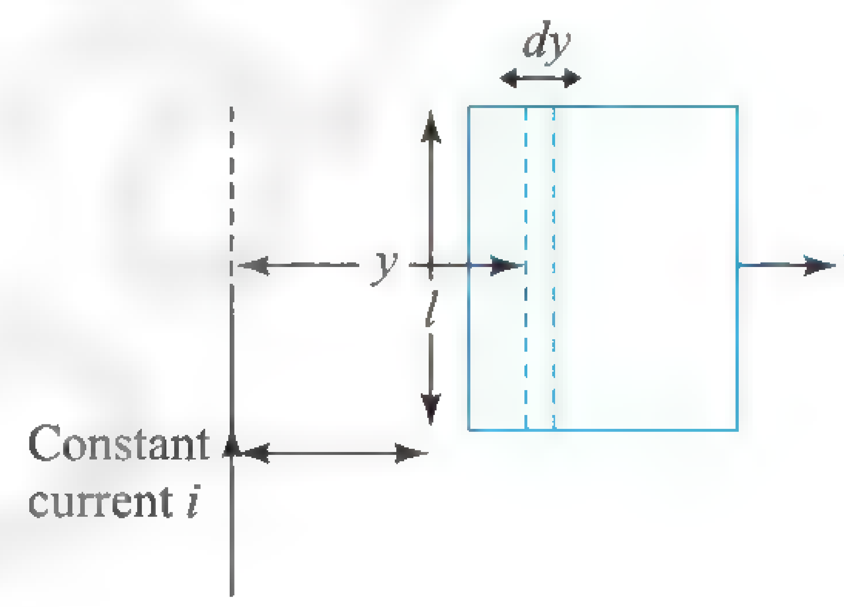
$\therefore$ The rate at which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$ and this will be induced e.m.f.

8. $E = B_1\,lv - B_2\,lv = \dfrac{\mu_0 i}{2\pi x}lv - \dfrac{\mu_0 i}{2\pi(x+b)}lv = \dfrac{\mu_0 ilbv}{2\pi x(x+b)}$



Alternatively:

Consider a small segment of width dy at a distance y from the wire. Let flux through the segment be $d\phi$.



$$\therefore\quad d\phi = \frac{\mu_0 i}{2\pi y}l\,dy$$

$$\therefore\quad \phi = \frac{\mu_0 il}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 il}{2\pi}\left(\ln(x+b) - \ln x\right)$$

Now,

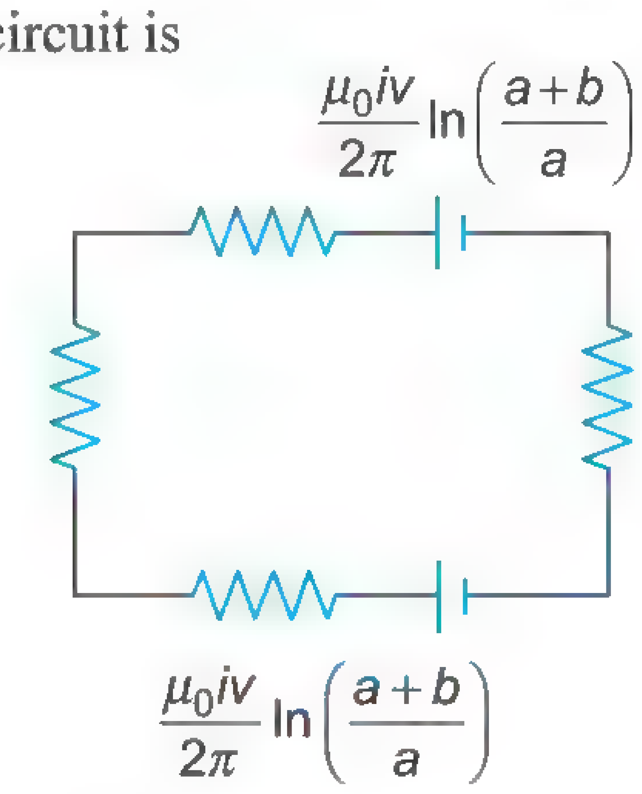
$$\frac{d\phi}{dt} = \frac{\mu_0 il}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]$$

$$= \frac{\mu_0 il}{2\pi}\left[\frac{(-b)}{x(x+b)}\right]v = \frac{-\mu_0 iblv}{2\pi x(x+b)}$$

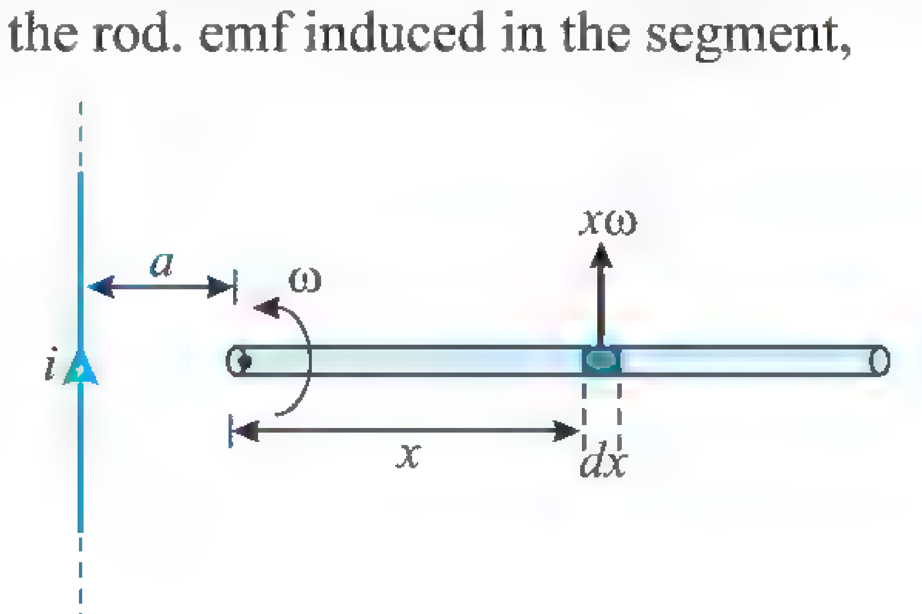
$$\therefore\quad \text{Induced emf} = \frac{\mu_0 iblv}{2\pi x(x+b)}$$

9. Net emf $= 0$

The equivalent circuit is



10. Consider a small segment of rod of length dx, at a distance x from one end of the rod. emf induced in the segment,



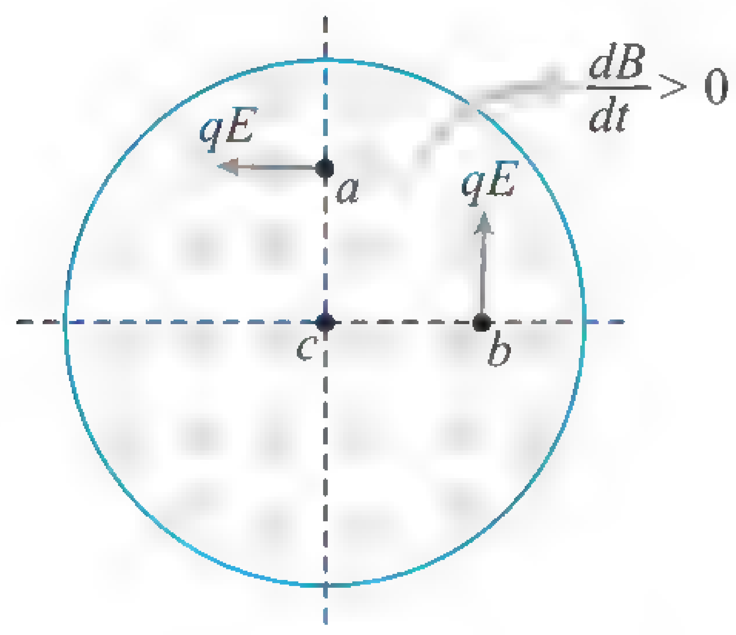
$$d\varepsilon = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

$$\therefore \quad \varepsilon = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}(x\omega)\,dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a\ln\left(\frac{\ell+a}{a}\right)\right]$$

Exercise 4.4

1. Since the field is increasing in inward direction, so electric field will be in anticlockwise direction. So, force on a and b are as shown in figure.

$$E2\pi r = \pi r^2 \frac{dB}{dt} \Rightarrow E = \frac{r}{2}\frac{dB}{dt}$$



So force on a and b: $F = qE = \dfrac{qr}{2}\dfrac{dB}{dt}$

At centre, electric field will be zero, so no force on charge at c.

2. $\oint \vec{E}\cdot d\vec{s} = -\dfrac{d\phi}{dt} = -A\dfrac{dB}{dt}$

For loop a: $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d(\vec{B}_1\cdot\vec{A}_1)}{dt} = -\dfrac{d(B_1A_1\cos 180°)}{dt}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = A_1\frac{dB_1}{dt} = \pi r_1^2(-8.0\text{ mT/s})$$

$$= -\pi(20.0\times10^{-2})^2(8.0\times10^{-3})$$

$$= -3.2\pi\times10^{-4}\text{ V}$$

For loop b: $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d(\vec{B}_2\cdot\vec{A}_2)}{dt} = -A_2\dfrac{dB_2}{dt}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = -\pi\left(\frac{25}{100}\right)^2\times\left(-8.0\times10^{-3}\right) = +5.0\pi\times10^{-4}\text{ V}$$

For loop c: $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d(\vec{B}\cdot\vec{A})}{dt} = -\dfrac{d(\vec{B}_1\cdot\vec{A}_1 + \vec{B}_2\cdot\vec{A}_2)}{dt}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = -\left[(-3.2\pi\times10^{-4}) + (5.0\pi\times10^{-4})\right]$$

$$= -1.8\pi\times10^{-4}\text{ V}$$

3. $\oint_1 \vec{E}\cdot d\vec{l} = mag$

$$\oint_3 \vec{E}\cdot d\vec{l} = 3(mag)$$

It means field in b and c should be out of the page.

$$\oint_4 \vec{E}\cdot d\vec{l} = 0$$

It means field in e will be opposite to that in c, i.e., into the page. Similarly field into d is also into the page.

4. (a) At point P_2 the induced electric field strength is given as

$$E = \frac{R^2}{2r_2}\left(\frac{dB}{dt}\right)$$

$$F = qE = \frac{eR^2}{2r_2}(6t^2 - 8t)$$

Substituting the values we get

$$F = \frac{(1.6\times10^{-19})(2.5\times10^{-2})}{2\times5\times10^{-2}}[6(2)^2 - 8(2)]$$

$$\Rightarrow \quad F = 8.0\times10^{-21}\text{ N}$$

(b) At P_1 being a point inside of the magnetic field, the induced electric field at this point is given as

$$E = \frac{r_1}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{r_1}{2}[6t^2 - 8t]$$

$$\Rightarrow \quad E = \frac{0.02}{2}[6(3)^2 - 8(3)]$$

$$\Rightarrow \quad E = 0.3\text{ V/m}$$

5. (a) As the loop is lying outside the magnetic field region. There is no flux linked to the loop. It means no emf is induced.

(b) Electric field at a distance r from center O outside the magnetic fieldis given by

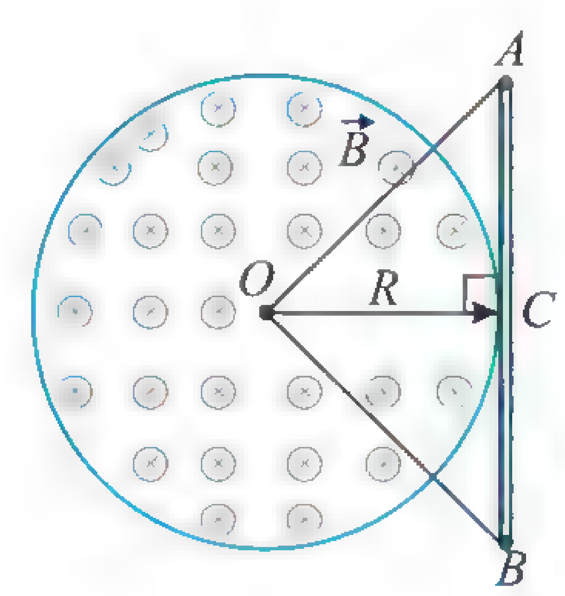
$$E = \frac{R^2}{2r}\frac{dB}{dt} \text{ (in tangential direction)}$$

Emf induced in arc AB $\varepsilon_{AB} = E.(b\theta) = \left(\dfrac{R^2}{2b}\dfrac{dB}{dt}\right)(b\theta) = \dfrac{R^2\theta\beta}{2}$

(c) Electric field lines are normal to arms BC. There is no emf in this arm.

6. (a) As emf induced in time varying magnetic field region is given by $e = \oint \vec{E}\cdot\vec{dl}$. Here the electric field lines are perpendicular to the rod at all points in Fig. (a), hence no emf will be induced.

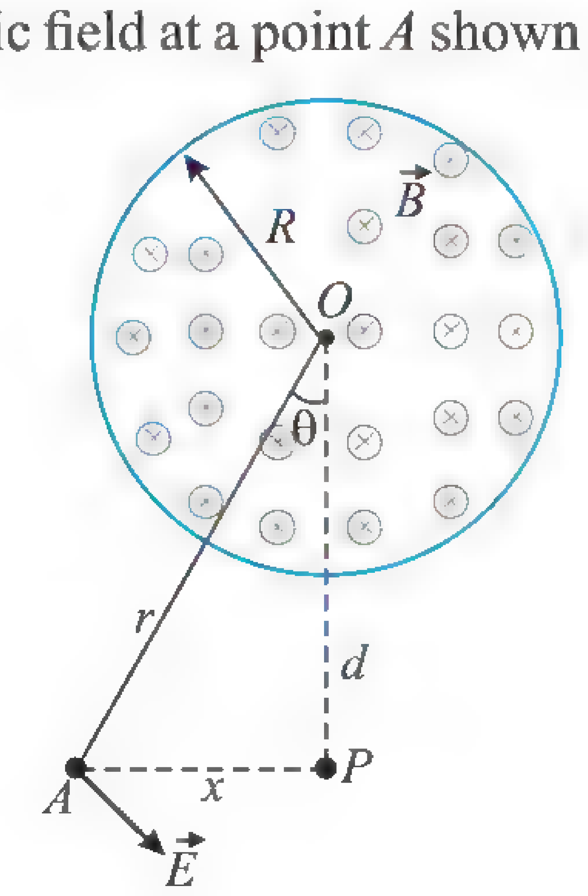
(b) Emf induced in AB = rate of change of flux through the triangular region.



$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\pi R^2}{4}\cdot\alpha$$

7. (a) Point P is situated outside the time varying magnetic field region. At this point the induced electric field is perpendicular to OP at all points in this path. The force and displacement both are perpendicular to each other. Hence the work done in moving it to infinity is zero.

(b) Induced electric field at a point A shown in the figure is given by



$$E = \frac{R^2}{2r}\left(\frac{dB}{dt}\right) = \frac{R^2\beta}{2r},$$

which is constant hence the electric force on the charge along AP is

$$F_e = qE\cos\theta = \frac{qR^2\beta\cdot d}{2r^2}$$

It means the external agent must apply equal and opposite force to keep the charge moving without gaining any kinetic energy. Work done by the external agent in small displacement dx will be

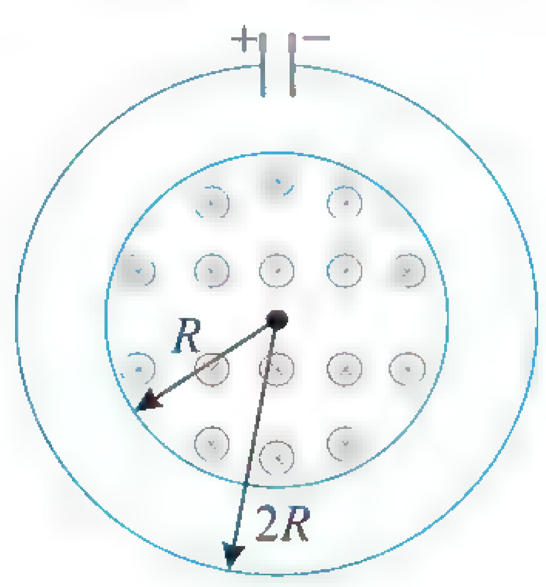
$$dW = \frac{qR^2\beta d}{2r^2}dx$$

But $x = d\tan\theta \Rightarrow dx = d\sec^2\theta\, d\theta$ and $r = d\sec\theta$

$$dW = \frac{qR^2\beta d}{2(d\sec\theta)^2}\cdot d\sec^2\theta\, d\theta = \frac{qR^2\beta}{2}d\theta$$

$$\therefore \quad W = \frac{qR^2\beta}{2}\int_0^{\pi/2} d\theta = \frac{\pi qR^2\beta}{4}$$

8. (a) The flux passing through the loop is



$$\phi = \pi R^2 B = \pi R^2 B$$

Where $B = \mu_0 ni$

Then $\varepsilon = \left|\frac{d\phi}{dt}\right| = \pi R^2\frac{dB}{dt} = \frac{\pi R^2\mu_0 n i_0}{T}$

The charge in the capacitor is $q = CV = \frac{\varepsilon_0 A}{d}\varepsilon$

$$q = \frac{\pi R^2\mu_0 n i_0\varepsilon_0 A}{dT}$$

(b) The static electric field in the capacitor is

$$E_{\text{static}} = \frac{q}{A\varepsilon_0} = \frac{\mu_0\pi R^2 n i_0}{dT}$$

(c) The induced electric field inside the capacitor ,

$$E_{\text{induced}} = \frac{R^2}{2r}\left(\frac{dB}{dt}\right) = \frac{R^2}{2.2R}\left(\frac{\mu_0 n i_0}{T}\right)$$

$$\Rightarrow \quad E_{\text{induced}} = \frac{R\mu_0 n i_0}{4T}$$

(d) $E_{\text{net}} = |\vec{E}_S + \vec{E}_i| = \left|\frac{\mu_0\pi R^2 n i_0}{dT} - \frac{R\mu_0 ni}{4T}\right|$

$$E_{\text{net}} = \frac{\mu_0 niR}{T}\left[\frac{\pi R}{d} - 1\right]$$

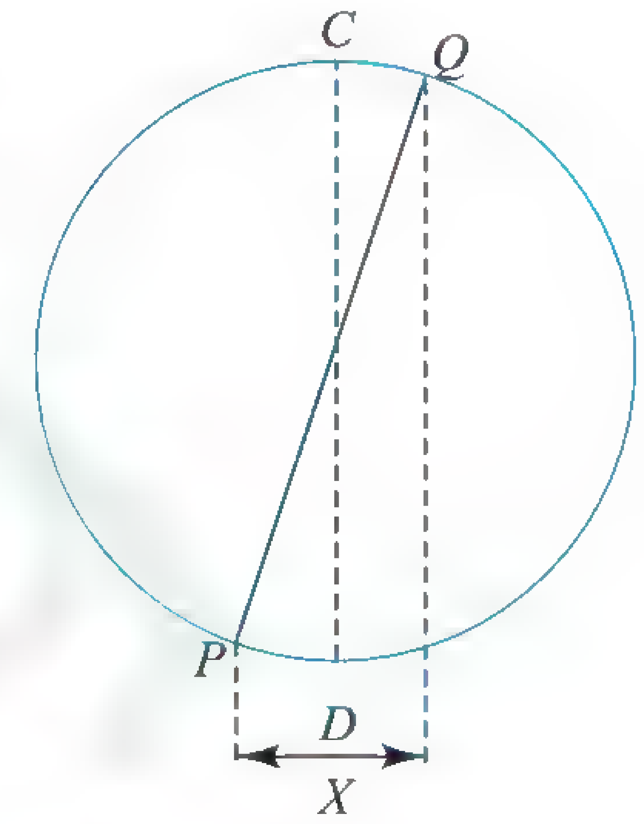
Exercises

Single Correct Answer Type

1. (1) Consider the force on an electron in PQ. This electron experiences a force towards Q. Free electrons in PQ tend to move towards N. So M will be positively charged.

2. (2) Apply Lenz's law.

3. (2) When the ring falls vertically, there will be an induced emf across A and B ($e = Bv(2r)$).



Note that there will be a potential difference across any two points on the ring, and the line joining these has a projected length in the horizontal plane. For example, between points P and Q there is a projected length x in the horizontal plane.

$\therefore$ P.D. across P and Q is

$V = B\,vx$

But for points C and D, $x = 0$

Therefore, P.D. = 0

Hence (2)

4. (3) Because A and C are at equal distance from B, and their flux across B is in opposite direction, so at any time flux in B will be zero. Hence no emf is induced.

5. (2) In the r–t graph, it is clear that from a to b there is no change in radius and hence no change in area and magnetic flux. Same is the situation from c to d.

Now, $|e| = \frac{d}{dt}(\phi)$

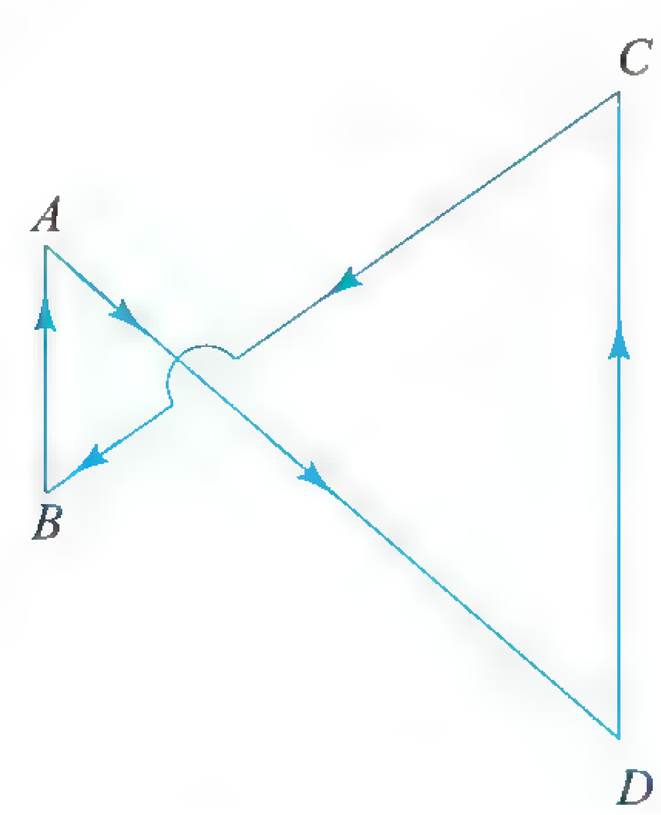
$$|e| = B\frac{d}{dt}(\pi r^2) = B\pi 2r\frac{dr}{dt}$$

Since $r \propto t$, $\therefore \frac{dr}{dt}$ = constant

$\therefore \quad |e| \propto r$

6. (1) Magnetic field in $\otimes$ direction is increasing. Therefore, induced current will produce magnetic field in $\odot$ direction. Thus, current in both the loops should be anticlockwise. But as the area of the loop on the right side is more, induced emf in this side will be more compared to the left side loop.

Therefore, net current in the complete loop will be in a direction shown below:



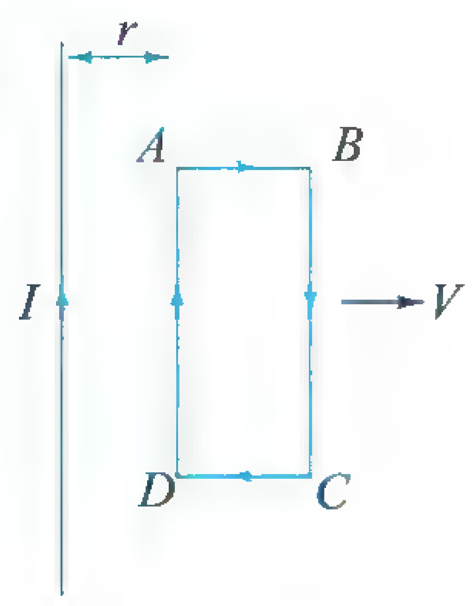
7. (4) When the coil is within the field, there is no change in the magnetic flux passing through it. Thus, no current will be induced and the acceleration will be g. But according to Lenz's law, the induced current will oppose its motion when it enters or leaves the field. Therefore, acceleration will be less than g.

8. (4) Magnetic flux in $\otimes$ direction through the coil is increasing. Therefore, induced current will produce magnetic field in $\odot$ direction. Thus, the current in the loop is anticlockwise. Magnitude of induced current at any instant of time is

$$i = \frac{e}{R} = \frac{Bv(FG)}{\rho(FG + GD + DF)}$$

When the wire AH moves downwards FG, GD and DF all increase in the same ratio. Therefore, i is constant.

9. (4)



As the flux decreases, to maintain flux, current in the loop is clockwise. Force on DA due to the long wire is towards left while on BC is towards right.

10. (4) Induced emf depends upon vertical edge.

11. (4) $Bl\ Vt$ = constant

$$B = \frac{C}{\ell Vt}$$

12. (3) When the electron is closest, flux due to magnetic field of electron's motion is maximum through the loop. So slope of flux–time graph will be zero or induced emf will be zero.

13. (2) As anticlockwise direction is positive, so area vector outwards is positive. So net flux through the given loop is

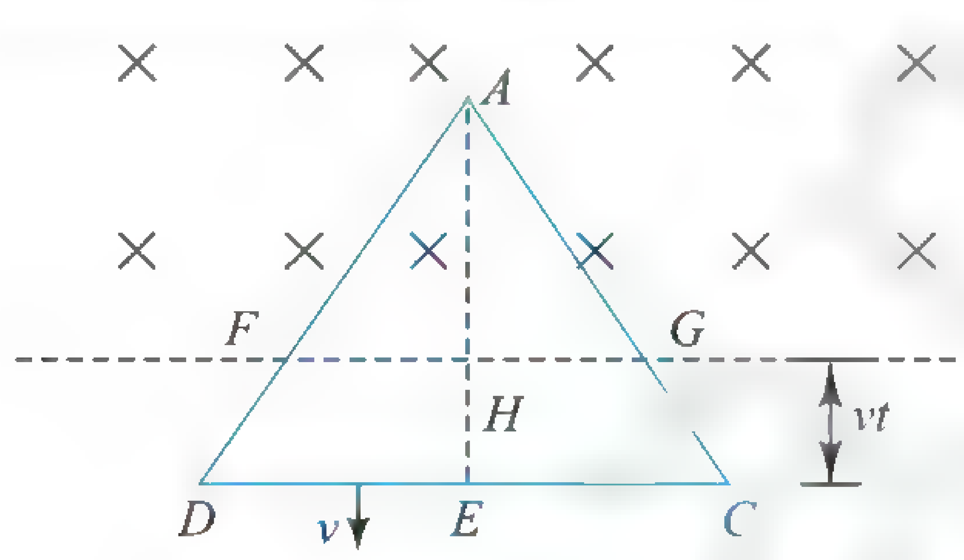
$$\phi = -BA - BA + BA = -BA$$

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\phi}{dt} = -\frac{d(-BA)}{dt} = A\frac{dB}{dt} = A(-\alpha)$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{r} = -\alpha A$$

14. (1) From Lenz's law if one rod is moved away from the second rod then the second rod will be attracted towards the first rod so as to oppose the change in flux.

15. (2) Let $2a$ be the side of the triangle and b be the length AE.



$$\frac{AH}{AE} = \frac{GH}{EC} \quad \Rightarrow \quad GH = \left(\frac{AH}{AE}\right)EC$$

or $\quad GH = \dfrac{(b - vt)}{b}.a = a - \left(\dfrac{a}{b}vt\right)$

$\therefore \quad FG = 2GH = 2\left[a - \dfrac{a}{b}vt\right]$

$\therefore \quad$ Induced emf, $e = Bv\,(FG) = 2Bv\left(a - \dfrac{a}{b}vt\right)$

$\therefore \quad$ Induced current, $i = \dfrac{e}{R} = \dfrac{2Bv}{R}\left[a - \dfrac{a}{b}vt\right]$

or $\quad i = k_1 - k_2\,t$

Thus i–t graph is a straight line with negative slope and positive intercept.

16. (3) The change in magnetic flux is zero, hence the current in the ring will be zero.

17. (2) $P = Fv = BI\ell v = 1.25 \times 10^{-3} \times 50 \times 0.1 \times 1$ W

$= 6.25 \times 10^{-3}$ W $= 6.25$ mW

18. (1) $|\vec{E}|$ = Magnitude of induced emf

$$= \frac{B\ell^2}{2}\omega,\ \ell = \sqrt{2}\,r$$

19. (3) $BI\ell = mg \quad$ or $\quad B\dfrac{Bv\ell}{R}\ell = mg \quad$ or $\quad v = \dfrac{mg\,R}{B^2\ell^2}$

20. (1) Speed of the loop should be

$$v = \frac{\ell}{t} = \frac{0.5}{2} = 0.25 \text{ m s}^{-1}$$

Induced emf, $e = Bv\ell = (1.0)\ (0.25)\ (0.5) = 0.125$ V

$\therefore \quad$ Current in the loop, $i = \dfrac{e}{R} = \dfrac{0.125}{10}$

$= 1.25 \times 10^{-2}$ A

The magnetic force on the left arm due to the magnetic field is

$F_m = i\ell B = (1.25 \times 10^{-2})\ (0.5)\ (1.0)$

$= 6.25 \times 10^{-3}$ N

To pull the loop uniformly an external force of 6.25×10^{-3} N towards right must be applied.

$\therefore \quad W = (6.25 \times 10^{-3}\text{N})\ (0.5\text{m}) = 3.125 \times 10^{-3}$ J

21. (3) Use Lenz's law. Induced emf of the current opposes the change in flux through it.

22. (3) Magnetic field due to larger loop

$$= \frac{\mu_0 1}{2r} = \frac{4\pi \times 10^{-7} \times 1}{2 \times 0.1}\text{ T } = 2\pi \times 10^{-6}\text{ T}$$

Now, magnetic flux linked with the smaller loop, ϕ

$= NBA \cos \omega t = 1 \times 2\pi \times 10^{-6} \times 5 \times 10^{-4} \cos \omega t$

$= \pi \times 10^{-9} \cos \omega t$ weber

23. (1) $ma_0 = eE \quad \Rightarrow \quad E = \dfrac{ma_0}{e}$

24. (1) $I = \dfrac{B\ell v}{R}$

$$I = \frac{5 \times 10^{-2} \times 0.3 \times 0.2}{5}\text{ A } = 0.6\text{ mA.}$$

Area and flux are decreasing. So, current flows to increase the flux. Clearly, current should be clockwise. So, it flows from B to C through 5 Ω.

25. (2) ϕ (flux linked) $= a^2B \cos 0° - b^2B \cos 180°$

$= (a^2 - b^2)B$

$$E = -\frac{d\phi}{dt} = -(a^2 - b^2)\frac{dB}{dt}$$

$= (a^2 - b^2)\,B_0\,\omega \cos \omega t$

where $B = B_0 \sin \omega t$, $B_0 = 10^{-3}$ T, $\omega = 100$

$\therefore \quad I_{max} = (a^2 - b^2)\dfrac{B_0\omega}{R}$

and $\quad R = (4a + 4b)\,r = 4\,(a + b)\,r$

$\therefore \quad I_{max} = \dfrac{(a - b)B_0\omega}{4r} = \dfrac{(1 - 0.4) \times 10^{-3} \times 100}{4 \times 5 \times 10^{-3}} = 3\text{A}$

26. (1) $\vec{\ell}, \vec{v}$ and $\vec{B}$ are coplanar.

27. (1) Component of weight along the inclined plane $= mg \sin\theta$

Again, $F = BI\ell = B\dfrac{B\ell v}{R}\ell = \dfrac{B^2\ell^2 v}{R}$

Now, $\dfrac{B^2\ell^2 v}{R} = mg \sin\theta \quad$ or $\quad v = \dfrac{mgR\sin\theta}{B^2\ell^2}$

28. **(2)** $e=\left(\vec{v}\times\vec{B}\right).\vec{\ell}$

$$e=\left[\hat{i}\times\left(3\hat{i}+4\hat{j}+5\hat{k}\right)\right].5\hat{j}$$

$\Rightarrow \quad e=-25$ V

29. **(1)** Let v be the velocity of conductor at any time, then induced emf:

$$e=Blv \qquad \text{...(i)}$$

Charge on capacitor: $q=Ce=CBlv$

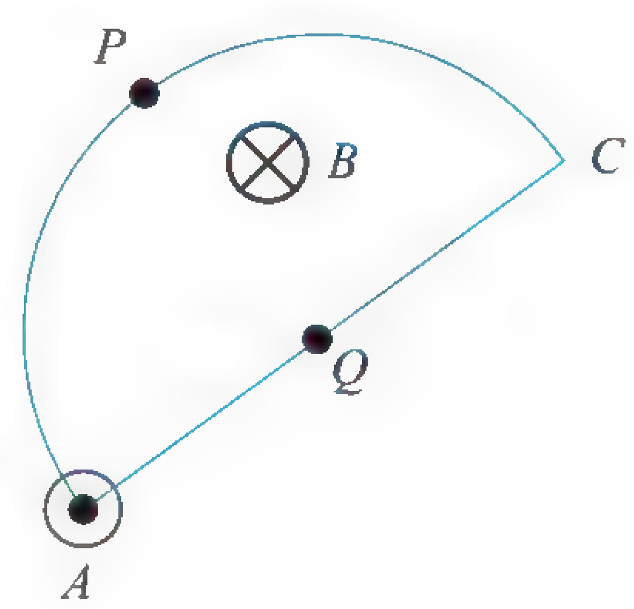
Current in circuit: $I=\dfrac{dq}{dt}=CBl\dfrac{dv}{dt}$

for conductor: $mg-IBl=\dfrac{mdv}{dt}$

$$\Rightarrow \quad mg-CB^2l^2\frac{dv}{dt}=\frac{mdv}{dt} \quad \Rightarrow \quad \frac{dv}{dt}=\frac{mg}{m+CB^2l^2}$$

This is the acceleration of conductor which is constant.

30. **(2)** We connect a conducting wire from A to C and complete the semicircular loop.



The emf in the semicircular loop is zero because its magnetic flux does not change.

$\therefore$ emf of section APC + emf of section $CQA=0$

$\therefore$ emf of section APC = emf of section $AQC=2BR^2\omega$

31. **(1)** Induced electric field at point P:

$$E=\frac{R}{2}\frac{dB}{dt} \text{ towards right}$$

Acceleration of electron: $a=\dfrac{eE}{m}=\dfrac{eR}{2m}\dfrac{dB}{dt}$ towards left

32. **(4)** Given that $\phi=at(T-t)$

Induced emf, $E=\dfrac{d\phi}{dt}=\dfrac{d}{dt}[at(T-t)]$

$$=at(0-1)+a(T-t)$$
$$=a(T-2t)$$

So, induced emf is also a function of time.

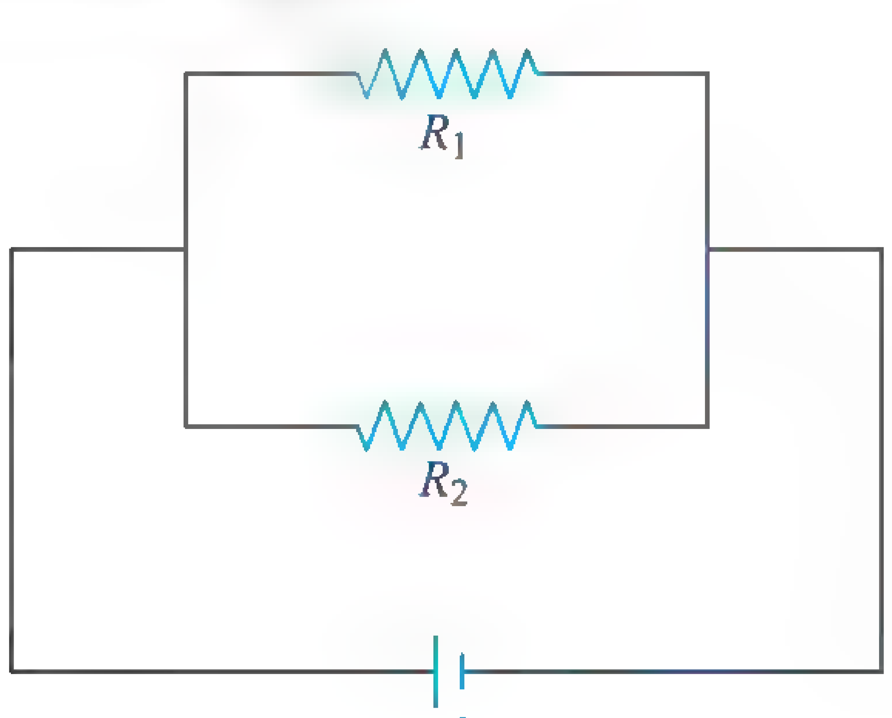
$\therefore$ Heat generated in time T is

$$H=\int_0^T\frac{E^2}{R}dt=\frac{a^2}{R}\int_0^T(T-2t)^2\,dt=\frac{a^2T^3}{3R}$$

33. **(4)** $W=F\ell(NIB\ell)\ell=NB\ell^2\left(\dfrac{BVN\ell}{R}\right)$

$$=\frac{N^2B^2\ell^3}{R}V=\frac{N^2B^2\ell^3}{R}\frac{\ell}{t}=\frac{N^2B^2\ell^4}{Rt}=0.1\text{ mJ}$$

34. **(1)** The equivalent diagram is



The induced emf across the centre and any point on the circumference is

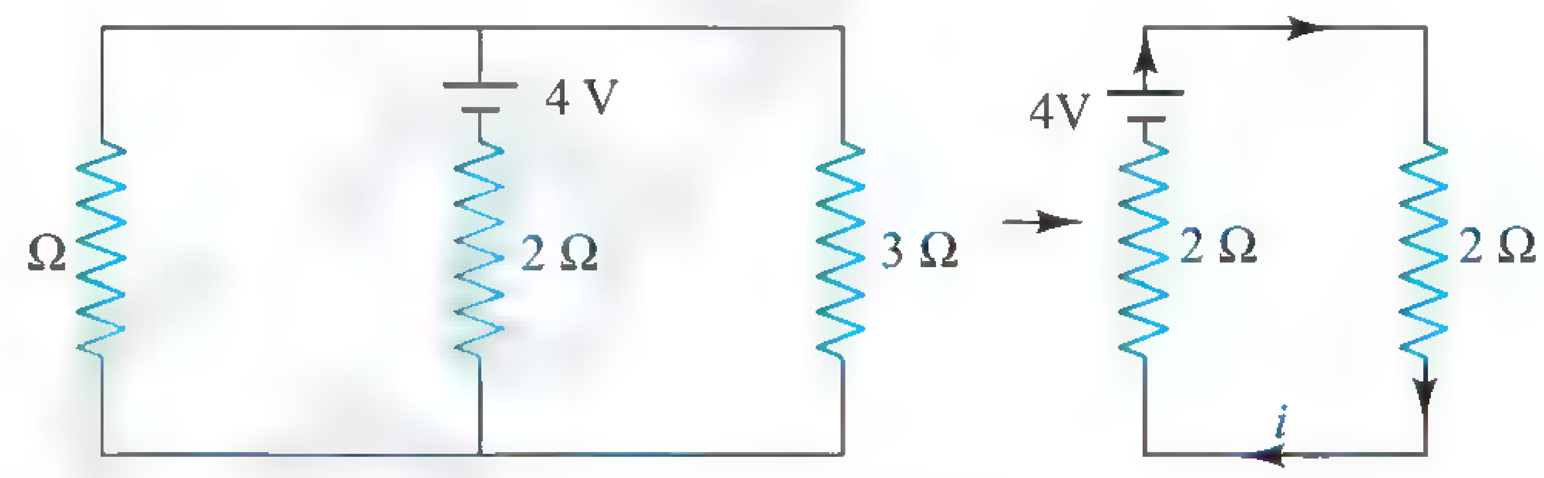
$$|\vec{e}|=\frac{1}{2}B\omega\ell^2=\frac{B\omega r^2}{2}$$

$\therefore$ Current through $R_1=\dfrac{B\omega r^2}{2R_1}$

35. **(3)** Motional emf

$$e=Bvl$$
$$e=(2)(2)(1)=4\text{ V}$$

This acts as a cell of emf $E=4$ V and internal resistance $r=2\ \Omega$. The simple circuit can be drawn as follows:



Therefore, current through the connector

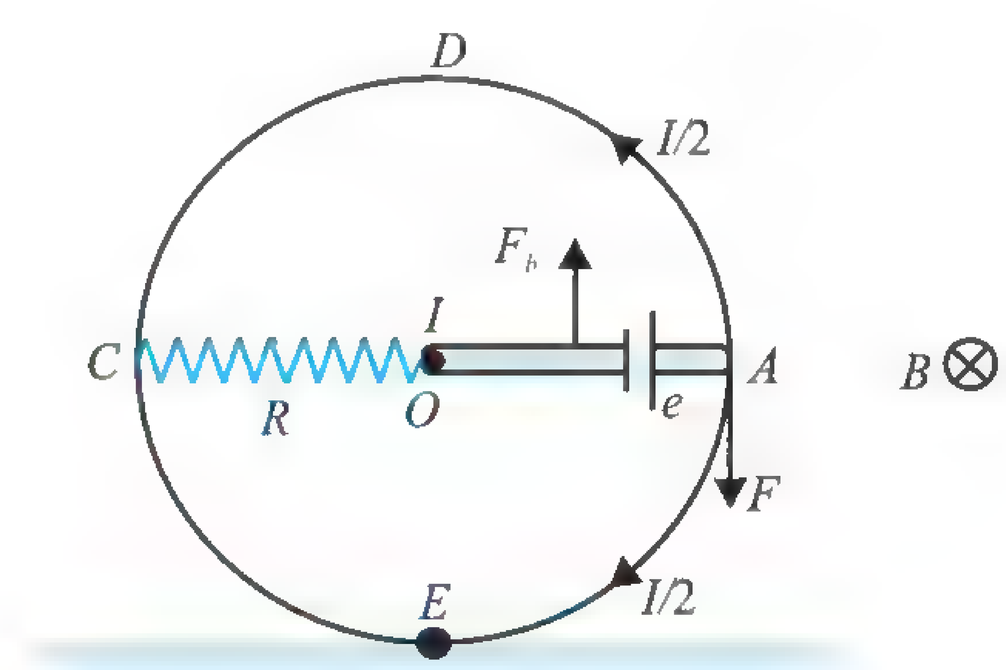
$$i=\frac{4}{2+2}=1\text{ A}$$

Magnetic force on connector

$$F_m=ilB=(1)(1)(2)=2\text{ N} \qquad \text{(towards left)}$$

Therefore, to keep the connector moving with a constant velocity, a force of 2 N will have to be applied towards right.

36. **(4)** Induced emf in the spoke is shown in figure below.



$$e=\frac{1}{2}B\omega r^2,\ I=\frac{e}{R}=\frac{B\omega r^2}{2R}$$

There will be no induced emf separately in parts ADC or AEC.

$$F_b=IrB=\frac{B^2\omega r^3}{2R}$$

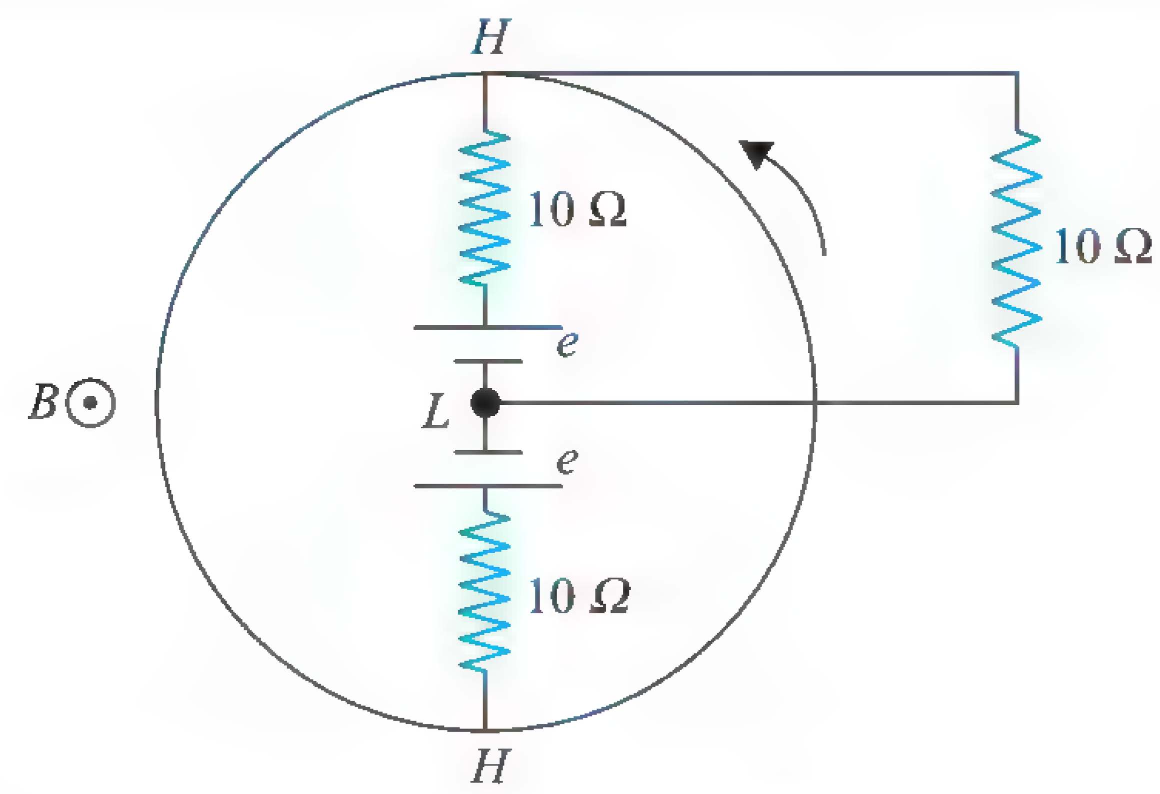
balancing torque about E: $Fr=F_br/2$

$$\Rightarrow \quad F=\frac{F_b}{2}=\frac{B^2\omega r^3}{4R}$$

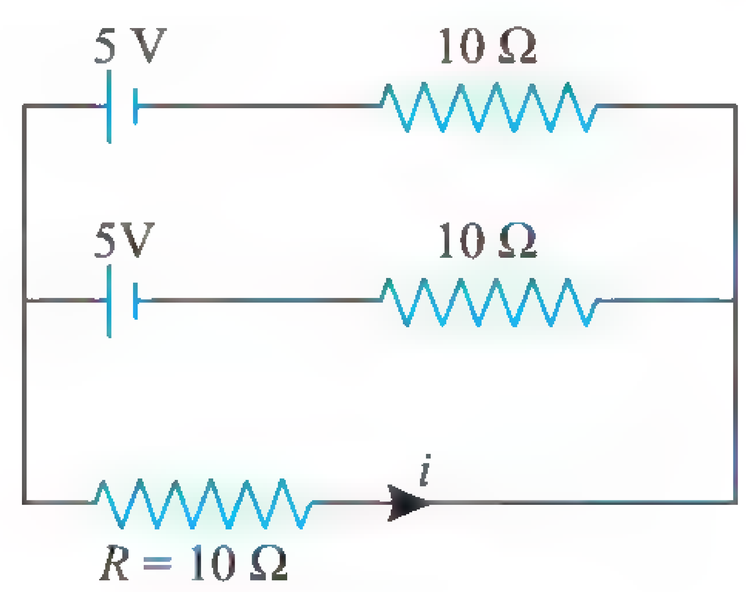
Note: Force due to currents $I/2$ will act on circular parts also, but their torque about E will be zero.

37. **(3)** Emf induced between centre of the ring and the rim is

$$e=\frac{1}{2}B\omega R^2=\frac{1}{2}(50(20)(0.1)^2=5\text{ V}$$



Now the circuit can be drawn as follows:



$\therefore \quad i = \dfrac{5}{10+5} = \dfrac{1}{3}$ A

38. (1) Force on the wire $= ilB$,

$\therefore$ Acceleration $= \dfrac{ilB}{m}$

$\therefore$ Velocity $= \dfrac{ilBt}{m}$

39. (2) Let E be the electric field at a distance r from the centre of the disc. Then

$$eE = m\omega^2 r$$

or $\quad E = \dfrac{m\omega^2 r}{e}$

$$\therefore \quad P.D. = \int_{r=0}^{r=a} E dr$$

$$= \int_0^a \frac{m\omega^2 r}{e} dr = \frac{m\omega^2 a^2}{2e}$$

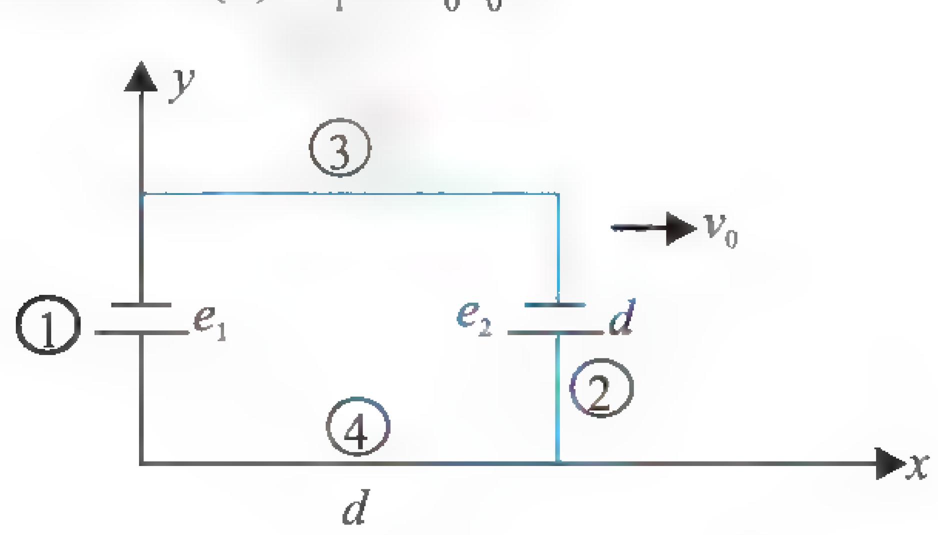
40 (4) Magnetic force $F = IlB$ acts upwards and weight acts downwards.

$a = \dfrac{mg - F}{m}$, depends upon mg and F.

41. (4) According to Lenz's law, emf of same magnitude in clockwise direction is induced in the two loops into which the figure is divided. So, current is induced in the clockwise direction in the outer boundary but no current is there in wire AB.

42. (1) Magnetic field at side (1): $B_1 = B_0$

Induced emf in (1): $e_1 = B_0 v_0 d$



Magnetic field at side (2): $B_2 = B_0\left[1 + \dfrac{d}{a}\right]$

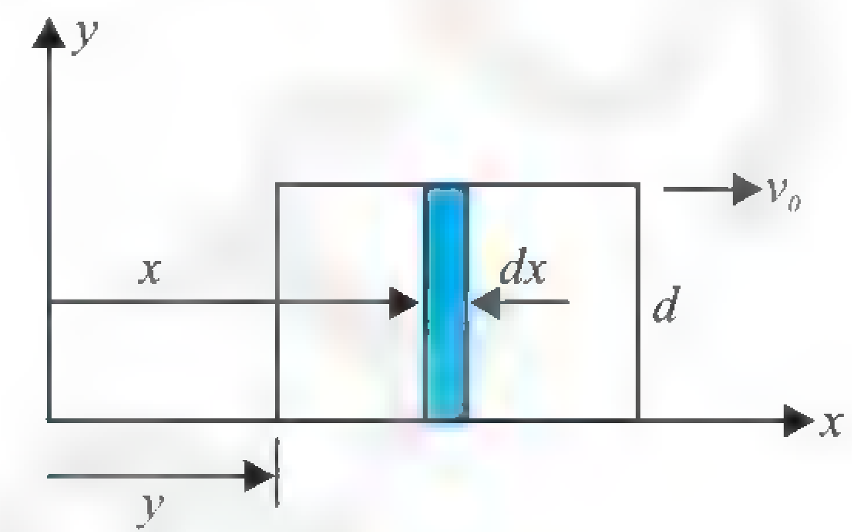
Induced emf in (2): $e_2 = B_0\left(1 + \dfrac{d}{a}\right) v_0 d$

Induced emf in 3 and 4 will be zero.

Net emf: $e = e_2 - e_1 = \dfrac{B_0 v_0 d^2}{a}$

Alternate method:

Let at any instant, the loop is at a distance y as shown below. Let us take anticlockwise direction to be positive.



Flux through a strip of width dx: $d\phi = B_0\left[1 + \dfrac{x}{a}\right](d)(dx)$

Net flux: $\phi = B_0 d \displaystyle\int_y^{y+d}\left(1 + \frac{x}{a}\right) dx$

$$\Rightarrow \quad \phi = B_0 d\left[x + \frac{x^2}{2a}\right]_y^{y+d}$$

$$\Rightarrow \quad \phi = B_0 d\left[d + \frac{1}{2a}\left[(y+d)^2 - y^2\right]\right]$$

$$\Rightarrow \quad \phi = B_0 d\left[d + \frac{1}{2a} d(2y + d)\right]$$

$$e = -\frac{d\phi}{dt} = -B_0 d\left[\frac{1}{2a} d \frac{2dy}{dt}\right]$$

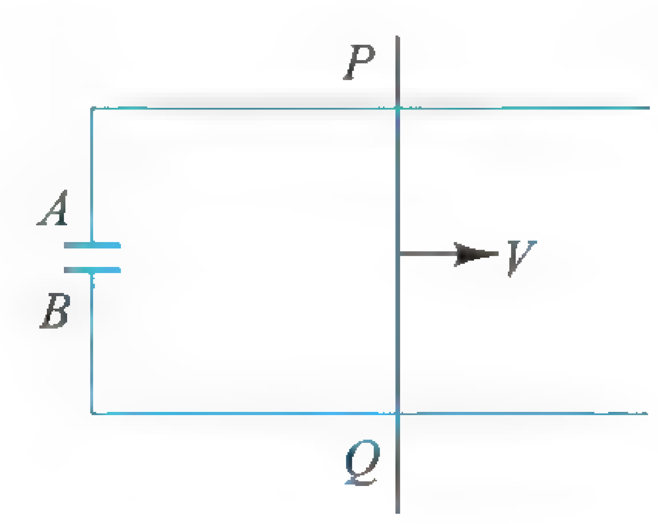
$$\Rightarrow \quad e = -\frac{B_0 d^2}{a}\frac{dy}{dt} = -\frac{B_0 d^2}{a} v_0$$

Negative sign indicates that emf induced is clockwise.

43. (1) $q = CV = C(Bvl)$

$= (10 \times 10^{-6})(4)(2)(1)$

$= 80\ \mu C$ = constant



Magnetic force on the electron in the conducting rod PQ is towards Q. Therefore, A is positively charged and B is negatively charged.

44. (1) Emf induced in coil, $\varepsilon = -\dfrac{\Delta\phi}{\Delta t} = -\dfrac{(\phi_2 - \phi_1)}{t_2 - t_1}$

$$\phi_2 = BA_2 - B = \frac{4a}{2 \times 3} \times \frac{4a}{3} \times \sin 60° = \frac{4\sqrt{3}}{9} Ba^2$$

$$\phi_1 = Ba^2$$

Work done, $W = \varepsilon q = \varepsilon I \Delta t = Ba^2\left(1 - \dfrac{4\sqrt{3}}{9}\right) i$

45. (4) Potential difference across capacitor

$$V = Bvl = \text{constant}$$

Therefore, charge stored in the capacitor is also constant. Thus, current through the capacitor is zero.

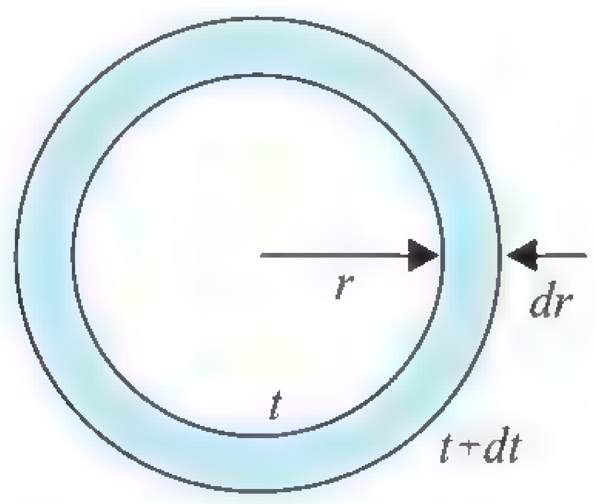
46. **(4)** Let radius of the loop is r at any time t and in further time dt, radius increases by dr.

Then change in flux: $d\phi = (2\pi r dr)B$

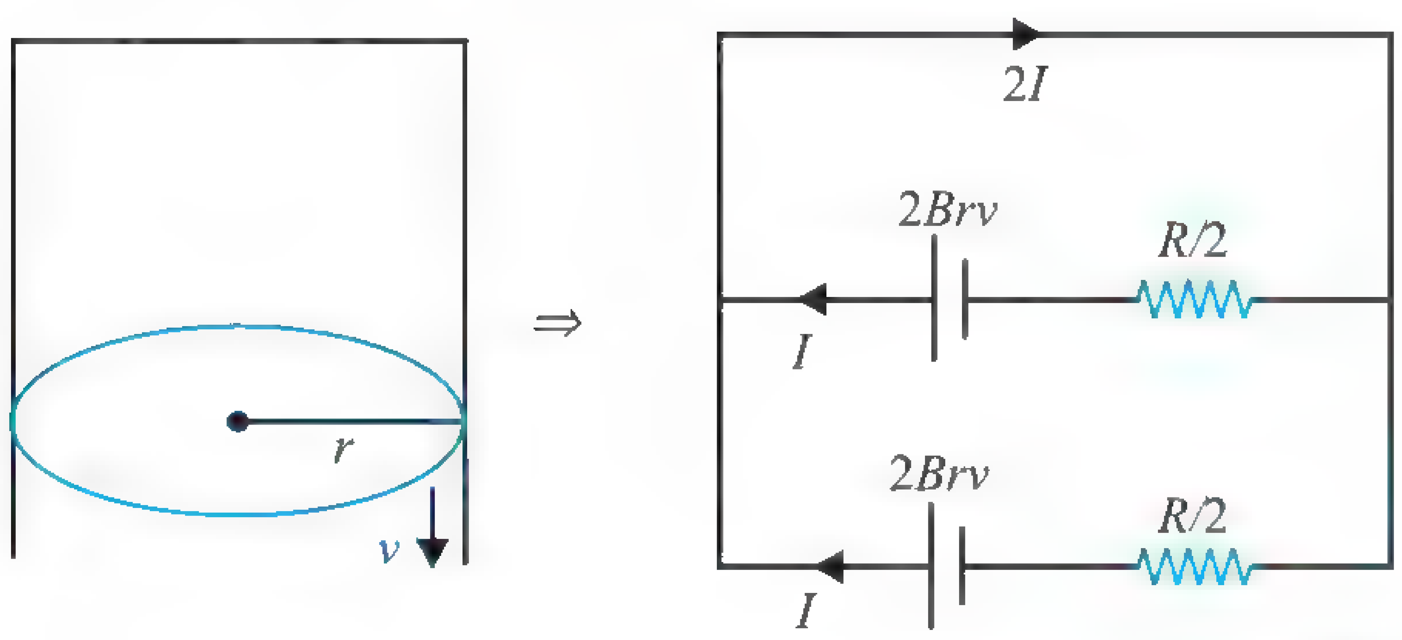
$$\Rightarrow \quad e = \frac{d\phi}{dt} = 2\pi r\left(\frac{dr}{dt}\right)\frac{k}{r}$$

$$\Rightarrow \quad e = 2\pi ck \text{ (constant)} \qquad \left[\because \frac{dr}{dt} = c, B = \frac{k}{r}\right]$$

Then change in flux: $d\phi = (2\pi r dr)B$



47. **(4)** $I = \frac{2Brv}{R/2} = \frac{4Brv}{R}$



Current in the top horizontal $= 2I = \frac{8Brv}{R}$.

48. **(4)** Volume of the balloon at any instant, when radius is r,

$$V = \frac{4}{3}\pi r^3$$

Time rate of change of volume,

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Time rate of change of radius of balloon,

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Flux through rubber band at the given instant,

$$\phi = B(\pi r^2)$$

Induced emf $= -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2) = -2\pi r B\frac{dr}{dt}$

$$= -2\pi r B\left(\frac{1}{4\pi r^2}\frac{dV}{dt}\right) = -\frac{B}{2r}\frac{dV}{dt}$$

As volume of the balloon is decreasing, $\frac{dV}{dt}$ is negative.

$$E_{\text{induced}} = -\frac{(0.04)}{2\times10\times10^{-2}}\times(-100\times10^{-6}) = 20\,\mu\text{V}$$

49. **(2)** When the copper rod is rotated, flux linked with the circuit varies with time.

Therefore, an emf is induced in the circuit.

At time t, plane of semi-circle makes angle ωt with the plane of rectangular part of the circuit. Hence, component of the magnetic induction normal to plane of semi-circle is equal to $B\cos\omega t$.

Flux linked with semicircular part is

$$\phi_1 = \frac{1}{2}\pi a^2 B\cos\omega t$$

Let area of rectangular part of the circuit be A.

$\therefore$ Flux linked with this part is

$$\phi_2 = BA$$

$\therefore$ Total flux linked with the circuit is

$$\phi = \frac{1}{2}\pi a^2 B\cos(\omega t) + BA$$

$\therefore$ Induced emf in the circuit,

$$e = -\frac{d\phi}{dt} = \frac{1}{2}\pi\omega a^2 B\sin(\omega t)$$

Since resistance of the circuit is negligible, therefore, potential difference across the capacitor is equal to induced emf in the circuit.

$\therefore$ Charge on the capacitor at time t is $q = Ce$

$$= \frac{1}{2}\pi\omega a^2 CB\sin(\omega t)$$

But current $I = \frac{dq}{dt} = \frac{1}{2}\pi\omega^2 a^2 CB\cos(\omega t)$

50. **(2)** Initial flux: $\phi_i = -\int_a^{2a}\frac{\mu_0 Ia}{2\pi}\frac{dx}{x} = -\frac{\mu_0 Ia}{2\pi}\ln 2$

Final flux: $\phi_f = \frac{\mu_0 Ia}{2\pi}\ln 2$

Charge flown: $q = \frac{\Delta\phi}{r} = \frac{\phi_f - \phi_i}{r} = \frac{\mu_0 Ia}{\pi r}\ln 2$

51. **(3)** $e = \int_{2l}^{3l} B\omega x\,dx = \frac{5B\omega l^2}{2}$

52. **(3)** $\phi_A = \frac{\mu_0 i\pi R^2}{2\pi(R^2+x^2)^{3/2}}\pi r^2$

$$\Rightarrow \quad E_A = -\frac{d\phi}{dt} = \frac{-\mu_0 i\pi}{2}R^2r^2(-3/2)(R^2+x^2)^{-5/2}2x$$

E_A is maximum when $\frac{dE_A}{dx} = 0$

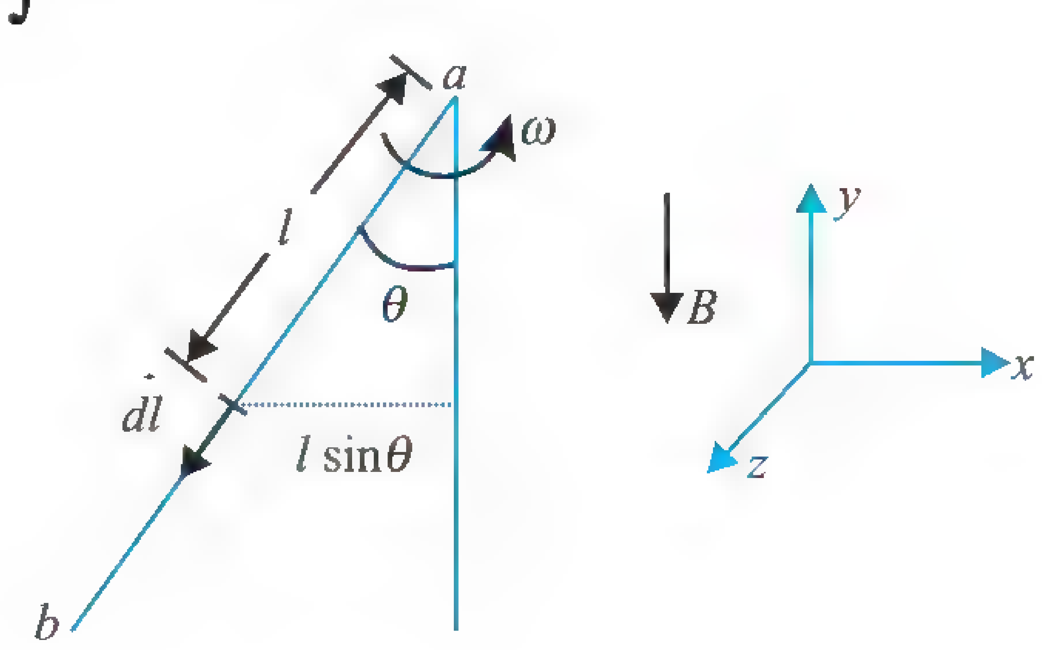
$$\Rightarrow \quad \frac{d}{dx}\frac{x}{(R^2+x^2)^{5/2}} = 0$$

or $\quad (R^2+x^2)^{5/2} - \frac{5x}{2}(R^2+x^2)^{3/2}2x = 0$

or, $\quad R^2 + x^2 - 5x^2 = 0$

or, $\quad x = \frac{R}{2}$

53. **(4)** $\varepsilon = \int(\vec{v}\times\vec{B}).\vec{dl}$



$$\Rightarrow \quad \varepsilon = \int\left[(l\sin\theta)\omega\hat{k})\times B(-\hat{j})\right]\cdot\left[dl\sin\theta(-\hat{i}) + dl\cos\theta(-\hat{j})\right]$$

$$\Rightarrow \quad \varepsilon = \int [B\omega l\sin\theta\hat{i}]\cdot[dl\sin\theta(-\hat{i}) + dl\cos\theta(-\hat{j})]$$

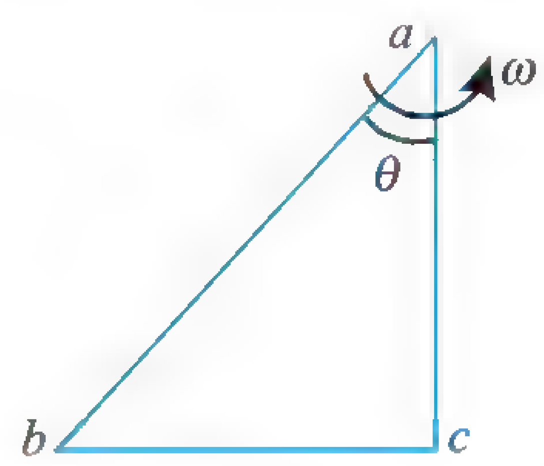
$$= -B\omega\sin^2\theta\int_0^L l dl = -\frac{1}{2}B\omega\sin^2\theta L^2$$

$$\Rightarrow \quad \varepsilon = V_b - V_a = -\frac{1}{2}B\omega L^2\sin^2\theta$$

Negative sign indicates that end b will be negative w.r.t. a.

Alternate Method:

Consider two more conductors ac and bc. This completes a closed loop. The net emf induced in this closed loop should be zero, as net flux through this loop always remains zero.



$$e_{ab} + e_{bc} + e_{ca} = 0$$

but $e_{bc} = \frac{1}{2}B\omega(L\sin\theta)^2$, $e_{ca} = 0$ putting the values, we get

$$e_{ab} = -\frac{1}{2}B\omega(L\sin\theta)^2$$

54. (4) $e = \frac{d\phi}{dt} \Rightarrow i = \frac{e}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$

where πr^2 = area of the loop of radius r and R = resistance of the loop of length $(2\pi r)$ and area of cross section πa^2.

$$R = \frac{\rho\ell}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$

$$i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt} \Rightarrow i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

55. (4) Induced electric field (in clockwise sense):

$$E = \frac{a^2}{2R}\frac{dB}{dt}$$

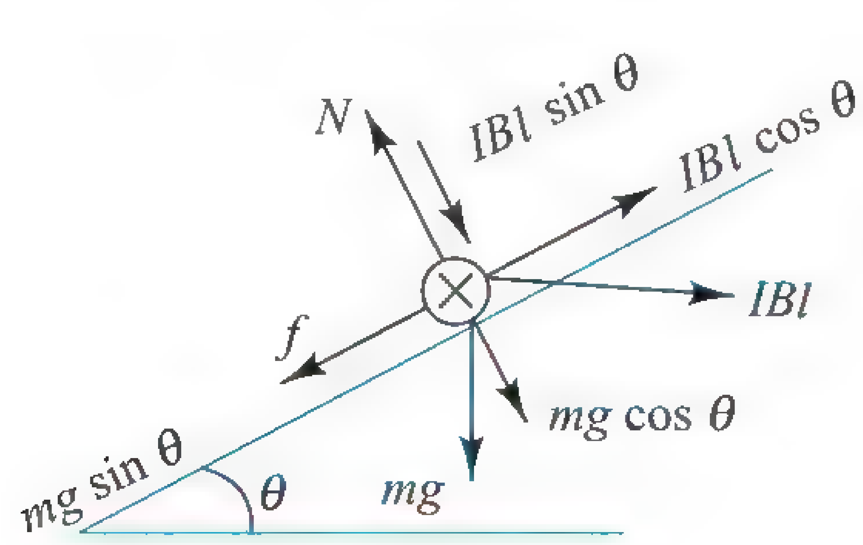
$$I\omega = \int \tau dt$$

$$\Rightarrow \quad mR^2\omega = \int qERdt = \int \frac{qa^2}{2R}\frac{dB}{dt}Rdt = \frac{\lambda 2\pi R}{2R}a^2R\int dB$$

$$\Rightarrow \quad \omega = \frac{B_0\pi a^2\lambda}{mR}$$

$$\Rightarrow \quad \vec{\omega} = -\frac{B_0\pi a^2\lambda}{mR}\hat{k} \text{ (because clockwise sense)}$$

56. (4) The front view of the arrangement is shown in figure.



From initial condition, $mg\sin\theta = \mu mg\cos\theta$

$\Rightarrow \quad \mu = \tan\theta$

$ma = IB\ell\cos\theta - mg\sin\theta - \mu N$

$N = mg\cos\theta + IB\ell\sin\theta$

$$\Rightarrow \quad a = \frac{IB\ell}{m}\cos\theta - 2g\sin\theta - \frac{IB\ell}{m}\frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{IB\ell}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta$$

Now, $s = \frac{1}{2}at^2 = \frac{1}{2}\left[\frac{IB\ell}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta\right]t^2$

57. (4) Induced emf between O and Q is

$$\frac{1}{2}B\omega(2r)^2 = B\omega 2r^2 = 2Bvr.$$

But net induced emf in the ring will be zero. Hence no current flows in the ring.

58. (3) $\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$ where $\phi = \int B2\pi r dr = \frac{2\pi}{3}B_0 tr^3$

$$E2\pi r = -\frac{2\pi B_0 r^3}{3} \Rightarrow E = -\frac{B_0 r^2}{3}$$

59. (3) $\phi = BA = \frac{\mu_0 i}{2\pi vt}a^2$

$$i = -\frac{1}{R}\frac{d\phi}{dt} = \frac{\mu_0 ia^2}{2\pi vt^2 R} = \frac{\mu_0 ia^2 v}{2\pi r^2 R}$$

60. (1) The mutual inductance M between solenoid P and Q is given by emf induced in Q due to changing current in $P = M \times$ (rate of change of current I_P in P)

$2\times 10^{-3} = M\times(5)$

$\therefore \quad M = 4.0\times 10^{-4}$ H

Simiarly, emf induced in P due to changing current in $Q = M\times$ (rate of change of current I_Q in Q)

$\Rightarrow$ induced emf in $P = (4.0\times 10^{-4})\times(2) = 8.0\times 10^{-4}$ V

61. (4) In first and third part, radius remains same, so there is no change in flux, hence no emf is induced. For second part:

$$e = B\frac{dA}{dt} = B\frac{d(\pi r^2)}{dt} = B2\pi r\frac{dr}{dt}$$

dr/dt is constant, but r increases linearly, hence induced emf also increases linearly.

62. (1) $e = \frac{d\phi}{dt} = A\frac{dA}{dt} = \frac{1}{2}2\times 2\sin 60°\sqrt{3} = 3$ V

Induced emf in each side $= e/3$

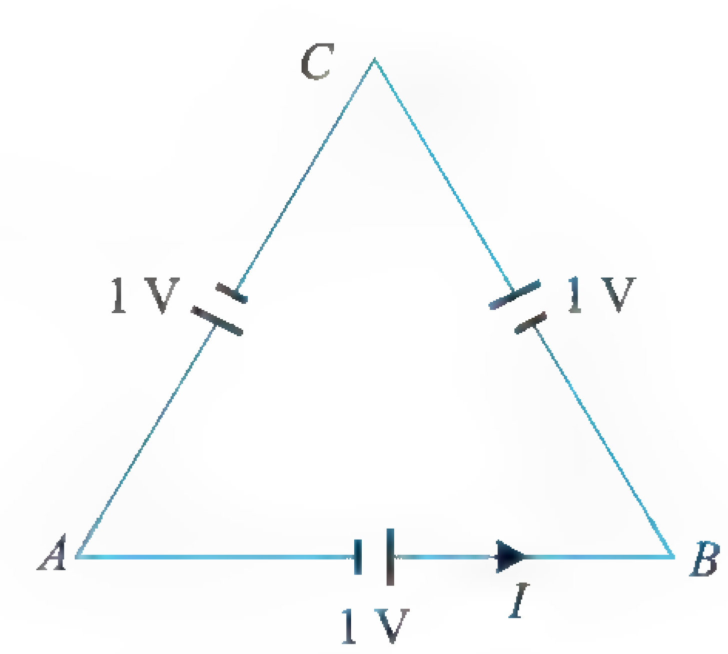
$= 1$ V

$$I = \frac{e}{R} = \frac{3}{1+2+2} = \frac{3}{5}\text{ A}$$

$$V_A + 1 - I\times 1 = V_B$$

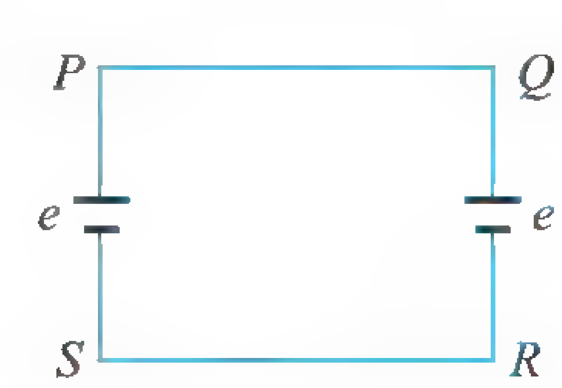
$$\Rightarrow \quad V_A - V_B = I - 1 = \frac{-2}{5}$$

$= -0.4$ V



63. (4) Since entire loop is in magnetic field, so no net emf will be induced in the loop, hence no current will flow in the loop.

So $V_P = V_Q$, $V_S = V_R$, $V_P > V_S$, $V_P > V_R$, $V_Q > V_R$



64. (3) Induced emf's in the two loops will oppose each other. So net induced emf:

$$e = \frac{d}{dt}[(b^2 - a^2)B] = (b^2 - a^2)\frac{dB}{dt}$$

$$= (0.20^2 - 0.10^2)\,\omega B_0 \cos\omega t$$

$$= (0.04 - 0.01) \times 100 \times 10 \times 10^{-3} \cos\omega t$$

$$= 30 \times 10^{-3} \cos\omega t$$

Resistance: $R = 50 \times 10^{-3}\,[4 \times 20 + 4 \times 10] = 60 \times 10^{-3}\ \Omega$

$$I = \frac{e}{R} = 0.5\cos\omega t$$

So amplitude of current = 0.5 A

65. (3) Given, $R = R_0 + t$

$$A = \pi R^2 = \pi(R_o + t)^2;\ \frac{dA}{dt} = 2\pi(R_o + t)$$

Therefore, emf induced in the loop is

$$e = B\frac{dA}{dt} = 2\pi(R_o + t)B$$

Since area is increasing, induced emf is in anticlockwise sense to oppose the change in flux.

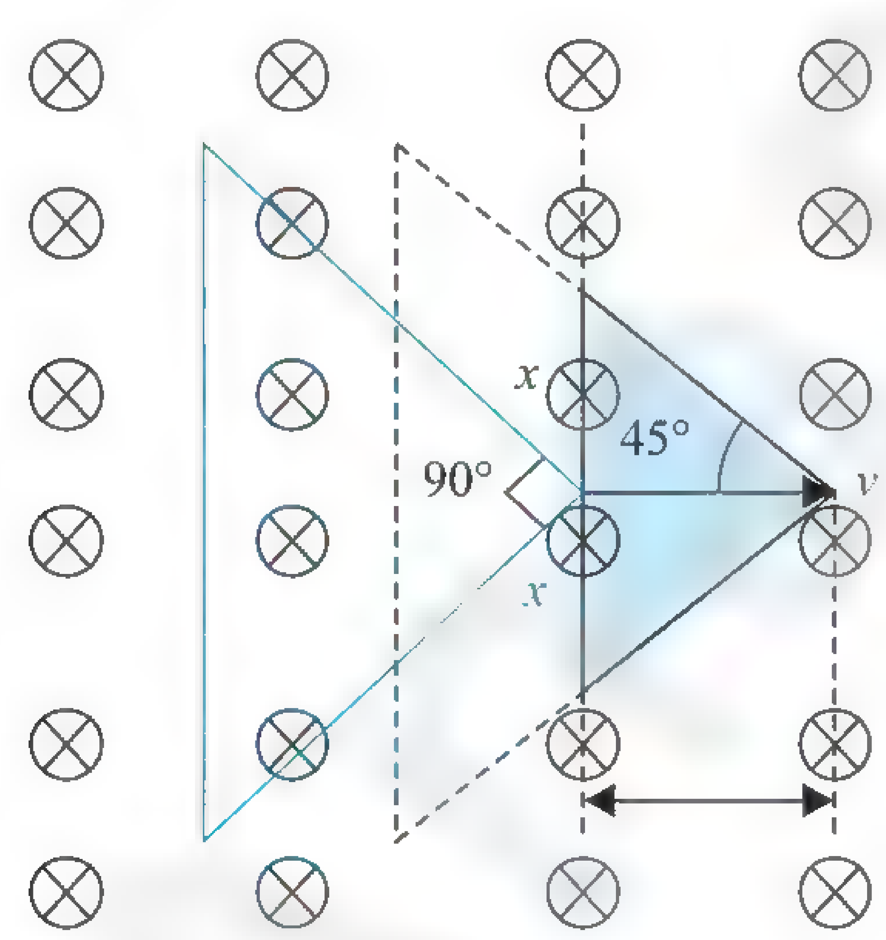
66. (3) For $0 < t < t_1$, $\frac{dB}{dt}$ is positive and hence, $e = -A\frac{dB}{dt}$ is constant and negative.

For $t_1 < t < t_2$, $\frac{dB}{dt}$ is zero and hence, $e = -A\frac{dB}{dt}$ is also zero.

For $t_2 < t < t_3$, $\frac{dB}{dt}$ is negative and hence, $e = -A\frac{dB}{dt}$ is constant and positive.

67. (3) Emf across the capacitor is constant and equal to Blv. In steady state, no current flows through the circuit

68. (4) $\phi = BA = B\left[\frac{1}{2}(2x)(x)\right]$



$$\phi = Bx^2$$

$$|\varepsilon| = \frac{d\phi}{dt} = 2B\,x \cdot v$$

$$I = \frac{\omega}{(R)} = \left(\frac{2Bv}{R}\right)x$$

69. (2) $Q = EI = (Blv)I = Fv \quad \Rightarrow \quad F = \frac{Q}{v}$

70. (4) As rod is moving with constant velocity, the potential difference across capacitor will remain constant. Hence no current will flow through capacitor branch.

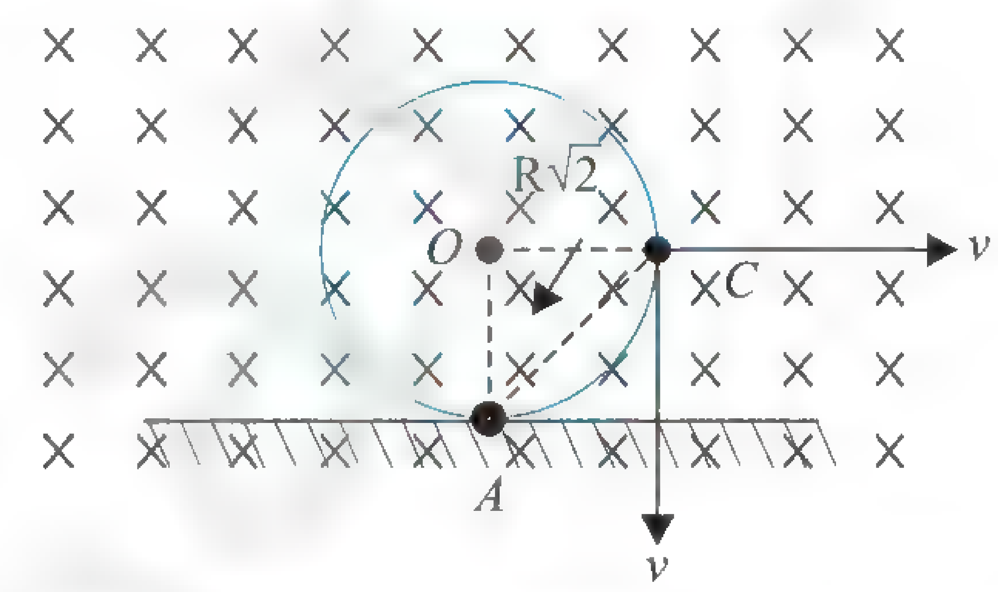
71. (2) $e = Bv\ell = \frac{B \times a \times \omega \times a}{2}$

$$i = \frac{Ba^2\omega}{2R}$$

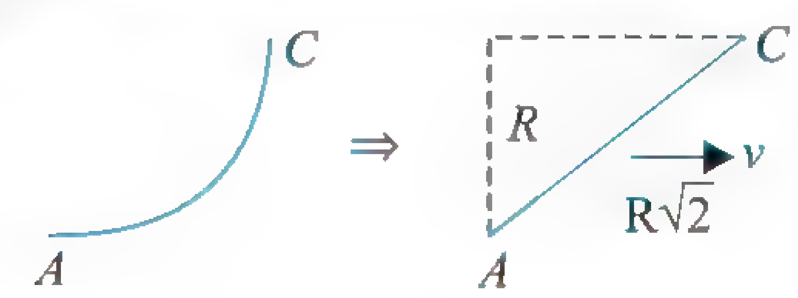
$$F = i\ell B = \frac{Ba^2\omega}{2R} \times a \times B = \frac{B^2a^3\omega}{2R}$$

towards right of OA

72. (4) Point C has two components of velocity: one is along the velocity of ring and other is along the tangent to the ring at that point. That component which is tangent to ring will not create any emf, because it is parallel to the length.



Since magnetic field is constant, so circular part of the ring between AC can be replaced by rod AC as shown.



So emf across $AC = BvR$

73. (1) As the loop is very small the distance of every point in its plane can be taken to be equal to $d = 1$ m

$$I = 3.36\,(1 + 2t) \times 10^{-2} \text{ ampere}$$

$$\frac{dI}{dt} = 2 \times 3.36 \times 10^{-2} \text{ m/sec}$$

Magnetic induction at every point on the loop, $B = \frac{\mu_0 I}{2\pi d}$

Magnetic flux linked with loop at any instant,

$$\phi = BA = \frac{\mu_0 I}{2\pi d} \cdot \pi r^2$$

Induced emf, $e = \frac{d\phi}{dt} = \frac{\mu_0 r^2}{2d}\left(\frac{dI}{dt}\right)$

Induced current, $I = \frac{e}{R} = \frac{\mu_0 r^2}{R \times 2d}\left(\frac{dI}{dt}\right)$

$$= \frac{4\pi \times 10^{-7} \times (10^{-3})^2 \times 2 \times 3.36 \times 10^{-2}}{8.4 \times 10^{-4} \times 2 \times 1}$$

$$= 5.024 \times 10^{-11} \text{ amp}$$

74. (3) Consider an element of width dy at a distance y from origin.

Induced emf across its ends is Bv_0dy, where $B = \frac{B_0}{L}y$.

Integrating from 0 to L, $E = \frac{B_0 v_0 l}{2}$

75. (4) $V_R - V_P = \int(\vec{v} \times \vec{B}) \cdot d\vec{e}$

$$= \int(v\hat{i} + v\hat{j}) \times (-3x\hat{k}) \cdot dx\hat{i}$$

$$= -3v\int x(-\hat{j} + \hat{i}) \,.\, dx\hat{i}$$

$$= -3v\int_0^{1.5L} x\,dx = \frac{-27}{8}vL^2$$

$$V_Q - V_R = \int (v\hat{i} + v\hat{j}) \times (-3(1.5L)\hat{k}) \,.\, dy\hat{j}$$

$$= -4.5vL \int (-\hat{j} + \hat{i}) \,.\, dy\hat{j}$$

$$= 4.5vL \int_0^{1.5L} dy = \frac{27}{4} vL^2$$

From above:

$$V_P - V_Q = \frac{27}{8} vL^2 - \frac{27}{4} vL^2 = \frac{-27}{8} vL^2$$

76. (3) In steady state, velocity of both wires will become same.

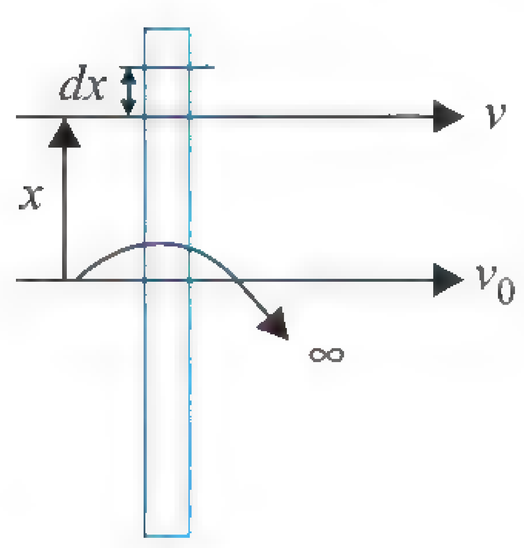
$$2\,mv = mv_0 \quad \Rightarrow \quad v = v_0/2$$

$$\text{Loss in } KE = \frac{1}{2} mv_0^2 - 2\left[\frac{1}{2} mv^2\right] = \frac{1}{4} mv_0^2$$

77. (1) $v = v_0 + wx = v_0 + \frac{v_0}{l} x$ due to mutual induction

$$\int de = \int B\,v dx$$

$$e = \int_0^{l/2} B\left(v_0 + \frac{v_0 x}{l}\right) dx = \frac{5}{8} B_0 v_0 l$$



78. (1) Flux through the ring increases in upward direction. To oppose the same, it is thrown up. As the ring moves up, the intensity of magnetic field due to electromagnet at upper points is less since

$$B \propto \left(\frac{1}{r^3}\right)$$

79. (2) $F = Bil \quad \Rightarrow \quad i = \frac{F}{Bl} = 2.5$ A

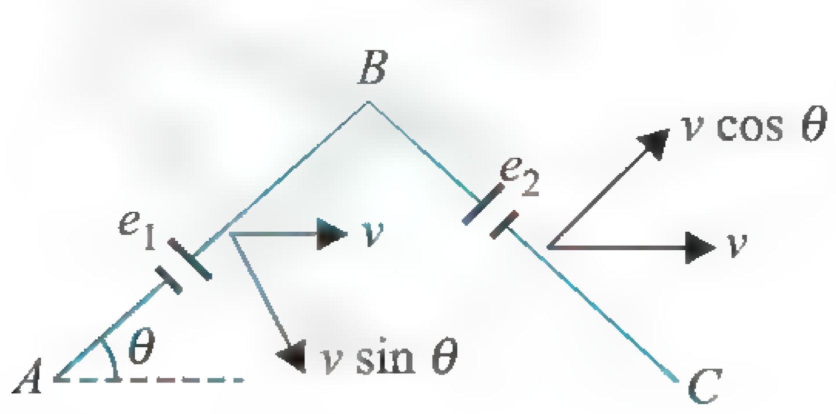
80. (3) We know that the moment of inertia of a symmetrical planar body about any axis lying in its plane and passing through its centre is same. Further, angular velocity is also same in both cases. Thus, induced emf is also same.

81. (2) At mean position, velocity is maximum and hence, rate of change of flux is also maximum.

82. (2) Let $AB = BC = l$

Velocity $= v$

Magnetic field $= B$



$$E_1 = Bv\, l \sin \theta$$

$$E_2 = Bv\, l \cos \theta$$

For induced emf at A > induced emf at B.

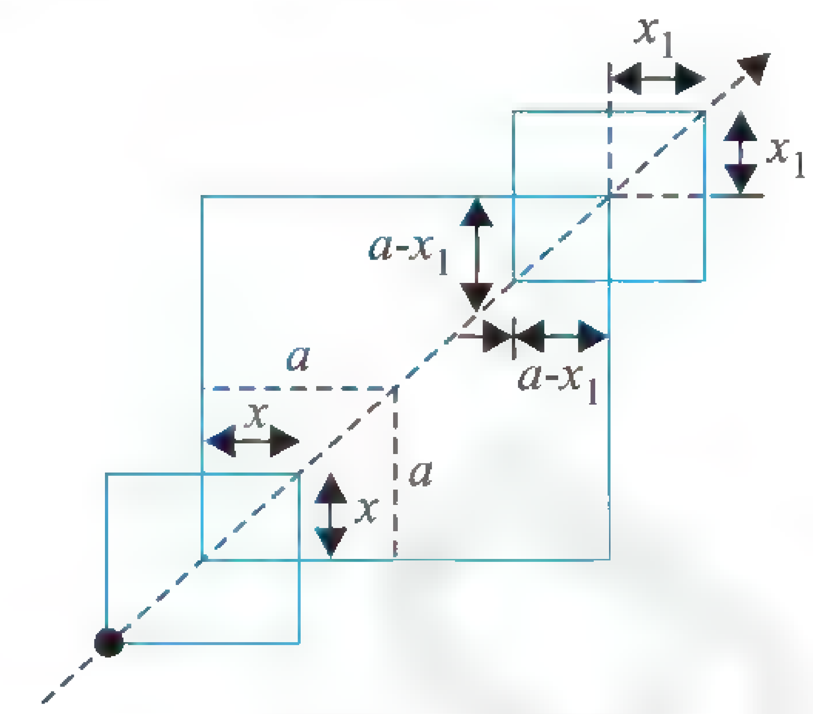
$$E_2 > E_1$$

$$Bv\, l\, (\cos \theta - \sin \theta) > 0$$

$$\cos \theta > \sin \theta \quad \Rightarrow \quad \tan \theta < 1$$

$$\theta < 45°$$

83. (4) The graph is to be drawn qualitatively



As shown, for $0 \le x \le a$

$$\phi = Bx^2$$

$$e = \frac{-d\phi}{dt} = -2Bx \cdot \frac{dx}{dt} = -2Bxv$$

At $\quad x = 0, e = 0$

At $\quad x = a/2, e = -Bav$

As, $\quad x \to a$ but $x \ne a$, $e \approx -2\,Bav$

As, $\quad x = a$, $e = 0$ as $\phi =$ constant

Distance along diagonal $x_0 = x\sqrt{2}$, thus, $x_0 = f(x)$.

When the loop reaches completely inside the field, it starts coming out of the field after some moment. When the loop is coming out, x_1 should be selected carefully, for which

$$\phi = B(a - x_1)^2$$

$$e = \frac{-d\phi}{dt} = -2B(a - x_1) \cdot (v)$$

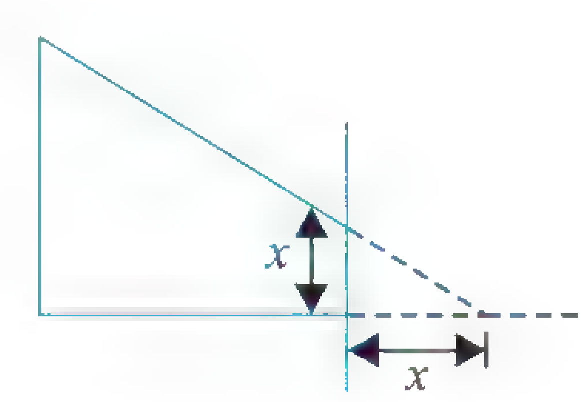
$$= 2B\,(a - x_1)\, v$$

At $\quad x_1 = 0, e = 2\,Bav$

$\quad x_1 = a, e = 0$

Thus, before the loop enters the field completely, $e = -2Bav$. When the loop is completely inside the field, $e = 0$ (ϕ = constant). As the loop starts coming out of the field ($e = 2Bav$). Option (4) is correct.

84. (3) For $0 \le x \le a$, $\phi = \frac{1}{2} Bx^2$



$$e = -\frac{d\phi}{dt} = -Bav$$

At $\quad x = 0, e = 0$

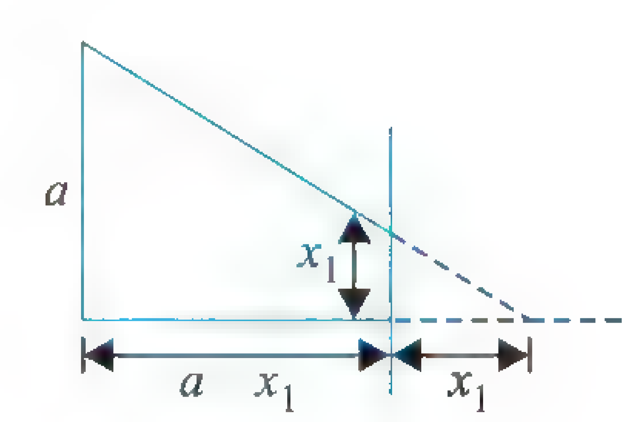
At $\quad x = a/2, e = -\frac{Bav}{2}$

At $\quad x = x \to a, e \approx -Bav$

As $\quad x = a, e = 0$

(ϕ = constant)

For $\quad a \le x \le 2a$



$$\phi = \frac{1}{2} Ba^2 = \text{constant}$$

For $\quad 2s \le x \le 3a$, $\phi = B\left[\frac{a^2}{2} - \frac{x_1^2}{2}\right]$

$$e = \frac{(-)d\phi}{dt} = -\frac{d}{dt}\left[\frac{B}{2}(a^2 - x_1^2)\right] = Bx_1 v$$

At $\quad x_1 = 0, e = 0$

At $\quad x_1 = a/2,\ e = \frac{B}{2} av$

At $\quad x_1 = a, e = 0$, loop is outside the field.

85. **(2)** $\dfrac{B^2 v_1 l_1^2}{R} - mg = ma_1$ ...(i)

$$\frac{B^2 v_1 l_2^2}{R} - mg = ma_2 \quad \text{...(ii)}$$

$$a_1 - a_2 = \frac{B^2}{mR}(v_1 l_1^2 - v_2 l_2^2)$$

For $a_1 > a_2$, $v_1 l_1^2 > v_2 l_2^2$

$$\frac{l_1}{l_2} > \left(\frac{v_2}{v_1}\right)^{1/2}$$

86. **(1)** $\dfrac{mgh - \frac{1}{2}mv_1^2}{mgv_1} = \dfrac{mgh - \frac{1}{2}mv_2^2}{mgv_2}$

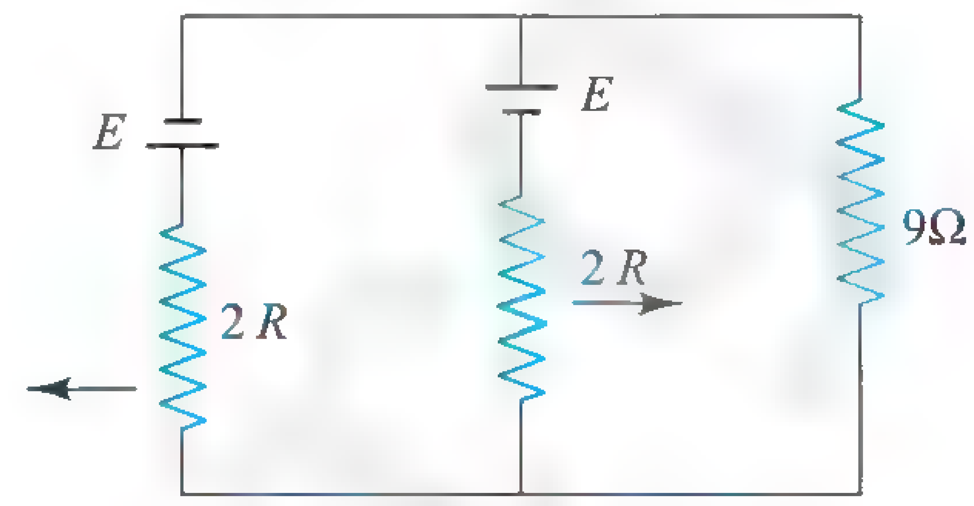
$$ghv_2 - \frac{1}{2}v_1^2 v_2 = ghv_1 - \frac{1}{2}v_2^2 v_1$$

$$gh(v_1 - v_2) = \frac{1}{2}v_1 v_2 (v_1 - v_2)$$

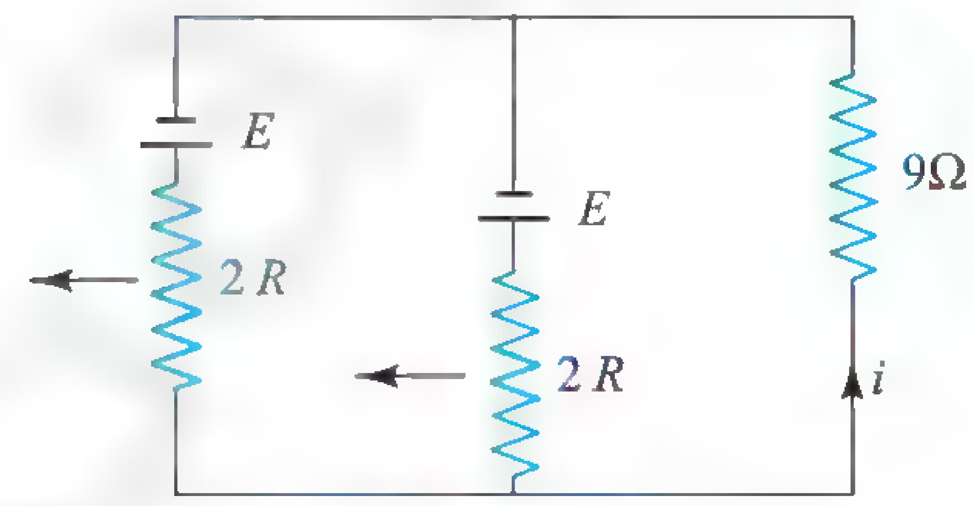
$$gh + \frac{1}{2}v_1 v_2 = 0$$

Multiple Correct Answers Type

1. **(1), (2)**
   As $\vec{B} \perp \vec{A}$, hence $\phi = 0$ and $e = 0$.
2. **(1), (2), (3), (4)**
   $\phi = BA\cos\theta$, $\theta = \omega t$
   $$e = \frac{-d\phi}{dt} = BA\omega\sin\theta$$
   at $\theta = \dfrac{\pi}{2}$, $\phi = 0$ and e is maximum. At $\theta = 0$, ϕ is maximum and e is zero. Emf is maximum when $\theta = \dfrac{\pi}{2}$ and for this plane of loop is parallel to magnetic field. Clearly phase difference between flux and emf is $\pi/2$.
3. **(2), (3)**
   Thought based.
4. **(1), (3)**
   Current induced in both A and B will be in same direction. So they will attract each other.
   A is closer to magnet, so rate of change of flux in A will be more. So more current is induced in A.
5. **(1), (3)**
   Use Lenz's law. The motion of ring will be opposed.
6. **(1), (3), (4)**
   No emf is induced because flux through the ring does not change. Hence no current flows in the ring. Emf is induced between A and D because length AD is perpendicular to v. But no emf is induced across CE because length CE is parallel to v.
7. **(1), (2), (3), (4)**
   Due to rotation, emf $= \dfrac{Br^2\omega}{2}$
   Due to translation indeed emf $= Bvr$
   where r is the separation.
8. **(2), (3)**
   Each wire can be replaced by a battery whose emf is equal to
   $$Blv = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2} = 20 \times 10^{-4}\text{ V}$$
   The polarity of the battery can be given by Fleming's right hand rule. When both wire move in opposite direction, the circuit diagram looks like as shown in Fig. (a).
   The effective emf of the two batteries shown in the diagram is zero. So, choice (2) is correct and choice (4) is wrong.
   When both wires move towards left, the circuit diagram looks like as shown in Fig. (b).



(a)



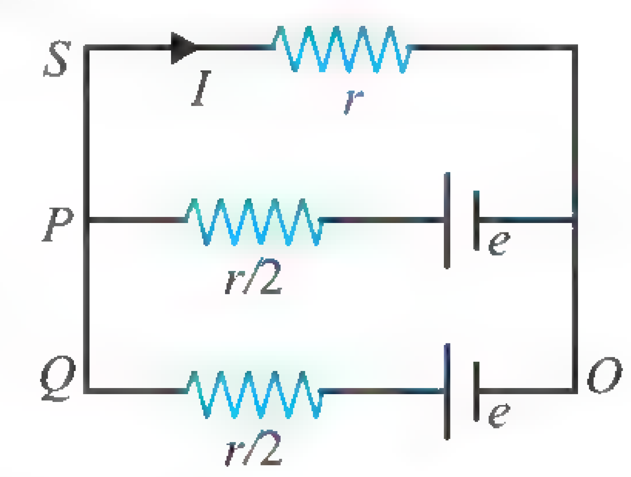
(b)

Effective emf of two batteries shown is E (= 20×10^{-4} V) and internal resistance is 1 Ω.
Hence, current in the circuit is

$$i = \frac{20 \times 10^{-4}}{10} = 0.2\text{ mA}$$

Hence, choice (3) is correct and choice (1) is wrong.

9. **(1), (2), (3), (4)**
   Use concept of motional emf
10. **(2), (4)**
    Equivalent circuit:



Current is from S to O, i.e., from circumference to centre

$$I = \frac{e}{r + r/4} = \frac{4}{5r}\left(\frac{1}{2}B\omega a^2\right) \Rightarrow I = \frac{2B\omega a^2}{5r}$$

11. **(1), (2), (3)**
    $$\phi = 4t^n + 6$$
    $$\frac{d\phi}{dt} = 4nt^{n-1}$$
    $$|e| = 4nt^{n-1},\ |e| = \frac{4n}{t^{1-n}}$$
12. **(2), (4)**
    Flux remains constant here, so emf induced is zero.
13. **(2), (4)**
    $$i = \frac{dq}{dt} = \frac{d}{dt}(CvBl) = CBl\frac{dv}{dt} = CBla$$
    $\therefore\ F - CB^2l^2 a = ma$
    $$\Rightarrow\ a = \frac{F}{m + B^2l^2C}$$

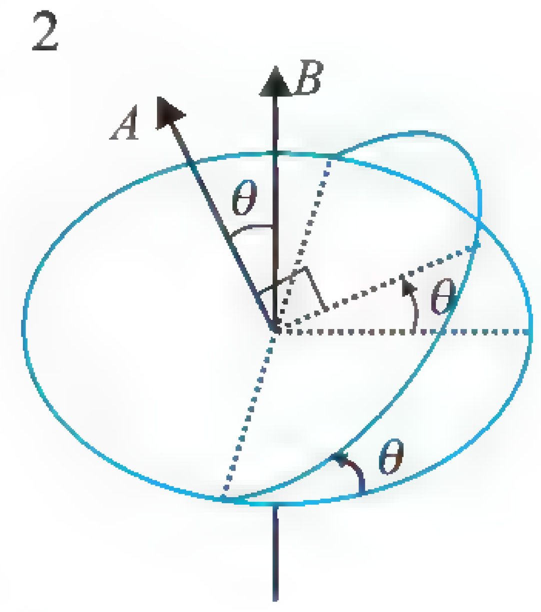
$\Rightarrow$ emf increases

$\Rightarrow$ charge increases.

14. (1), (4)

$\theta = \omega t$. Only half circular part will be involved in inducing emf, so

effective area $A = \dfrac{\pi a^2}{2}$



$$\phi = BA\cos\theta$$

$$e = -\frac{d\phi}{dt} = +BA\sin\theta\left(\frac{d\theta}{dt}\right) \Rightarrow e = \frac{B\pi a^2}{2}\omega\sin\theta$$

$$I = \frac{e}{R} = \frac{B\pi a^2\omega}{2R}\sin\theta$$

Clearly $I = 0$, when $\theta = 0°$ and when $\theta = \dfrac{\pi}{2}$, $I = \dfrac{B\pi a^2\omega}{2R}$

15. (1), (4)

Apply Lenz's law

16. (2), (3)

$$|\varepsilon_i| = \left|-\frac{d\phi}{dt}\right| = |\varepsilon_i| \propto \left|-\frac{d\phi}{dt}\right|$$

and also area under the curve gives

Change in flux $= \phi_f - \phi_i = \int d\phi = \int \varepsilon_i dt$

Total charge that will flow $= \Delta\phi/R$

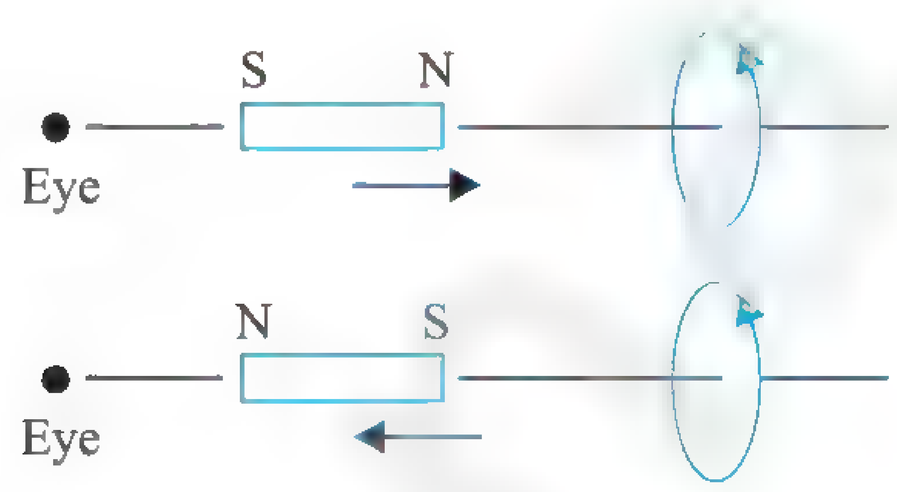
17. (1), (3), (4)

Apply Lenz's law

18. (1), (2), (3), (4)

Since line joining the ends of the ring is parallel to B, hence no emf will be induced in any case.

19. (2), (3)



20. (1), (4)

If $\dfrac{di}{dt} = 0$, it means current is constant, its flux through loop will remain constant. Hence no emf is induced.

If $\dfrac{di}{dt} > 0$, current increases so flux increates in loop in inward direction, so induced current should be anticlockwise which produces flux in outward direction.

21. (1), (2), (4)

As long as the rod moves at a constant speed, all the work done by external person pulling the rod is dissipated as heat in the resistor. As per work–energy theorem, there is no change in kinetic energy of rod.

Here, force applied by external agent is $F = BIl$.

However, if applied external force is doubled then a part of external power increases the velocity of rod and the rest is dissipated as heat. Obviously, Lenz's law is not satisfied under such circumstances.

Further, $P = E^2/R = B^2l^2v^2/R$. Thus, if resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

22. (1), (2), (3)

$E = NAB\omega\cos\theta$, where θ is the angle between plane of coil and magnetic field. Substituting values, options (1), (2) and (3) are correct.

23. (1), (2), (3), (4)

While the frame enters the magnetic field, magnetic flux through the frame is increasing. Hence from Lenz's law, induced current will be anticlockwise.

So, choice (1) is correct.

After 1 sec, Distance moved by frame

$$S = \frac{1}{2}at^2$$

$$S = \frac{1}{2} \times 1 \times 1^2 = 0.5 \text{ m}$$

and speed $v = at = 1 \times 1 = 1$ m/s

Induced emf $= Bv\dfrac{L}{5}$

$= 5 \times 1 \times 1 = 5$ V

Hence induced current

$$i = \frac{\varepsilon}{R} = \frac{5}{4} = 1.25 \text{ A}$$

(resistance of loop $= 4\ \Omega$)

So, choice (2) is correct.

Force $F = iB\dfrac{L}{5} = 1.25 \times 5 \times 1 = 6.25$ N

Since magnetic moment and magnetic field are anti-parallel so torque will be zero. So, choice (4) is correct.

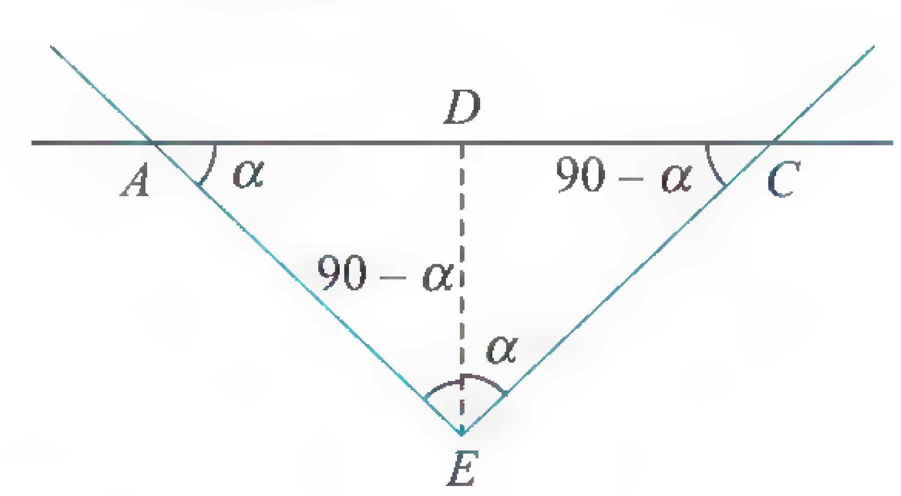
Hence choices (1), (2), (3) and (4) are correct.

24. (1), (4)

PQ is parallel to magnetic field, hence no emf is induced in it. QR is $\perp$ perpendicular to magnetic field. Induced emf $= Bvl = B_0\omega aa = B_0\omega a^2$

25. (2), (4)

Let $\angle CAE = \alpha =$ constant



At time t, $DE = vt$

$AD = DE\cot\alpha$

and $DC = DE\tan\alpha$

$AC = DE(\tan\alpha + \cot\alpha)$

$= vt(\tan\alpha + \cot\alpha) = l$ (say)

Induced emf $e = Bv\ell$

$e = Bv^2t(\tan\alpha + \cot\alpha)$

$e \propto v^2$ and $e \propto t$

26. (1), (3)

According to Lenz's law, current I is in clockwise direction

$$A'B' = 2A'O = \frac{2}{\tan 60°}\left(\frac{r\sqrt{3}}{4}\right) = \frac{r}{2}$$

For equilibrium $mg = I\left(\frac{r}{2}\right)B$

$$I = \frac{2mg}{rB}$$

If loop is displaced by x,

F = Restoring force

$$= -I\left[\left(\frac{2}{\tan 60°}\right)\left(\frac{r\sqrt{3}}{4}+x\right)\right]B + mg$$

$$= -\frac{IrB}{2} + mg - \frac{2IB}{\sqrt{3}}x$$

$$= -\frac{2IB}{\sqrt{3}}\cdot x$$

$F \propto (-x)$ x, motion is SHM.

$$a = -\frac{2IB}{m\sqrt{3}}\cdot x$$

$$T = 2\pi\left[\frac{m\sqrt{3}}{2IB}\right]^{1/2} = \pi\left[\frac{r\sqrt{3}}{g}\right]^{1/2}$$

Linked Comprehension Type

For Problems 1–2

1. (1) **2. (4)**

Current should enter the bar from P so that magnetic force is upwards.

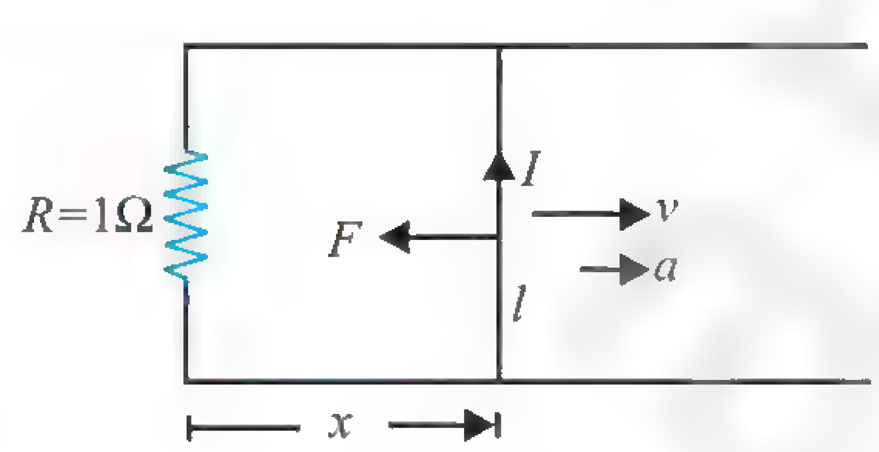
$$ilB = mg \quad \text{or} \quad \frac{V}{5}lB = mg$$

or $$m = \frac{150\times 0.6\times 1.5}{5\times 10} = 2.7 \text{ kg}$$

For Problems 3–5

3. (1) **4. (4)** **5. (1)**

Let anticlockwise direction is positive



$\phi = BA \cos 180° = cxlx \cos 180° = -clx^2$

$$e = -\frac{d\phi}{dt} = cl2x\frac{dx}{dt}, \quad I = \frac{e}{R} = \frac{2clxv}{1} = 2clxv$$

This is positive so current is anticlockwise.

$$F = m(-a) \Rightarrow IBl = -mv\frac{dv}{dx} \Rightarrow 2c^2l^2x^2v = -mv\frac{dv}{dx}$$

$$\Rightarrow \quad 2c^2l^2\int_0^x x^2dx = -\int_{10}^{5} mdv \Rightarrow 2c^2l^2\frac{x^3}{3} = m(5)$$

$$\Rightarrow \quad x^3 = \frac{15m}{2c^2l^2} \Rightarrow x = \left(\frac{15}{2}\right)^{\frac{1}{3}}$$

Heat produced = loss in KE of rod $= \frac{1}{2}\times 1[10^2 - 5^2] = \frac{75}{2} = 37.5$ J

Magnetic force is not doing any work.

For Problems 6–7

6. (2) **7. (3)**

$$\varepsilon_{av} = -\frac{\Delta\Phi_B}{\Delta t} = -B\frac{\Delta A}{\Delta t} = -B\frac{(-\pi r^2)}{\Delta t}$$

$$= \frac{1\times\pi(0.10)^2}{0.314} = 0.1 \text{ V}$$

Since the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore, the current flows in clockwise direction

$$I_{av} = \frac{\varepsilon_{av}}{R} = \frac{0.1}{0.01} = 10 \text{ A}$$

For Problems 8–9

8. (2) **9. (3)**

Magnetic field on the axis of a circular coil is given by

$$B = \frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}$$

Since $x >> R$, therefore, magnetic field at the centre of the smaller loop is

$$B \approx \frac{\mu_0 iR^2}{2x^3}$$

Flux linked with coil is

$$\phi = B(\pi r^2) = \frac{\mu_0\pi iR^2r^2}{2x^3}$$

From Faraday's law we have

$$E = -\frac{d\phi}{dt} = \frac{3}{2}\frac{\mu_0\pi iR^2r^2}{x^4}v$$

For Problems 10–12

10. (4) **11. (1)** **12. (2)**

The large circuit is a circuit with a time constant of

$\tau = RC = (10\ \Omega)\ (20\times 10^{-6}\ \text{F}) = 200\ \mu\text{s}.$

Thus, the current as a function of time is

$$i = \left(\frac{100\text{ V}}{10\ \Omega}\right)e^{-\frac{t}{200\mu\text{s}}}$$

At $t = 200$ ms, we obtain

$i = (10\text{ A})\ (e^{-1}) = 3.7$ A.

Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop,

$$\Phi_B = \int_c^{c+a}\frac{\mu_0 ib}{2\pi r}dr = \frac{\mu_0 ib}{2\pi}\ln\left(1+\frac{a}{c}\right)$$

So the emf induced in the small loop at $t = 200$ ms

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{c}\right)\frac{di}{dt}$$

$$= -\frac{\left(4\pi\times 10^{-7}\frac{Wb}{A\times m^2}\right)(0.200\text{ m})}{2\pi}$$

$$\times\ln(3.0)\left(-\frac{3.7\text{ A}}{200\times 10^{-6}\text{s}}\right)$$

Thus, the induced current in the small loop is

$$i' = \frac{\varepsilon}{R} = \frac{0.81\text{ mV}}{25(0.600\text{ m})(1.0\ \Omega/\text{m})} = 54\ \mu\text{A}.$$

Initially current in large loop is maximum and afterwards decreases. Hence flux through the smaller loop decreases with time.

The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

For Problems 13–14

13. (2) **14. (1)**

Let us take anticlockwise direction as positive, then area vector in upward direction will be positive

$$\phi = BA = Bl\,x$$

$$e = -\frac{d\phi}{dt} = -\ell\left[B\frac{dx}{dt} + x\frac{dB}{dt}\right]$$

$$= -5\times10^{-2}\,[0.1\times(-5\times10^{-2}) + 5\times10^{-2}\times0.2]$$

$$= -250\times10^{-6}\text{ V} = -250\ \mu\text{V}$$

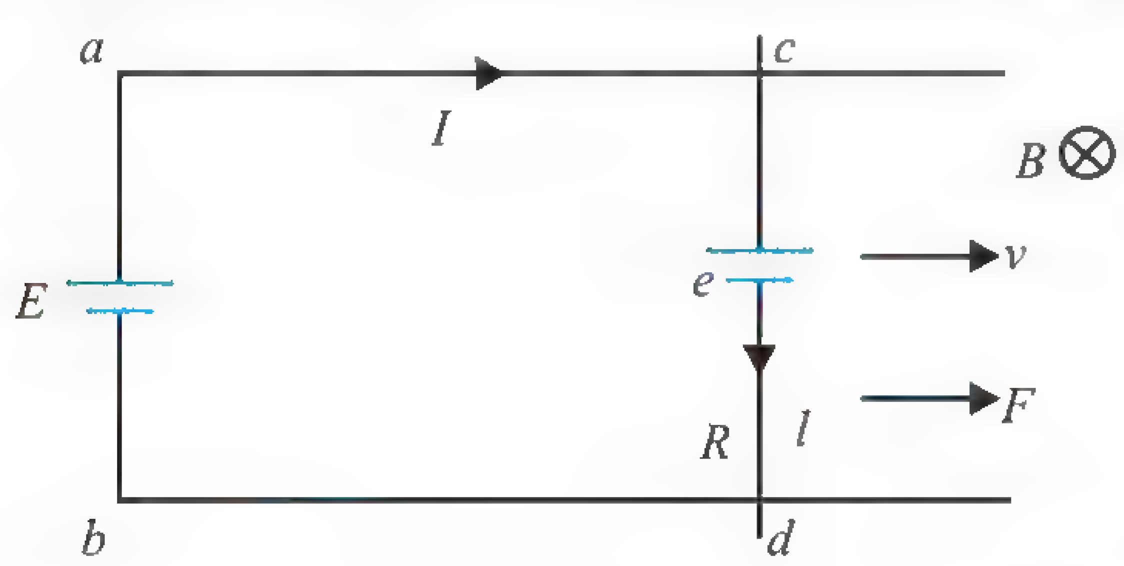
Emf is coming out to be negative, so it should be clockwise.

Current: $I = \dfrac{e}{R} = \dfrac{250\times10^{-6}}{10^{-4}} = 2.5\text{ A}$

For Problems 15–17

15. (1) **16. (3)** **17. (2)**

Due to applied emf, current will flow as shown. Due to this current and magnetic field B, force on the wire will act towards right and rod will start moving towards right. Let at any instant, its velocity is v. Due to this velocity, emf $e = Blv$ will be induced as shown.



Net emf: $E - B\ell v,\ I = \dfrac{E - B\ell v}{R}$ ...(i)

$$F = IB\ell = \left(\frac{E - B\ell v}{R}\right)B\ell \qquad \text{...(ii)}$$

$$\Rightarrow \quad m\frac{dv}{dt} = (E - B\ell v)\frac{B\ell}{R}$$

$$\frac{mR}{B\ell}\frac{dv}{dt} = E - B\ell v \ \Rightarrow\ \int_0^v \frac{dv}{E - B\ell v} = \int_0^t \frac{B\ell}{mR}dt$$

$$\Rightarrow \left[\frac{\ln(E - B\ell v)}{-B\ell}\right]_0^v = \frac{B\ell}{mR}t \ \Rightarrow\ \ln\left(\frac{E - B\ell v}{E}\right) = \frac{-B^2\ell^2}{mR}t$$

$$\Rightarrow \frac{E - B\ell v}{E} = e^{-t/\tau} \ \Rightarrow\ v = \frac{E}{B\ell}\left[1 - e^{-t/\tau}\right]$$

Velocity will increase upto $t = \infty$, and at $t = \infty$, velocity will be maximum and constant.

$$v_{t=\infty} = \frac{E}{B\ell} \quad \rightarrow \quad \text{This is the required terminal velocity.}$$

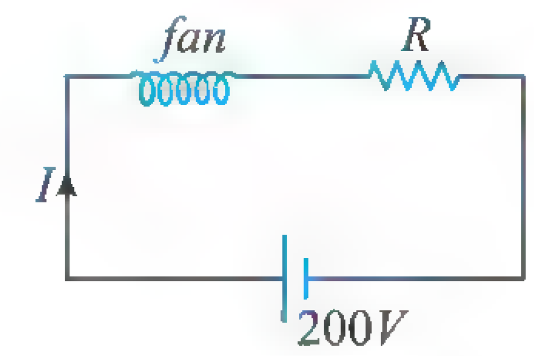
Also, when the velocity is constant, net force on the rod will become zero, so putting $F = 0$ in equation (ii), we get

$$v_{\text{terminal}} = \frac{E}{B\ell}$$

when the rod attains terminal velocity, then from equation (i), $I = 0$.

For Problems 18–20

18. (3) **19. (3)** **20. (2)**



The fan is running at 200 V, consuming 1000 W, then

$$I = \frac{1000}{200} = 5\text{ A}$$

But as coil resistance is 1 Ω, power dissipated by internal resistance as heat is $P_1 = I^2R = 25$ W.

If V is the net emf across the coil, then

$$\frac{V^2}{R} = 25\text{ W} \quad \text{or} \quad V = 5\text{ V}$$

Net emf = source emf – back emf

or $\quad V = V_s - e \ \Rightarrow\ e = 195\text{ V}$

The work done $P_2 = 1000 - 25 = 975$ W.

For Problems 21–23

21. (1) **22. (2)** **23. (3)**

$$\frac{dB}{dt} = 2\text{ T/s}$$

$$E = -\frac{AdB}{dt} = -800\times10^{-4}\text{ m}^2\times2 = -0.16\text{ V}$$

$$i = \frac{0.16}{1\,\Omega} = 0.16\text{ A, clockwise}$$

At $t = 2$ s, $B = 4$ T, $\dfrac{dB}{dt} = 2$ T/s

$$a = 20\times30\text{ cm}^2$$

$$= 600\times10^{-4}\text{ m}^2;\ \frac{dA}{dt} = -(5\times20)\text{ cm}^2/\text{s}$$

$$= -100\times10^{-4}\text{ m}^2/\text{s}$$

$$E = -\frac{d\phi}{dt} = -\left[\frac{d(BA)}{dt}\right] = -\left[\frac{BdA}{dt} + \frac{AdB}{dt}\right]$$

$$= -[4\times(-100\times10^{-4}) + 600\times10^{-4}\times2]$$

$$= -[-0.04 + 0.120] = -0.08\text{ V}$$

Alternative: $\phi = BA = 2t\times0.2\,(0.4 - vt)$

$$= 0.16\,t - 0.4\,vt^2$$

$$E = -\frac{d\phi}{dt} = 0.8\,vt - 0.16$$

At $\quad t = 2$s

$$E = 0.08\text{ V}$$

At $t = 2$ s, length of the wire = (2 × 30 cm) + 20 cm = 0.8 m

Resistance of the wire = 0.8 Ω

Current through the rod $= \dfrac{0.08}{0.8} = \dfrac{1}{10}$ A

Force on the wire is $= ilB$

$$= \frac{1}{10}\times(0.2)\times4 = 0.08\text{ N}$$

Same force is applied on the rod in opposite direction to make net force zero.

For Problems 24–26

24. (3)

25. (4) When $x = -\dfrac{9}{5}a : |x| > a + \dfrac{a}{2} = \dfrac{3}{2}a$, so the loop is outside the magnetic field therefore, both induced current and external force required are zero.

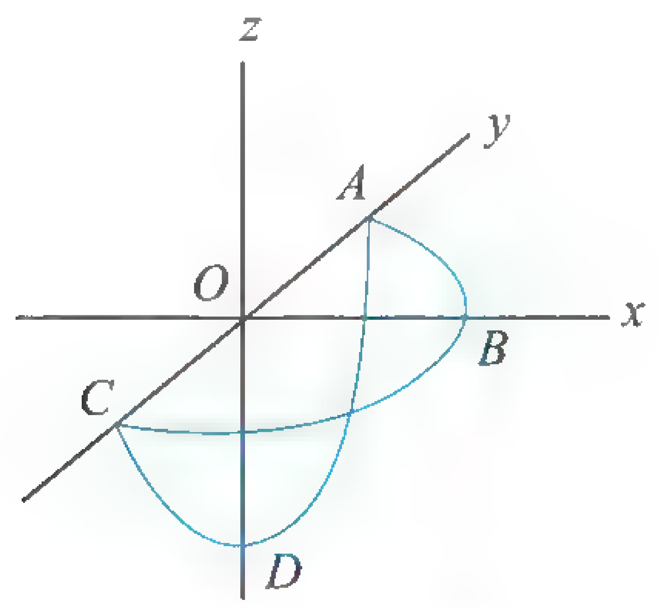
26. (3) At $x = -\frac{a}{4}$, loop is completely inside the field. Flux through the loop is $BA = Ba^2$. As $\frac{dB}{dt} = 0$, induced current is zero so is external force is required.

For Problems 27–29

27. (4) It is given $\vec{B} = \frac{(\vec{i} - \vec{k})}{\sqrt{2}}|B|$ and $\left|\frac{dB}{dt}\right| = 10^{-2}$ T/s

It is obvious that B is directed perpendicular to y-axis. Components B_x and B_z of the field B are $B_x = \frac{B}{\sqrt{2}}$ and $B_z = -\frac{B}{\sqrt{2}}$.

Then the flux (at instant t) through $ABC = \phi_1 = \left(\frac{\pi a^2}{2}\right)B_z$



Hence emf e_1 induced in loop ABC is

$$e_1 = -\frac{d\phi_1}{dt} = -\frac{\pi a^2}{2}\cdot\frac{dB_z}{dt} = +\frac{\pi a^2}{2\sqrt{2}}\left(\frac{dB}{dt}\right)$$

This emf will be directed clockwise (looking along negative z-axis) $\overline{ABC}$.

Similarly the flux (at instant t) through $ADC = \phi_2 = \frac{\pi a^2}{2}Bx$.

Hence emf e_2 induced in loop ADC is

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{\pi a^2}{2}\frac{dB_x}{dt} = -\frac{\pi a^2}{2\sqrt{2}}\left(\frac{dB}{dt}\right)$$

This emf will be directed clockwise (looking along positive x-axis) $\overline{CDA}$.

It is easy to see that these two emfs along the two semicircles will add up.

Hence induced emf in the closed loop will be

$$e = |e_1| + |e_2| = \frac{\pi a^2}{2\sqrt{2}}\frac{dB}{dt} + \frac{\pi a^2}{2\sqrt{2}}\frac{dB}{dt}$$

It is given $\frac{dB}{dt} = 10^{-2}$ T/s and $a = 0.5$ m

Hence $e = \frac{\pi(0.5)^2}{\sqrt{2}}\cdot 10^{-2}\text{ V} = \frac{5\pi}{2\sqrt{2}}\times 10^{-3}\text{ V} = \frac{5\pi}{2\sqrt{2}}$ mV

This will be in clockwise sense, when looking along B.

28. (3) It is given that $\vec{B} = \frac{(\vec{i} - \vec{j})}{\sqrt{2}}|B|$

This direction is perpendicular to z-axis.

Hence there will be no flux in the semicircular section ABC.

The induced emf in the other semicircular section ADC will be

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\vec{B}\cdot\vec{A}\right]$$

$$= -\frac{d}{dt}\left[B\cdot\frac{\pi a^2}{2}\cdot\cos\frac{\pi}{4}\right]$$

since the field makes an angel of $\pi/4$ with the yz plane

$$e = \frac{\pi a^2}{2\sqrt{2}}\cdot\frac{dB}{dt} = \frac{\pi a^2}{2\sqrt{2}}\cdot 10^{-2}\text{ V}$$

Putting $a = 0.5$ m, $e = \frac{\pi(0.5)^2}{2\sqrt{2}}10^{-2}\text{ V} = \frac{5\pi}{4\sqrt{2}}$ mV

This will be in the clockwise sense looking along positive x-axis i.e., $\overline{CDA}$.

29. (2) Since the field is directed along the z-axis, there will be no flux through the semicircular section ADC.

The flux through section $ABC = \phi = B.A$ or $\phi = B\cdot\frac{\pi a^2}{2}$ direction along z-axis.

The induced emf $e = -\frac{d\phi}{dt} = -\frac{\pi a^2}{2}\left|\frac{dB}{dt}\right|$

Substituting values, $a = 0.5$, $\frac{dB}{dt} = +10^{-2}$ T/s

$$|e| = \frac{\pi a^2}{2}\left|\frac{dB}{dt}\right| = \frac{\pi(0.5)^2}{2}10^{-2}\text{ V} = \frac{5\pi}{4}\text{ mV}$$

When looking along field direction, (i.e.,) along z-axis, this will be counter clockwise sense, since the field increases with time.

For Problems 30–31

30. (1) **31. (3)**

i. Velocity of wire frame when it starts entering the magnetic field.

$$V_1 = \sqrt{2gh} = \sqrt{2(10)(5)} = 10\text{ m/s}$$

and the time taken is, $t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(5)}{10}} = 1$ s

ii. When the frame has partially entered the field, the induced emf produced is $\varepsilon = Blv$

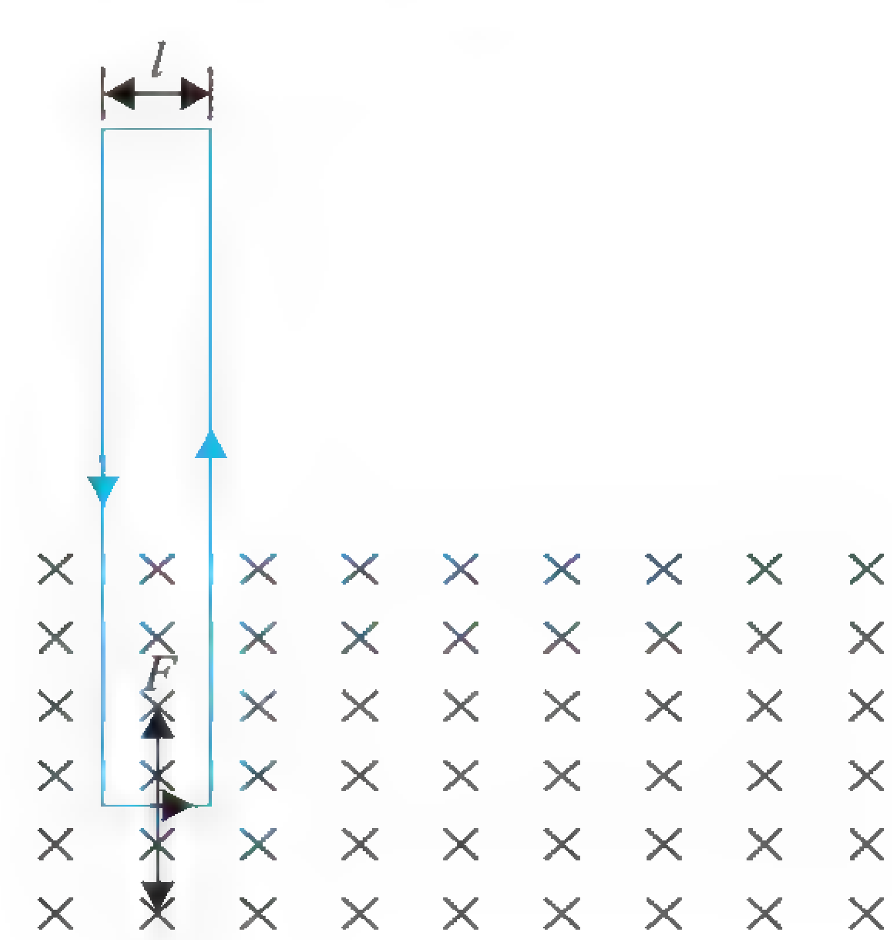
$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}\text{ (anticlockwise)}$$

Ampere's force, $F = \frac{B^2l^2v}{R}$ (upward)

Putting $m = 0.5$ kg, $B = 1$ T, $\ell = 0.25$ m, $v = 10$ m/s, $R = 1/8\ \Omega$

We get, $F = \frac{(1)^2(0.25)^2(10)}{1/8} = 5$ N

Since, $mg = (0.5)(10) = 5$ N



Therefore, using Newton's second law, the acceleration of the wire frame while entering the magnetic field is zero. Thus, time taken to completely enter into the field is $t_2 = \frac{2}{10} = 0.2$ s

iii. When the frame has completely entered the field, the current becomes zero and thus, the ampere's force also become zero. The frame accelerates under gravity only.

$$t_3^2 + 2t_3 - 3 = 0 \quad \text{or,} \quad t_3 = 1 \text{ s}$$

The total time taken is

$$T = t_1 + t_2 + t_3 = 1 + 0.2 + 1 = 2.2 \text{ s}$$

For Problems 32–33

32. (1)

33. (2) Since magnetic field strength B varies with time, therefore an emf is induced in the loop and a current starts flowing through the loop.

Due to flow of current heat is generated.

At an instant t, flux linked with the loop

$$\phi = \pi a^2 B_0 (tT - t^2)$$

Induced emf, $e = -\frac{d\phi}{dt} = -\pi a^2 B_0 (T - 2t)$

Induced current, $i = \frac{e}{R} = \frac{\pi a^2 B_0}{R}(2t - T)$

Thermal power generated at this instant,

$$P = \frac{\pi^2 a^4 B_0^2}{R}(2t - T)^2$$

During an elementary time interval dt, heat generated

$$= Pdt = \frac{\pi^2 a^4 B_0^2}{R}(2t - T)^2 \cdot dt$$

Total heat generated from $t = 0$ to $t = T$.

$$Q = \int Pdt = \frac{\pi^2 a^4 B_0^2}{R} \int_0^T (2t - T)^2 dt$$

$$= \frac{\pi^2 a^4 B_0^2 T^3}{3R}$$

The current reverses its sign when its magnitude reduces to zero. Let this happen at instant $t = t_0$. Substituting t by t_0 in equation (i).

$$\frac{\pi a^2 B_0}{R}(2t_0 - T) = 0$$

$$t_0 = \frac{T}{2}$$

Substituting I by dq/dt in equation (i),

$$dq = \frac{\pi a^2 B_0}{R}(2t - T)dt$$

$\therefore$ Charge that flows from $t = 0$ to $t = T/2$,

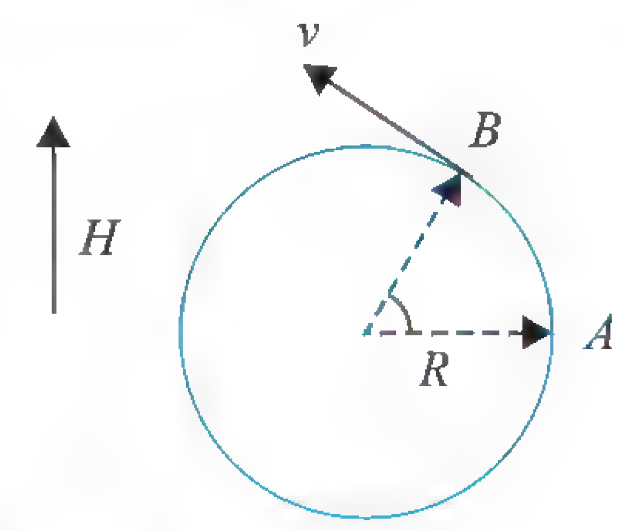
$$q = \frac{\pi a^2 B_0}{R} \int_0^{T/2} (2t - T)dt = -\frac{\pi a^2 B_0 T^2}{4R}$$

or magnitude of charge that flows $= \frac{\pi a^2 B_0 T^2}{4R}$

For Problems 34–35

34. (3)

35. (2) Path of the car is a circle of radius R as shown in figure.



Let at instant $t = 0$, the car be at A when the axis of ring was parallel to horizontal component H of earth's magnetism.

Since, car completes n revolutions per minute, therefore, angular velocity of car is $\omega = \left(\frac{2\pi n}{60}\right)$

Hence, at time t, its position B is given by,

$$\theta = \omega t = \frac{2\pi nt}{60} = \frac{\pi nt}{30}$$

Axis of the ring has also turned through the same angle θ. Therefore, component of H along ring axis is now equal to $H \cos \theta$.

Hence, flux linked with the ring at this instant is,

$$\phi = \pi a^2 (H \cos \theta) = \pi a^2 H \cos\left(\frac{\pi nt}{30}\right)$$

Induced emf,

$$e = -\frac{d\phi}{dt} = \frac{\pi^2 a^2 nH}{30} \sin\left(\frac{\pi nt}{30}\right)$$

Resistance of the ring $= 2\,\pi al$.

Due to induced emf, a current starts to flowing through the ring and due to this current heat is produced in the ring. At time t, thermal power,

$$P = \left(\frac{e^2}{\text{Resistance}}\right)$$

Period of one complete revolution of the car is,

$$T = \frac{60}{n} \text{ sec}$$

Average power,

$$P_{\text{avg}} = \frac{\int_0^T p\,dt}{T} = \frac{\pi^3 a^3 H^2 n^2}{3600\,\lambda} \text{ Js}^{-1}$$

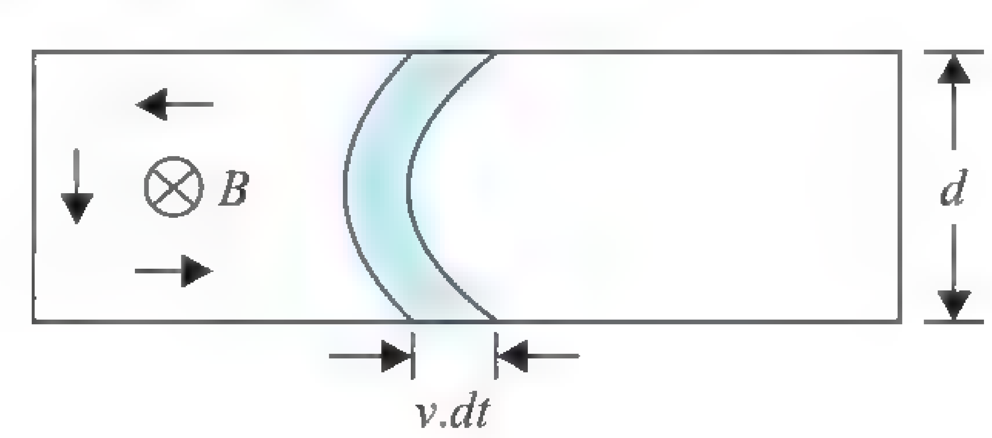
For Problems 36–37

36. (4)

37. (4) When ring moves to the right, emf is induced in each of the two semi-circles. During an elemental time interval dt, displacement of the ring is $v\,dt$.

Consider two semi-circular separately.

During this interval, left semi-circle cuts flux area shown as shaded part in figure. This area is $d\,(v\,dt)$. Therefore, flux cut by semi-circle during this interval is,

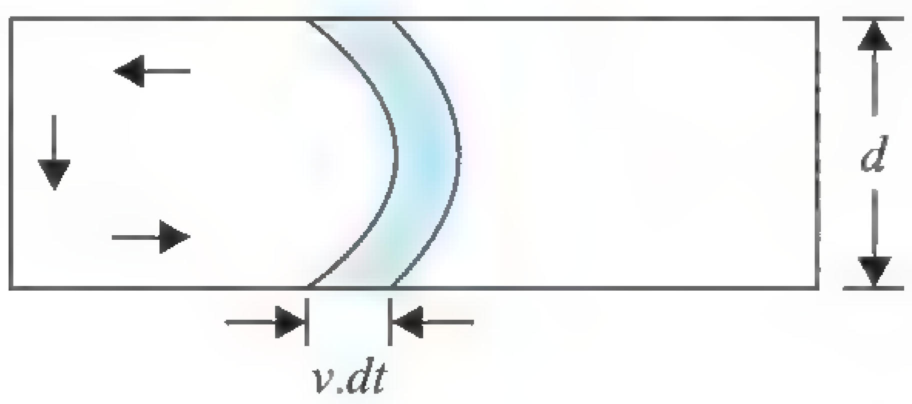


$$d\phi = Bd(vdt)$$

Hence, emf induced in it is $e = \frac{d\phi}{dt} = Bvd$

This emf tries to force an anticlockwise current in the circuit as shown in figure.

Similarly, emf induced in right semi- circle is also equal to $e = Bvd$ and it also tries to force current in the same direction as shown in figure.



Hence, these two semi-circles are two identical electrical sources connected in parallel with each other. EMF of each source is $e = Bvd$ and internal resistance is, $r = (\pi d/2)\lambda$

$\therefore$ Equivalent internal resistance of parallel combination of two sources is,

$$\frac{r}{2} = \frac{1}{4}\pi d\lambda$$

Since, rails have negligible resistance, therefore, equivalent resistance of the circuit is,

$$R = \frac{r}{2} = \frac{1}{4}\pi\lambda d$$

Hence, induced current through rails is

$$I = \frac{e}{R}$$

But current through each semi-circle is equal to $I/2$.

Force required to maintain velocity of ring at constant = Retarding force acting on ring due to induced current = 2 × Retarding force on each semi-circle.

$$= 2 \times B\frac{1}{2}d = Bld = \frac{4B^2vd}{\pi\lambda}$$

For Problems 38–41

38. (4) **39. (1)** **40. (3)** **41. (1)**

Given, $\vec{A} = \pi a^2 \hat{j}$ and $B = (\hat{i} + t^2 \hat{j})$

Associated flux is $\phi = \vec{B}.\vec{A} = \pi a^2 t^2$

Differentiating, $e = -2\pi a^2 t$

Thus, induced electric field is

$$E = \frac{e}{2\pi a} = at$$

If induced current is I, Torque due to its weight is πga

Torque due to applied field is $\vec{\tau} = I\vec{A} \times \vec{B} = -\pi a^2 I\hat{k}$

where, $I = e/R$

Ring just begins to topple when torque due to field becomes numerically equal to torque due to weight of ring.

$\pi a^2 \cdot \frac{2\pi a^2 t}{R} = \pi ga$. Putting $a = 1$m and $R = 2\ \Omega$, we get $t = \frac{10}{\pi}$ s

Further, heat produced in time t is

$$H = \int \frac{e^2}{R}dt = \frac{4\pi^2 a^4}{R} \cdot \frac{t^3}{3}$$

Substituting the value of time,

$$H = \frac{2000}{3\pi} \text{ J}$$

Induced electric field at the circumference of ring at the instant ring start toppling is $E = at = \frac{10}{\pi}$

Matrix Match Type

1. i. → b., d.; ii. → a., c.; iii. → a., c.; iv. → b., d.

i. If current is increased, flux in the loop will increase in inside direction, then due to Lenz's law induced emf in the loop will be in anticlockwise direction. Due to this current, the current in the nearer side of loop to the wire will be in opposite direction to that of wire. Hence, there will be repulsion.

ii. This situation is opposite to part (i)

iii. If loop is moved away, then flux decreases and this becomes similar to part (ii)

iv. Similar to part (i)

2. i. → b., d.; ii. → b., c.; iii. → a., d.; iv. → a., c.

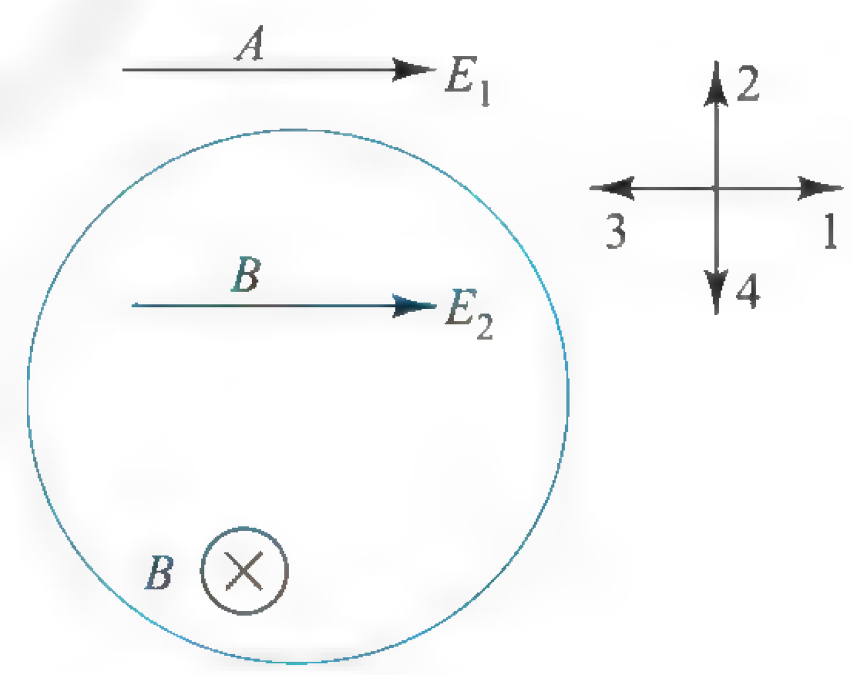
Since field is decreasing, so induced electric field at both points A and B will be in clockwise direction or towards 1. Hence, force on an electron will be along 3 at both points A and B.

For A: $E2\pi r = \pi a^2 \frac{dB}{dt}$

$$E \propto \frac{1}{r}$$

For B: $E2\pi r = \pi r^2 \frac{dB}{dt}$

$$E \propto r$$



3. i. → b., d.; ii. → a., d.; iii. → c.; iv. → c.

i. If loop is moved away, then flux through loop decreases in –ve z direction. To increase this decreasing flux, current induced should be in clockwise direction.

Now, induced current in AB will be parallel to I, so that there will be net attraction between wire and loop.

Hence **(i) → (b, d)**

ii. Explain in the similar way as in (i).

In cases (iii) and (iv), just after the rotation is started, velocity of each element of the loop will be parallel or antiparallel to the magnetic field produced by I. Hence, no emf is induced.

So **(iii) → (c), (iv) → (c)**

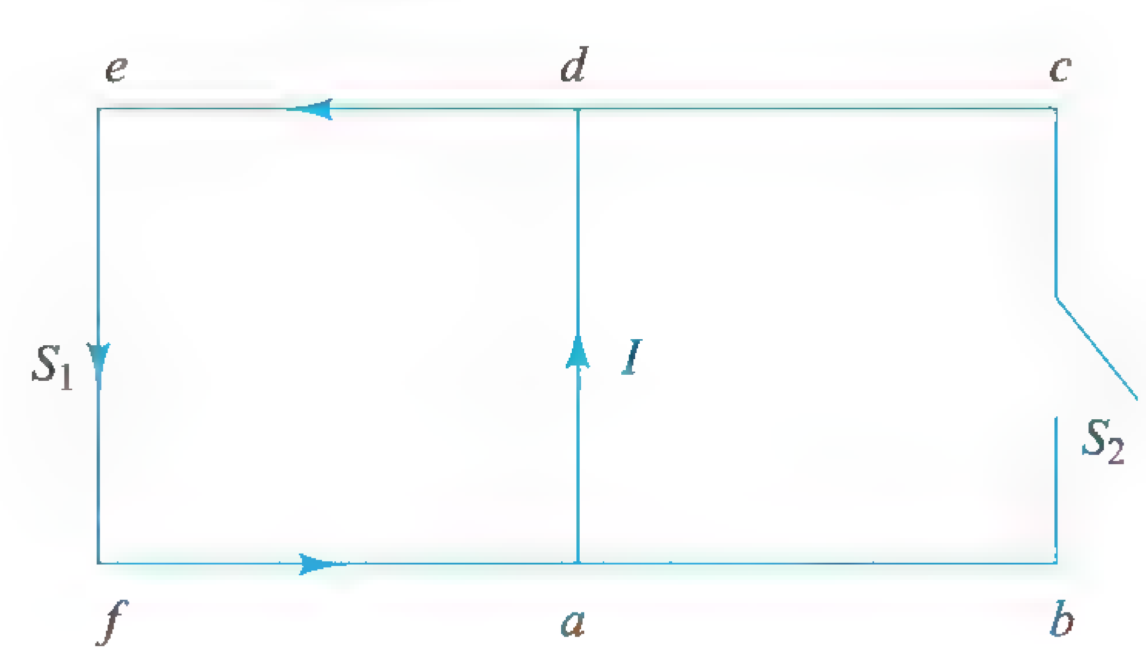
4. i. → b.; ii. → a.; iii. → d.; iv. → d.

i. $\frac{dB}{dt} = 10 \times 10^{-3}$ T/s

$$A = 2^2 = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

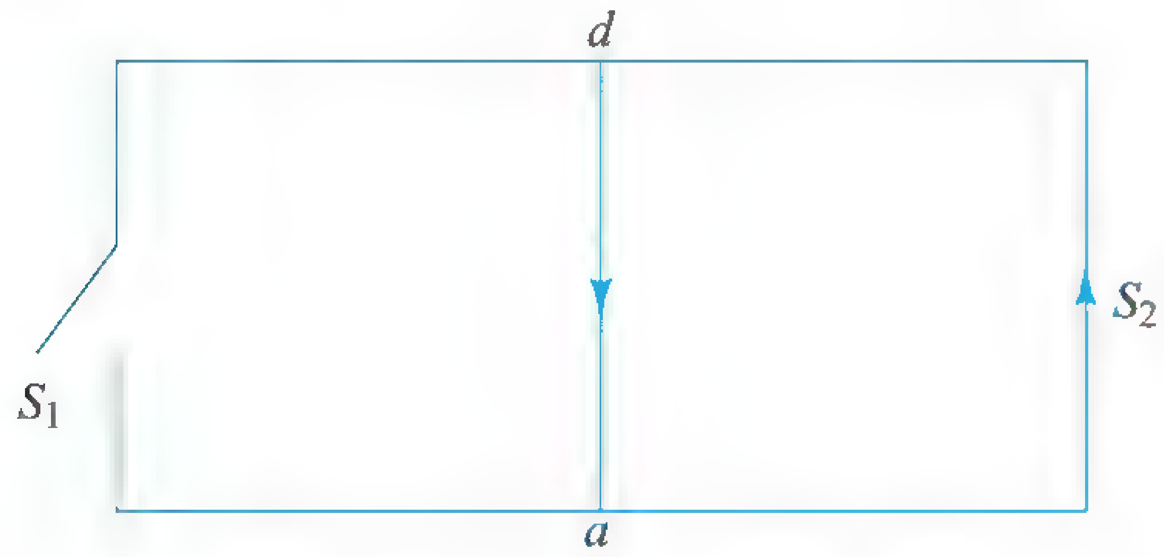
$$e = \frac{d\phi}{dt} = A\frac{dB}{dt} = 4 \times 10^{-4} \times 10 \times 10^{-3} = 4 \times 10^{-6} \text{ V}$$

$$I = \frac{e}{R} = \frac{4 \times 10^{-6}}{2 \times 4} = 5 \times 10^{-7} \text{ A}$$



The emf will be in anticlockwise direction, so current will be from a to d.

ii. Again, current will be in anticlockwise direction.



This makes the direction of current from d to a, magnitude same as that in part (i).

iii. If both are open, induced emf will develop, but no current will flow.

iv. If both are closed, then induced emf in the left part will tend to flow will from a to d and in right part, the current will tend to flow from d to a. So, from the principle of superposition, no current will flow in ad.

5. i. → c., d.; ii. → c., d.; iii. → b., d.; iv. → a., d.

i. $e_{OA} = \frac{1}{2} B\omega(OA)^2 = \frac{1}{2} B\omega(\sqrt{2}\,L)^2 = B\omega L^2$

ii. $e_{OD} = \frac{1}{2} B\omega(OD)^2 = \frac{1}{2} B\omega(\sqrt{2}\,L)^2 = B\omega L^2$

iii. $e_{OC} = \frac{1}{2} B\omega L^2$ or $e_{CD} = E_{OD} - E_{OC} = \frac{1}{2} B\omega L^2$

iv. $e_A - e_O = B\omega L^2$

$e_D - e_O = B\omega L^2$

$e_A - e_D = 0$

6. i. → b.; ii. → c.; iii. → a.; iv. → b.

i. At t = 1s, flux is increasing in the inward direction, hence induced emf will be in anticlockwise direction.

ii. At $t = 5$ s, there is no change in flux, so induced emf is zero.

iii. At $t = 9$ s, flux is increasing in upward direction, hence induced emf will be in clockwise direction.

iv. At t = 15 s, flux is decreasing in upward direction, so induced emf will be in anticlockwise direction.

7. i. → c.; ii. → a., b.; iii. → d.; iv. → c.

We know that $e = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$. If we take area vector in the upward direction, then anticlockwise direction will be positive. From 0 to t_1 and t_5 to t_6, dB/dt is +ve. Hence induced emf e is –ve. So, induced current will be in clockwise direction. From t_2 to t_4, dB/dt is –ve. Hence induced emf e is +ve. So, induced current will be in anticlockwise direction. From t_1 to t_2 and t_4 to t_5, dB/dt is zero. Hence, no emf is induced. Induced emf or current is maximum from 0 to t_1 and t_5 to t_6, because here magnitude of dB/dt is maximum.

8. i. → a., c.; ii. → a., c.; iii. → b., d.; iv. → a., b., c., d.

a. Speed of the charged particle cannot be changed by magnetic force because magnetic force does no work on charged particle. Only electric field in case (a) and induced electric field in case (c) can change speed of the charged particle.

b. Magnetic field cannot exert force on the charged particle at rest. Only electric field in case (a) and induced electric field in case (c) can exert force on charge initially at rest. In case (c) after the charged particle starts moving, the magnetic field can exert force on the charge.

c. A charged particle can move on a circle with a uniform speed due to uniform and constant magnetic field. Even within a region of non-uniform magnetic field, at all points on the circle, the field may be uniform, for example, on any circle concentric with a current-carying ring.

d. A moving charged particle is accelerated by electric field and also accelerated by magnetic field (provided v is not parallel to B).

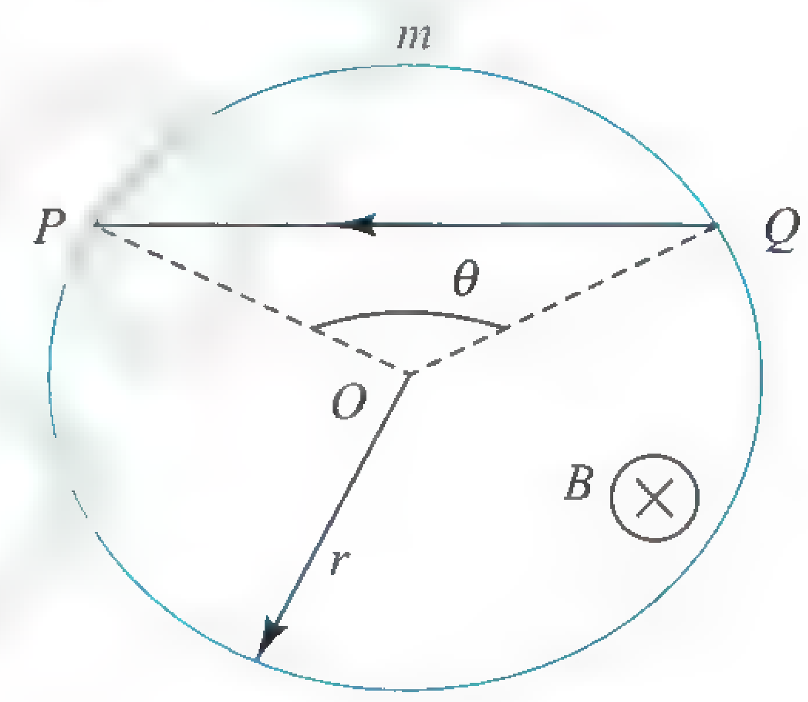
9. i. → b., c.; ii. → a., d.; iii. → a., c.; iv. → b., d.

i. Area $OPMQO = \frac{1}{2} r^2\theta$

Flux in this area, $\phi_1 = \frac{1}{2} r^2\theta B$

Induced emf in this area,

$$e_1 = \frac{d\phi_1}{dt} = \frac{1}{2} r^2\theta \frac{dB}{dt}$$



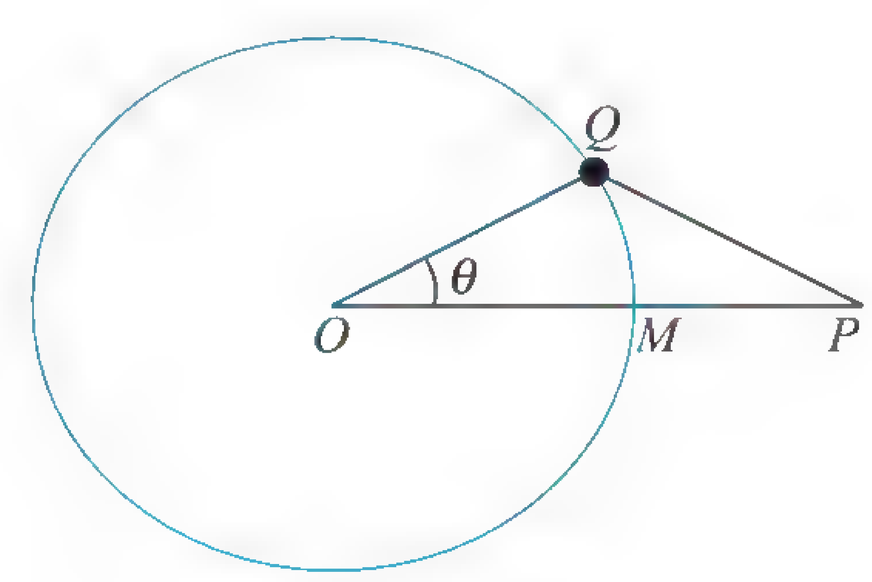
Area $OPQ = r\sin\left(\frac{\theta}{2}\right) r\cos\left(\frac{\theta}{2}\right) = \frac{1}{2} r^2 \sin\theta$

Induced emf in OPQ,

$$e_2 = \frac{d\phi_2}{dt} = \frac{1}{2} r^2 \sin\theta \frac{dB}{dt}$$

e_2 will be only in part PQ, because in OQ and OP induced emf will be zero. Clearly, $e_2 < e_1$, because area OPQ < area $OPMQO$. Since B is increasing, so e.m.f. will be in anticlockwise direction. Hence end P will be positive w.r.t. Q.

ii. Here emf in OPQ will be due to flux changing in area OMQ. This area is $\frac{1}{2} r^2\theta$. The entire emf will be in part PQ. End Q will be positive.



iii. Induced emf $= \frac{1}{2} r^2\theta \frac{dB}{dt}$. End P will be positive.

iv. Area in which flux is changing is lesss than $\frac{1}{2} r^2\theta$. End Q will be positive.

10. i. → c.; ii. → d.; iii. → a.; iv. → b.

Magnetic field is along x-axis because when the cube is moved along x-axis, there is no motional emf as $\vec{v} \times \vec{B} = 0$. When the block is moved along y-axis, force on the electrons is in direction $-(\hat{j} \times \hat{i}) = \hat{k}$

Therefore, electric field will be created along z-axis.

Now, $c\,v\,B = 24$ mV

$\Rightarrow c = 20$ cm

Similarly, $bvB = 36$ mV

$\Rightarrow\ b = 30$ cm

$\therefore\ a = 25$ cm

11. i. → a., d.; ii. → a., d.; iii. → d.; iv. → d.

i. Switch is closed, due to the heating effect, R increases as temperature increases, so current decreases and hence flux linked with B is decreasing with time, so, the induced current in R_1 will try to strengthen the original flux. Direction can be found by using right hand palm rules.

ii. As switch is opened, current in R decreases from finite to zero value, and hence flux also decreases (same as for a).

iii. Due to motion of A, flux linked with B is increasing and due to heating effect flux linked with B is decreasing. So, net effect on flux cannot be determined precisely, so current in R_1 can be in any direction and even it may be zero if effects have been all cancelled.

iv. The varying emf can be increasing or decreasing (explanation somewhat similar to r).

12. i. → c.; ii. → b.; iii. → a.; iv. → b.

Using Faraday's law, whenever there is a change in flux linked with coil, emf is induced the coil.

For $0 < x < b$

Flux linked with coil

$$e = -\frac{d\phi}{dt} = -\frac{dBIlx}{dt} - BIl\frac{dx}{dt}$$

$$e = -BIlv$$

When $b < x \le 2b$, there is no change in flux, hence no *emf* is induced

When $2b < x < 3b$ there is a decrease in flux hence $e = BIl\,v$

When $x > 3b$, again flux linked with coil is zero, hence no emf is induced.

13. i. → a., b.; ii. → a., b.; iii. → c.; iv. → a., d.

i. $\dfrac{V - Blv}{R} = i$, $F = \dfrac{V - Blv}{R} = Bl$

Here magnetic force is to the right side. Actually rod will first accelerate towards right for some time and then finally will attain constant velocity. Constant velocity will be attained when F becomes zero and that constant velocity will be $v = V/Bl$.

If we apply another force to the right then rod cannot move with constant velocity rather its acceleration will be more.

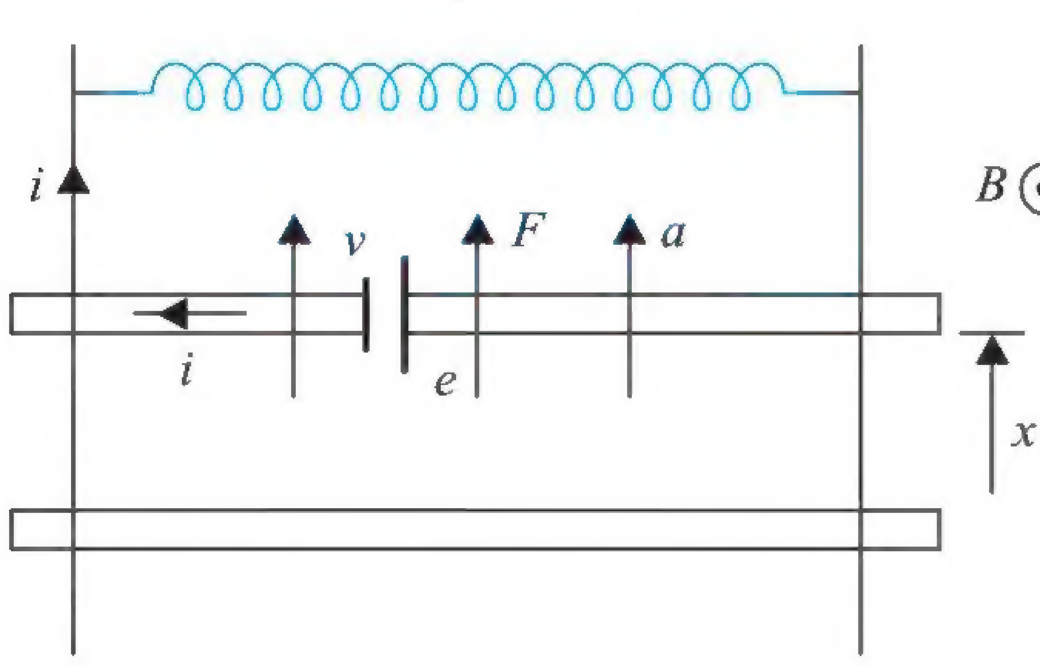
ii. Let at any time charge on the capacitor is Q and current in the circuit is i, then

$$F = ma \quad \Rightarrow \quad i\,l\,B = ma \quad \frac{lBdQ}{dt} = \frac{mdv}{dt}$$

$$\Rightarrow \quad \int_0^v mdv = -\int_{Q_0}^{Q} BldQ \Rightarrow v = \frac{Bl(Q_0 - Q)}{m}$$

After the whole charge of capacitor discharges, the rod achieves constant velocity.

iii. No energy dissipated due to no resistance. Let at on time current in the circuit is i.



$$F = Bil \Rightarrow ma \Rightarrow Bil \Rightarrow i = \frac{ma}{Bl}$$

$$e = -L\frac{di}{dt} \Rightarrow Blv = -L\frac{di}{dt}$$

$$\Rightarrow \quad Bl\frac{dx}{dt} = -L\frac{di}{dt} \Rightarrow Bl\int_0^x dx = -L\int_{i_0}^{i} di$$

$$\Rightarrow \quad Blx = -L(i - i_0)$$

Simplify to get $a = \dfrac{B^2 l}{ml}\left[x - \dfrac{Li_0}{Bl}\right]$

This represents SHM. So rod executes SHM.

iv. Induced emf will be developed which will oppose the motion of rod by applying a force to the left. And after some time, the rod will come to rest. But if a constant force to the right is applied immediately, then it can move with constant velocity provided the applied force is same as that of initial force developed due to motion.

Numerical Value Type

1. (2) $\Delta\phi = R(\Delta q) = R\int idt$

$= R$ [area under i–t graph]

$= \dfrac{1}{2}(4)(0.1)(10) = 2$ Wb

2. (5) The total flux through N turns of the coil,

$$\phi_{\text{total}} = NBA\cos\theta$$

According to Faraday's law of electromagnetic induction,

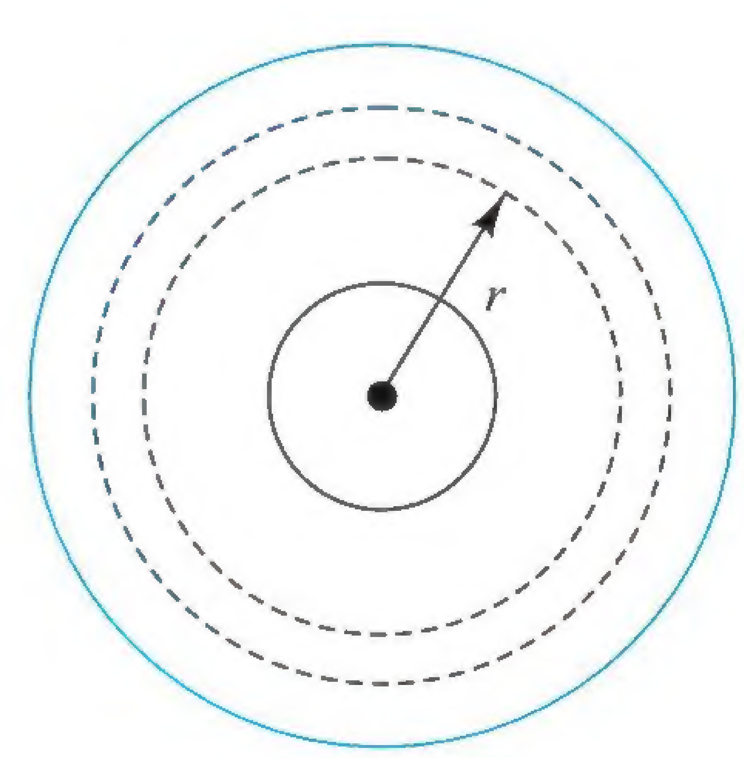
$$E_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\theta)$$

$$= -(NA\cos\theta)\frac{dB}{dt}$$

The current induced in the coil,

$$I_{\text{induced}} = \frac{E_{\text{induced}}}{R} = 5\text{ A}$$

3. (7)



The magnetic field inside is only due to the current of the inner cylinder.

$$B = \frac{\mu_0 i}{2\pi r}$$

Magnetic field energy density is not uniform in the space between the cylinders. At a distance r from the centre,

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

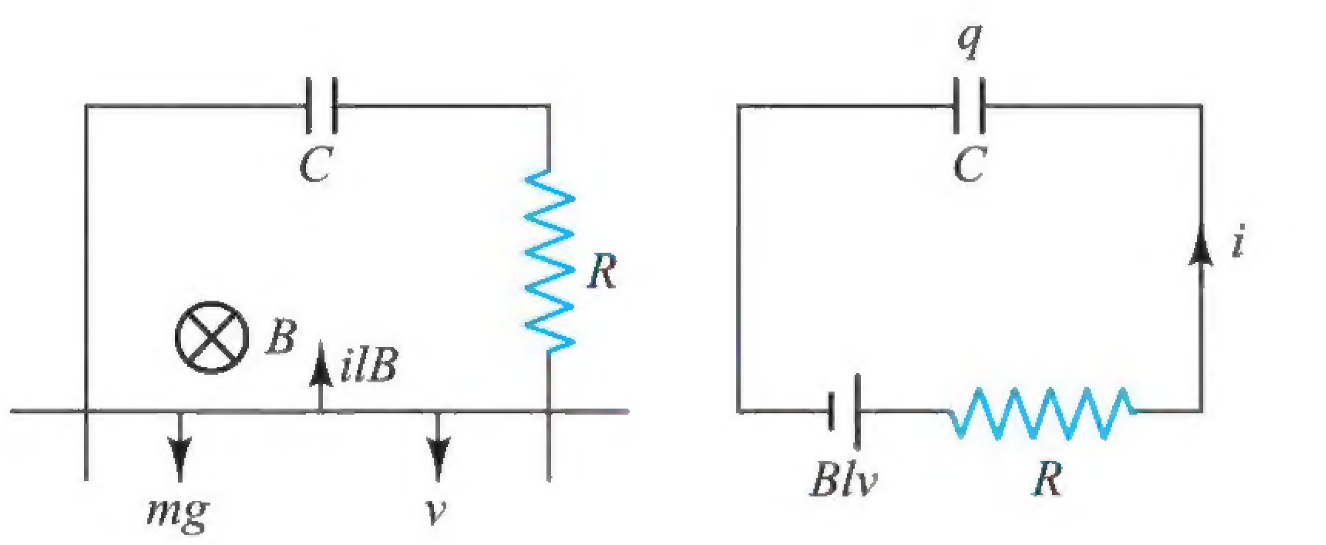
Energy in volume of element (length ℓ)

$$dU_B = u_B\, dV = \frac{\mu_0 i^2}{8\pi^2 r^2}\,(2\pi r\ell)dr = \frac{\mu_0 i^2 \ell}{4\pi}\frac{dr}{r}$$

$$U_B = \frac{\mu_0 i^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 \ell}{4\pi}\ell n\frac{b}{a}$$

Using values, we get $U = 7$ nJ

4. (5)



By Newton's law, $mg - ilB = m\dfrac{dv}{dt}$ ...(i)

Using KVL $Blv = iR + \dfrac{q}{C}$ ...(ii)

Differentiating equation (ii) w.r.t. time, we get

$$Bl\frac{dv}{dt} = R\frac{di}{dt} + \frac{i}{C} \quad \text{...(iii)}$$

Eliminating $\dfrac{dv}{dt}$ from equations (i) and (iii), we get

$$mg - ilB = \frac{m}{Bl}\left[R\frac{di}{dt} + \frac{i}{C}\right]$$

$$\Rightarrow \quad mg\,Bl - iB^2l^2 = m\left(R\frac{di}{dt} + \frac{mi}{C}\right) \quad \text{...(iv)}$$

I will be maximum when $\dfrac{di}{dt} = 0$. Use this in equation (iv)

$$\Rightarrow \quad mg\,B\ell C = i(B^2\ell^2 C + m)$$

$$\Rightarrow \quad i_{\max} = \frac{mgB\ell C}{m + B^2\ell^2 C}$$

5. (8) After long time, from conservation of momentum.

$$mv_0 = 2mv;\; v = \frac{v_0}{2} = 8 \text{ m s}^{-1}.$$

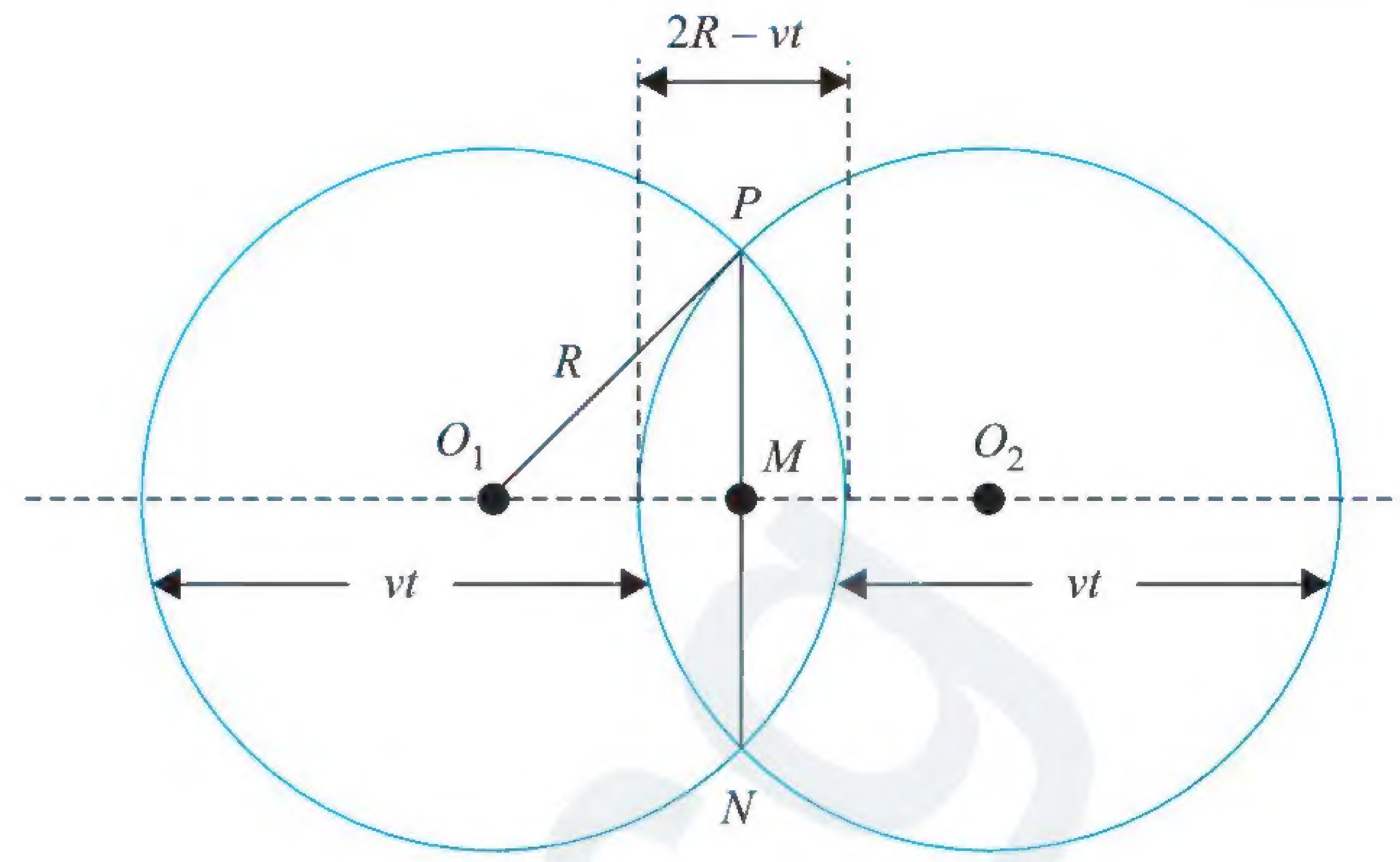
6. (7) Induced emf should be equal to 10 V

$$e = A\frac{dB}{dt} \Rightarrow 10 = \left(\frac{10}{100}\right)^2 \frac{7}{\Delta t} \Rightarrow \Delta t = 7 \text{ ms}$$

7. (1) The rate of electrical energy consumed in the bulb = rate of loss of gravitational PE of the mass = Mgv = 100 W. Hence

$$M = \frac{100}{10\times 10} = 1 \text{ kg}.$$

8. (8) $O_1M = (vt - R) + \left(\dfrac{2R - vt}{2}\right) = \dfrac{vt}{2}$



$$\ell = PN = 2\sqrt{R^2 - (O_1M)^2} = 2\sqrt{R^2 - \left(\frac{vt}{2}\right)^2} = \sqrt{4R^2 - v^2t^2}$$

so $\quad E = Bvl = Bv\sqrt{4R^2 - v^2t^2}$

given $R = 5$ m, $v = 2$ m/s, $t = 3$ s

$$E = 0.5\times 2\sqrt{4\times 25 - 4\times 9} = 8 \text{ V}$$

9. (5) $a = \dfrac{ilB}{m}$ and $v = \left(\dfrac{ilB}{m}\right)t = 5$ cm/s

10. (4) It is clear that the diameter AC traces out a circle as the given wire rotates about axis XY. The emf developed by induction between the ends A and C of the given semicircular wire is the same as if a straight wire AC were rotating perpendicular to field B.

Hence flux linked $= \phi_B = B\left(\dfrac{1}{2}(AC)^2\theta\right)$

where θ is the angle traced in time t.

By Faraday's law, the induced emf $e = \oint \vec{E}\cdot d\vec{l}$

But we also have, $\oint E\cdot dl = e = -\dfrac{d\phi_B}{dt}$

$$\oint E\cdot dr = -\frac{d}{dt}\left(\frac{1}{2}(d)^2\theta\right)B$$

$$= -\frac{1}{2}d^2B\left(\frac{d\theta}{dt}\right) = -\frac{B}{2}\omega d^2$$

But we desire only the $\displaystyle\int_C^A E\cdot dr$ which will be $\dfrac{1}{2}$ or $\oint E\cdot dr$.

Hence $\displaystyle\int_C^A E\cdot dr = -\frac{B}{4}\omega d^2$

11. (8) Dynamic emf

$$e_d = -\vec{l}.(\vec{v}\times\vec{B}) = \vec{l}.(\vec{B}\times\vec{v})$$

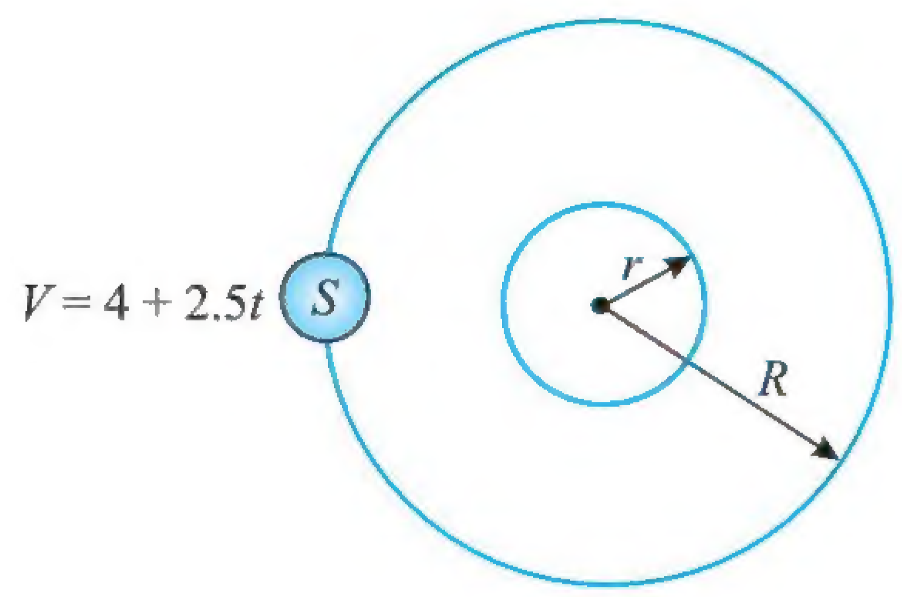
$$(\vec{B}\times\vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(32-0) - \hat{j}(16-0) + \hat{k}(12-16)$$

$$= 32\hat{i} - 16\hat{j} - 4\hat{k}$$

$$e_d = (2\hat{k})\cdot(32\hat{i} - 16\hat{j} - 4\hat{k}) = -8 \text{ volt}$$

12. (1.25)

The magnetic field at the centre O due to the current in the larger loop is $B=\dfrac{\mu_0 I}{2R}$.



If r is the resistance per unit length, then

$$I=\frac{\text{potential difference}}{\text{resistance}}=\frac{4+2.5t}{2\pi R\cdot\rho}$$

$$\therefore \quad B=\frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}$$

$\therefore$ $r << R$, so the field B can be taken almost constant over the entire area of the smaller loop.

$\therefore$ the flux linked with the smaller loop is

$$\phi=B\times\pi r^2=\frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}\cdot\pi r^2$$

Induced emf $e=\dfrac{d\phi}{dt}=\dfrac{\mu_0 r^2}{4R^2\rho}\times 2.5$

The corresponding current in the smaller loop is I' then

$$I'=\frac{e}{R}=\frac{\mu_0 r^2}{4R^2\rho}\times 2.5\times\frac{1}{2\pi r\rho}$$

$$=\frac{2.5\,\mu_0 r}{8\pi R^2\rho^2}=\frac{2.5\times 4\pi\times 10^{-7}\times 0.1}{8\pi\times(1)^2\times(10^{-4})^2}=1.25\,\text{A}$$

13. (1) The magnetic field is decreasing with time. Using Faraday's law(or Lenz's law) one can see that the emf induced in a closed path will be in clockwise sense. It means that the induced electric field is clockwise. Therefore, end A will reach point C.

Magnitude of induced electric field is

$$E=\frac{l}{2}\left|\frac{dB}{dt}\right|=\frac{l}{2}\frac{B_0}{2}=\frac{B_0 l}{4}=\frac{B_0\times 0.04}{4}=1\text{ V/m}$$

$$\tau=2Eq\times l$$

$$\therefore \quad \alpha=\frac{2Eql}{I}=\frac{2Eql}{2\times ml^2}=\frac{Eq}{ml}$$

$$\therefore \quad \theta=\frac{1}{2}\alpha t^2$$

$$r^2=\frac{2\theta}{\alpha}=\frac{2\times\frac{\pi}{2}}{\alpha}=\frac{\pi ml}{Eq}$$

$$=\frac{\pi l}{E}\times\frac{1}{q/m}=\frac{\pi\times 0.04}{1}\times\frac{1}{\frac{4\pi}{100}}$$

$\therefore$ $t^2=1$ $\quad\therefore t=1\text{s}$

Archives

JEE Advanced

Single Correct Answer Type

1. **(4)** According to Lenz's law, current will be in anticlock-wise sense as magnetic field is increasing into the plane of paper.
2. **(3)** True for induced electric field and magnetic field.
3. **(2)** If we rotate the magnet about the axis of the disc, it creates a state where the plate moves through the magnetic flux, due to which an electromotive force is generated in the plate and eddy currents are induced. These currents are such that it opposes the relative motion. It means the disc will rotate in the direction of rotation of magnet.

Multiple Correct Answers Type

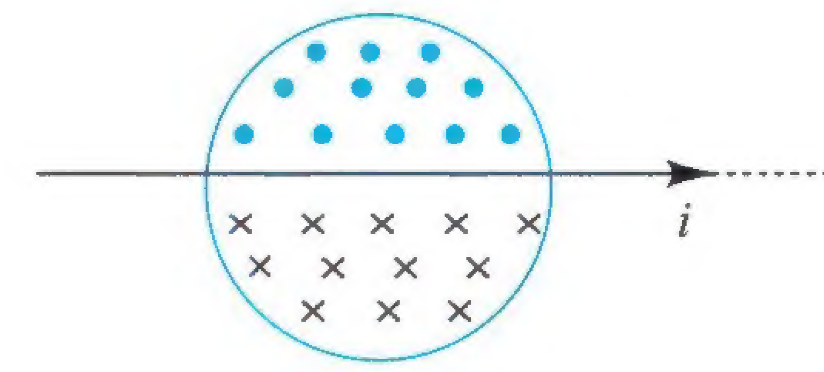
1. (2),(4)

As $\dfrac{d\phi}{dt}$ = emf is the same, the current induced in the ring will depend upon the resistance of the ring. Larger the resistivity, smaller the current.

2. (1),(3)

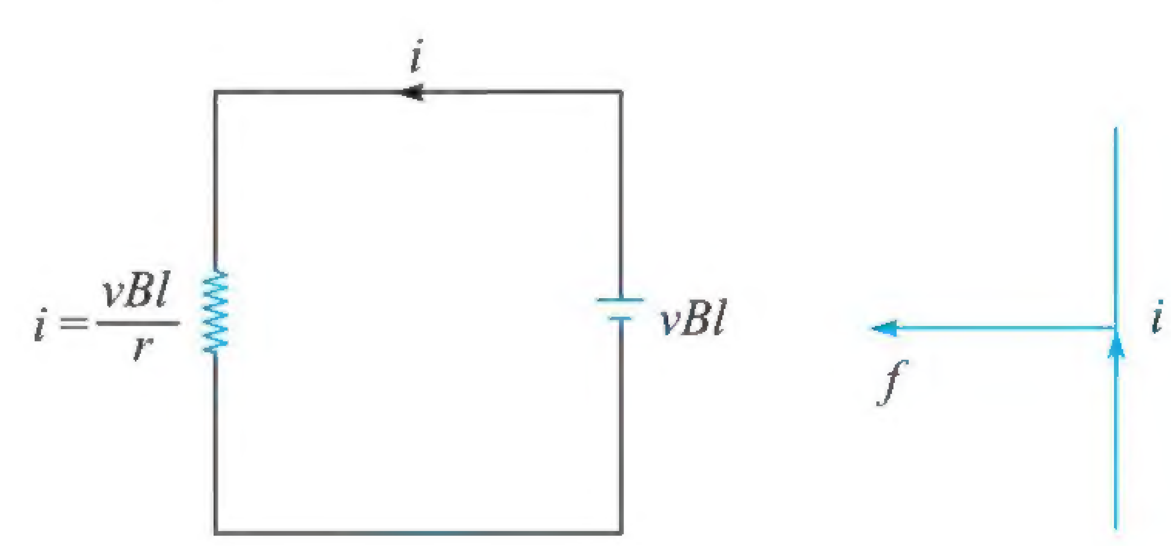
$(\phi)_{\text{loop}}=0$ for all cases

So induced emf = 0



3. (3),(4)

While entering i.e. $x < L$



$$f=i\ell B=\frac{B^2\ell^2 v}{R}$$

$$a_2=\frac{f}{m}=\frac{B^2\ell^2 v}{mR}=kv=-\frac{vdv}{dx}\left[k=\frac{B^2\ell^2}{m}\right]$$

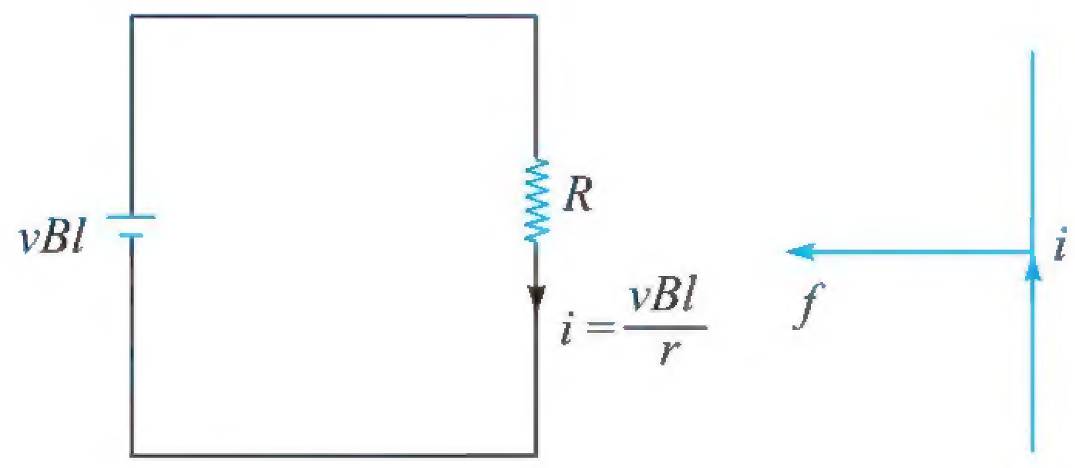
$$\int_{v_0}^{v}dv=-k\int_0^x dx\Rightarrow v=v_0-kx$$

$$f=\frac{B^2\ell^2}{R}(V_0-kx)=\alpha-\beta x$$

$$i=(v_0-kx)\frac{B\ell}{R}=i_0-\gamma x$$

For $3L > x > L$ $f=0$ $i=0$ v = constant.

For $4L > x > 3L$



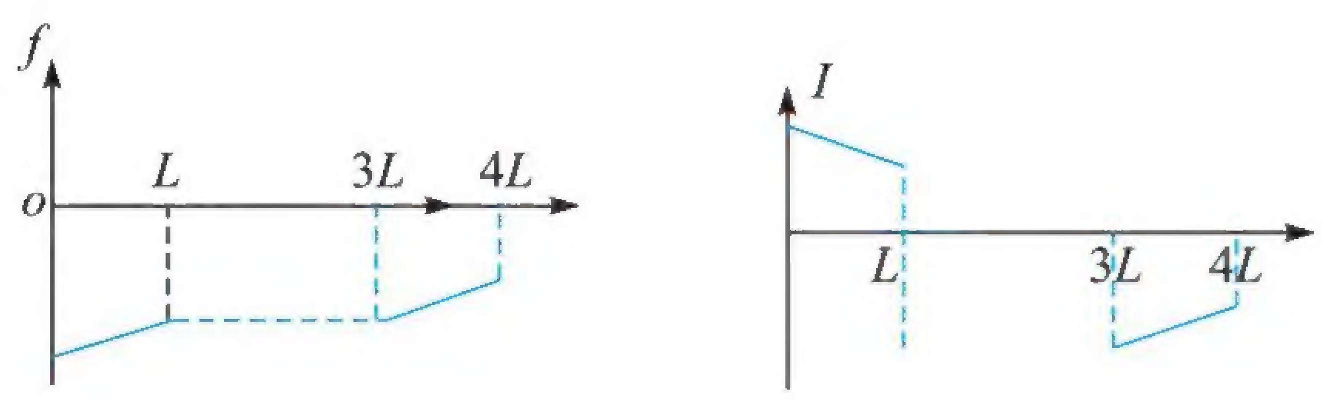
$$f = i\ell B = \frac{B^2\ell^2 v}{R}$$

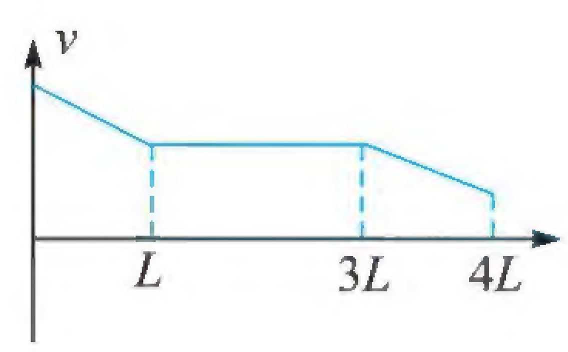
$$a = \frac{B^2\ell^2}{mR}v = kv = -\frac{vdv}{dx}$$

$$v = v'_0 - kx$$

$$f = \alpha' - \beta'.x$$

$$i = i'_0 - \gamma' x$$





4. (1),(2)

The angle that area vector makes with $\vec{B}$ at time t is $\theta = \omega t$.

$$|\phi_1| = BA\cos\omega t \Rightarrow |\varepsilon_1| = BA\omega\sin(\omega t)$$

$$|\phi_2| = 2BA\cos(\omega t) \Rightarrow |\varepsilon_2| = 2BA\omega\sin(\omega t)$$

Due to orientation of loops, the two EMFs will work against each other.

So, $|\varepsilon(\text{net})| = BA\ \omega\sin(\omega t)$

So, (1) is correct.

We can see that ε_{net} is maximum when $\theta = \frac{\pi}{2}$.

So, (2) is correct. Obviouly, (3) is incorrct. (4) is incorrect as the EMF is proportional to difference in areas.

5. (1),(3)

EMF developed across the emf of semi-circular rod =

$$\int_1^4 \frac{\mu_0 i}{2\pi r}drv = \frac{\mu_0 iv}{2\pi}\ln 4 = \frac{\mu_0 iv}{\pi}\ln 2$$

$$E = \frac{4\pi\times10^{-7}\times2\times3\times0.7}{\pi} = 24\times7\times10^{-8}\ \text{V}$$

$$i_{\text{max}} = \frac{E}{R} = \frac{24\times7\times10^{-8}}{1.4} = 1.2\times10^{-6}\ \text{A}$$

$$Q_{\text{max}} = C_0E = 24\times7\times10^{-8}\times5\times10^{-6} = 8.4\times10^{-12}\ \text{C}$$

Linked Comprehension Type

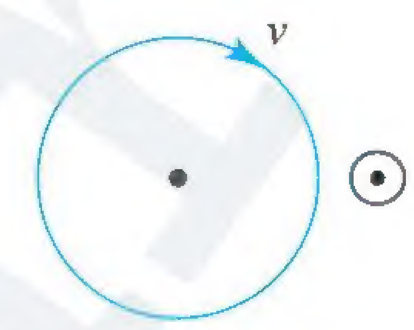
For Problems 1–2

1. (2) $E(2\pi R) = \pi R^2\frac{dB}{dt}$

$$E = \frac{RB}{2}$$

2. (2) $\Delta L = \int \tau dt$

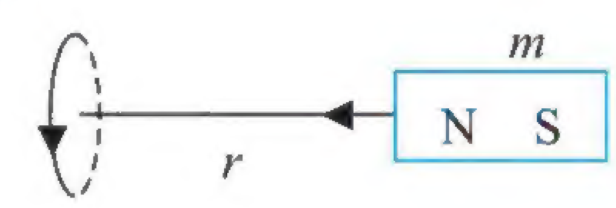
$$= Q\left(\frac{R}{2}B\right)R(1) = \frac{QR^2B}{2}, \text{ in magnitude}$$



$\Delta\mu = \gamma\Delta L = -\gamma\frac{BQR^2}{2}$ (taking in account the direction)

For Problems 3–4

3. (1)



Since it is a super conducting loop, so net flux (self flux + external flux) passing through it will remain constant.

$$(\phi_{\text{total}})_i = (\phi_{\text{total}})_f$$

$L(0) + (0)(pa^2) = Li - \left(\frac{\mu_0}{2\pi}\frac{m}{r^3}\right)$, Here L = self inductance of the super conducting loop.

$$\Rightarrow\quad i = \frac{\mu_0}{2\pi L}\left(\frac{m}{r^3}\right) \Rightarrow i \propto \frac{m}{r^3}$$

4. (3) The current carrying superconducting loop will also behave like a magnet, whose magnetic dipole moment

$$\Rightarrow\quad i = \frac{\mu_0}{2\pi L}\left(\frac{m}{r^3}\right) \Rightarrow i \propto \frac{m}{r^3}$$

$$\Rightarrow\quad m_1 \propto \frac{m}{r^3}$$

The repulsive force felt be the magnet will be

$$F = \frac{Km_1m_2}{r^4} = \frac{(K)\left(\frac{m}{r^3}\right)(m)}{r^4} \propto \frac{m^2}{r^7}$$

$$W_{\text{ext}} = -\int Fdr = -\int\frac{m^2}{r^7}dr$$

$$W_{\text{ext}} \propto \frac{m^2}{r^6}$$